- For plasma with collisional mean free path $\lambda >>$ gyro-radius r_g consideration of individual particle motion is appropriate on gyro-scales; that is what we did for drifts and mirroring

- For scales $>> \mathrm{Min}(r_g, \lambda)$ for which there are many particles, a "collective" approach is needed. This can either involve study of the evolution of particle distribution function $f(\vec{p}, \vec{x}, t)$ or, equations for moments of f; i.e. the fluid or MHD equations.

$$\vec{V}_{flow} = \int_0^\infty f(\vec{V}_p, \vec{x}, t)\, \vec{V}_p \, d^3\vec{V}_p$$

$$\frac{\partial_t \vec{V}_{flow}}{\partial t} = -\vec{V}_{flow} \cdot \nabla \vec{V}_{flow} + \frac{\vec{J} \times \vec{B}}{\rho} - \frac{\nabla P}{\rho}$$

$\frac{J \times B}{\rho}$ force for fluid is obtained by

$$m_e \partial_t \vec{V}_e = \text{other} \; \underline{-e\vec{E} - |e| \vec{V}_e \times \vec{B}}$$

$$m_i \partial_t \vec{V}_i = \text{other} \; \underline{+e\vec{E} + |e| \vec{V}_i \times \vec{B}}$$

$$\vec{J} = en(\vec{V}_i - \vec{V}_e)$$

$$\partial_t V_{flow} = \partial_t V_e + \partial_t V_i = \text{other} + 0 + \frac{\vec{J} \times \vec{B}}{n}$$

$$\underbrace{\partial_t V_i - \partial_t V_e}_{\approx\, 0} = \overbrace{2|e|\vec{E} + |e|(\vec{V}_i