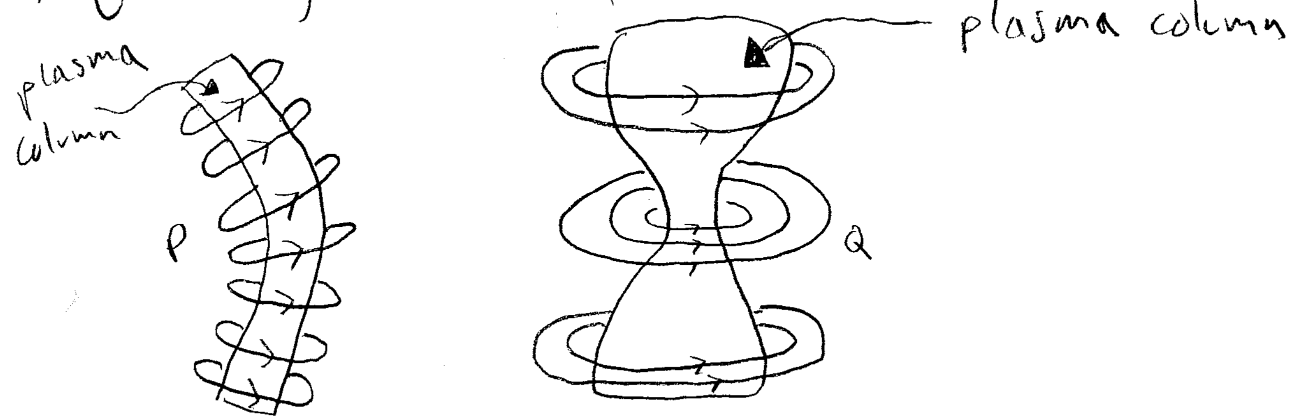
180

Apr 24 cont

Stability of plasma columns

physical considerations allow one to intuit the stability or instability of a plasma column. Detailed calcs required, but consider the perturbations below:



Crowding of B-field lines at point P enhances magnetic pressure there and pushes plasma column so as to enhance the kink $\longrightarrow$ kink grows, system unstable. $\boxed{\text{KINK INSTABILITY}}$

In the second fig., B_θ at Q is larger than its value away from the perturbed pinch. The extra tension pinches further and system is unstable to $\boxed{\text{SAUSAGE INSTABILITY}}$

An Axial field can supress these instabilities: As the kink bends the field column, the axial field tension resists the bending. Similarly, for the sausage case, the magnetic pressure associated with the axial field resists the pinching. $B_{axial} \gtrsim B_\theta$ is required to stabilize the instabilities.

Fusion & plasma confinement

Dominant fusion reaction desired is
2 deuterium atoms → tritium or Helium + energy

Coulomb forces repulse the deuterium atoms so the must have high enough relative velocity to penetrate coulomb barrier to fuse. This requires hot plasma

→ But high temp deuterium ($> 10^7$ K) canot be easily confined. It would burn container walls if too dense. And, if too diffuse, it would quickly lose heat content with the wall. ⇒ hope is to confine plasma with magnetic fields. Push for fusion devices was initiated by US, UK, USSR after WW II. ⟶

Expectation was that commercial production possible in a few years but its 60 years later and still a long way off.

Confining was more difficult than expected and a lot of energy is always lost in heating & setting up the configurations.

In order to get sufficient energy out of fusion, plasma must be confined for a time such that the product of the number density and confinement time τ, satisfies

$n\tau > 10^{16}$ sec cm^{-3}. (called Lawson criterion).

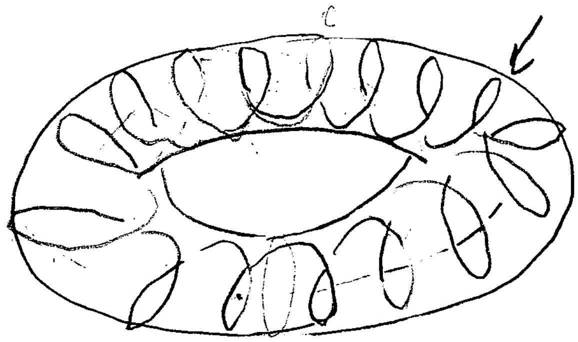
Typical magnetic devices have $n\tau \sim \left(\frac{10^{15}}{cm^3}\, 0.1\,sec\right) \sim \frac{10^{14}}{cm^3}\,sec$

too small by about 2-orders of magnitude.

In the laser lab, the idea is to make n very large even though τ is short. $\tau \sim 10^{-10}$ sec so n has to be 10^{26} cm^{-3}, but such high densities are not yet reached. The system falls short, in part due to the Rayleigh-Taylor Instability.

Magnetic confinement still seems like best hope. Devices are typically toroidal plasma columns, which avoid edges by allowing plasma to close on itself.

Key reason for the difficulty in confinement is plasma instabilities, which induce plasma in the core region to diffuse prematurely to the walls.

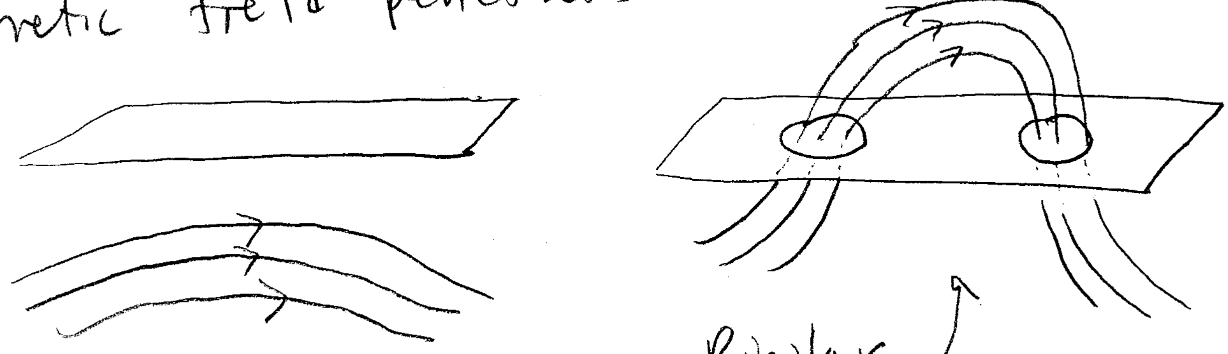


torus confinement
e.g. TOKOMAK

<u>core</u> is region inside the pinch where plasma is confined between the inner and outer walls.

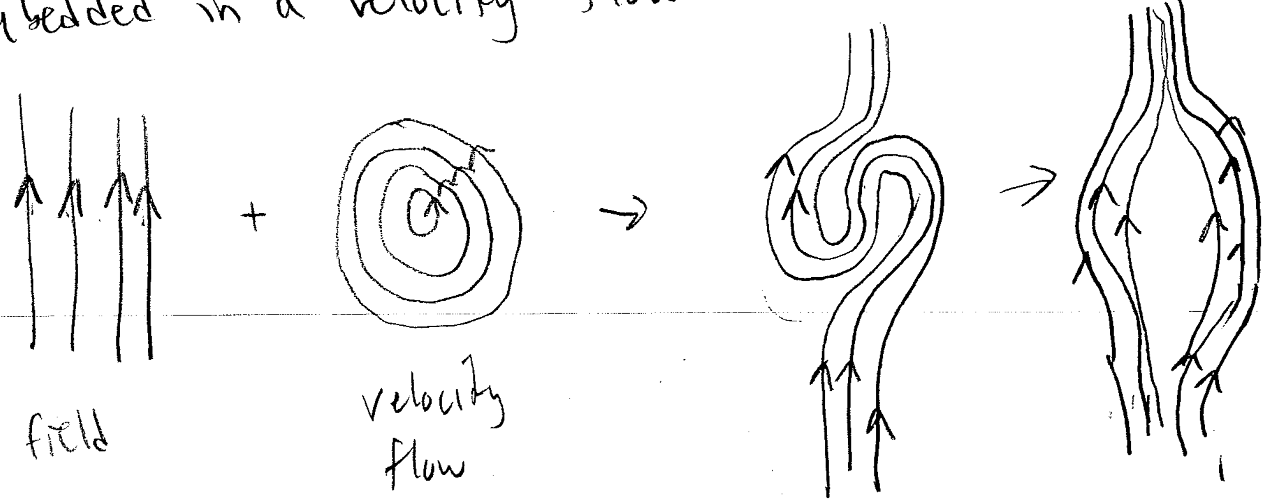
Bipolar magnetic regions and buoyancy

Hale (1908) realized sunspots were associated with magnetic fields, and in 1919 noticed that often two large sunspots appear side by side with opposite polarities. The obvious explanation is that the dual spots represent places where magnetic field penetrates the solar surface:

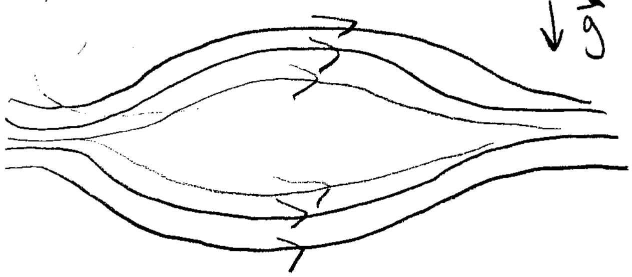


(a) (b)

Inside convection zone, field is pushed toward boundaries of convection cells for example, consider a region of field embedded in a velocity flow:



tilt the last figure, and imagine it is imbedded in sun:



$\downarrow \vec{g}$ $\uparrow \hat{r}$

we can see that we have segregated flux tubes.

the top part can represent fig ⓐ on the previous page. Now, why should such a structure become buoyant, and rise through solar surface to corona?

Consider a horizontal flux tube with gas pressure p_i inside the tube and let p_e be the external pressure →

equation of motion without velocity field but with gravity, pressure, and $\vec{B}$ is

$$\frac{1}{4\pi}(\vec{B}_0\cdot\vec{\nabla})\vec{B} = \nabla\left(\frac{B^2}{8\pi}+P\right) - \rho\vec{g}$$

consider flux tube of strength $\vec{B} = B_0\hat{x}$ in vertically stratified atmosphere. Left side vanishes in this situation. If tube is in pressure balance with surroundings then

$$P_e = P_i + \frac{B^2}{8\pi} \qquad (306)$$

where P_e is external pressure and P_i is internal gas pressure. Then $P_i < P_e$. If tube is in thermal equilib with surroundings then $\rho_i < \rho_e$ or

$$n_i kT = P_i = P_e - \frac{B^2}{8\pi} = n_e kT - \frac{B^2}{8\pi}$$

$$\Rightarrow n_i = n_e - \frac{B^2}{8\pi kT}, \qquad (307)$$

grav. force ↙ tube volume

thus $F_{buoy} = (n_e - n_i) m_H g V = \dfrac{B_0^2 m_H g V}{8\pi kT}$ (308)

is the upward force. Now $\frac{kT}{mg}$ is scale height so

$$F_{buoy} = \frac{B^2 V}{8\pi H} \qquad (309)$$

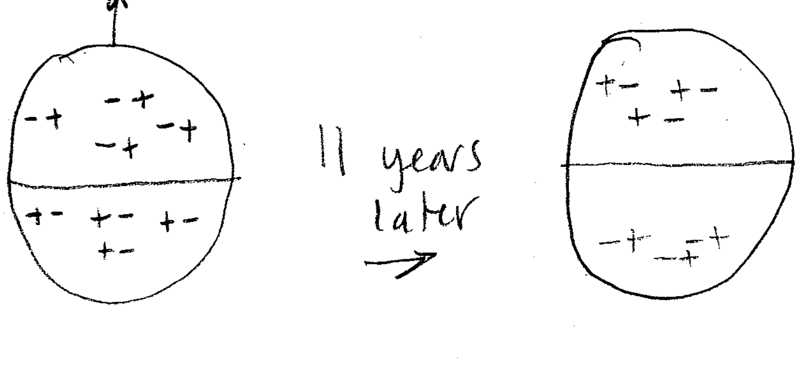
After rising distance H, tube gets kinetic energy

$F_{buoy}\cdot H = \dfrac{B^2}{8\pi}V = \frac{1}{2}\rho_i V u^2$, so u = velocity of tube

is $\Rightarrow$ $u = \left(\dfrac{B^2}{4\pi\rho_i}\right)^{1/2} = V_{A,tube}$ = Alfvén speed of tube (310) when the temperatures inside and outside the tube are not equal, the entire tube may not rise up, since then ρ_i at that location may not necessarily be $< \rho_e$, since $P_i \leq P_e$ can be satisfied by $\rho_e < \rho_i$ if $T_i < T_e$. (buoyancy requires $\rho_i < \rho_e$).

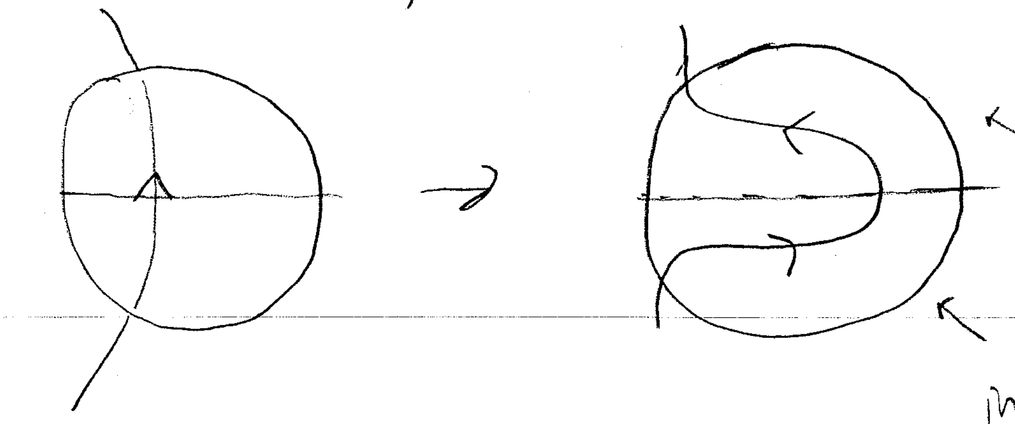
On the sun, most bipolar regions are roughly aligned parallel to the solar equator. In northern hemisphere, when + polarities are to right of negative polarity, in the south, − polarities are to the right of + polarities. Thus each of the northern & southern hemispheres typically show an opposite sign of leading & trailing polarity system, and the pattern reverses every 11 years:

e.g.



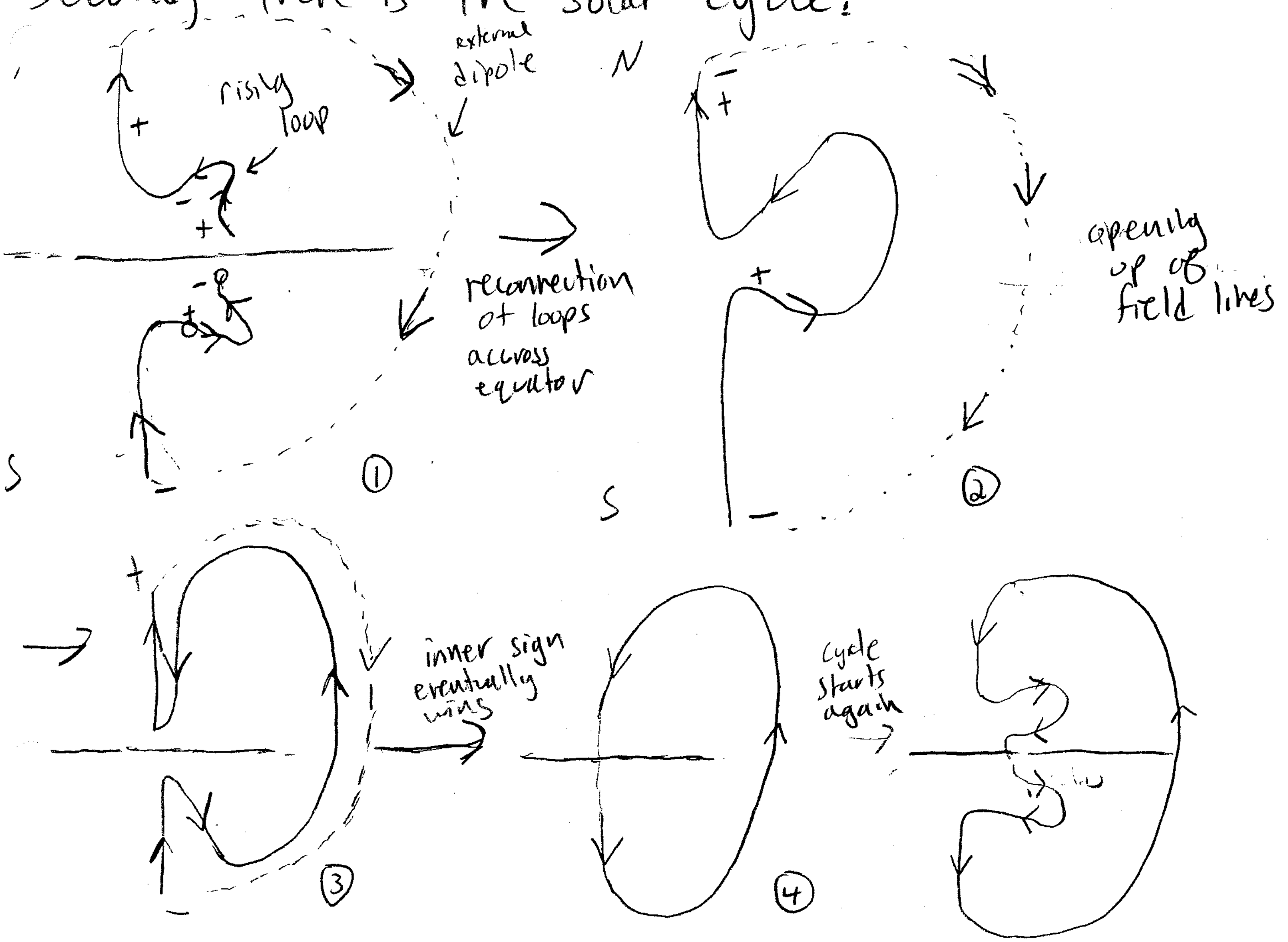
How can this situation arise? :

First, the sun is not rotating uniformly, but differentially, faster at equator, so that



thus buoyancy would naturally produce opposite trailing and leading polarity patterns in each hemisphere as on previous page.

Second, there is the solar cycle:



Angular momentum transport & magnetic fields:

Magnetic fields can help transport angular momentum. To see this, first prove a theorem:

Ferraro's law of isorotation: consider rotating object symmetric around rotation axis. Using cylindrical coords, this implies

$$V = r\Omega(r,z)\,\hat{e}_\theta \qquad (311)$$

independent of θ. Suppose object has axisymmetric poloidal field, frozen into plasma. Steady state is possible only if Ω is constant along field lines:

proof: A poloidal (r,z) field independent of θ can be written as curl of vector potential A_θ and in the form: (cylindrical coords)

$$\vec{B} = \left(\nabla \times \left(\tfrac{1}{r}\psi(r,z)\right)\hat{e}_\theta\right) \qquad (312)$$

Then $B_r = -\frac{1}{r}\frac{\partial \psi}{\partial z}$, $\quad B_z = \frac{1}{r}\frac{\partial \psi}{\partial r} \qquad (313)$

ψ is $\perp$ to field lines. Now let dr, dz represent displacements along streamlines of $\vec{B}$ (i.e. curves which have tangents $\parallel$ $\vec{B}$) $\rightarrow$

then $$\frac{dr}{B_r} = \frac{dz}{B_z} \quad . \tag{314}$$

From above we then have

$$\frac{\partial \psi}{dr} dr + \frac{\partial \psi}{\partial z} dz = 0$$ so that ψ

is constant along streamlines of $\vec{B}$.

now use induction equation in steady state with no diffusivity:

$$\nabla \times (\vec{v} \times \vec{B}) = 0 \tag{315}$$

for (312) & (311)

$\Rightarrow$ $$\nabla \times \left(r \Omega \hat{e}_\theta \times \left(\nabla \times \frac{1}{r} \psi \hat{e}_\theta \right) \right)$$

$$- \nabla \times \left(r \Omega \hat{e}_\theta \times \frac{1}{r} \frac{\partial \psi}{\partial z} \hat{e}_r \right) + \nabla \times \left(r \Omega \hat{e}_\theta \times \frac{1}{r} \frac{\partial \psi}{\partial r} \hat{e}_z \right)$$

$$+ \nabla \times \left(\frac{\cancel{r} \Omega}{\cancel{r}} \frac{\partial \psi}{\partial z} \hat{e}_z \right) + \nabla \times \left(\frac{\cancel{r} \Omega}{\cancel{r}} \frac{\partial \psi}{\partial r} \hat{e}_r \right)$$

$$\left[\frac{\partial}{\partial z} \left(\Omega \frac{\partial \psi}{\partial r} \right) - \frac{\partial}{\partial r} \left(\Omega \frac{\partial \psi}{\partial z} \right) \right] \hat{\theta} = 0$$

$$= \frac{\partial \Omega}{\partial z} \frac{\partial \psi}{\partial r} - \frac{\partial \Omega}{\partial r} \frac{\partial \psi}{\partial z} = 0 \quad \Rightarrow \Omega = f(\psi)$$

thus, $\Omega = f(\psi)$

and this means that the angular velocity is constantant along field lines, since ψ is a constant along field lines.

If Ω were to vary along field lines then poloidal lines would be continuously stretched to produce toroidal lines, and steady state is not possible without dissipation. When field lines are stretched, work is done on them. If field is strong, then field resists deformation and tries to impose rigid rotation.

Now this helps to explain why B-fields can transport ℓ momentum. We will consider examples of magnetic breaking and jets

$\rightarrow$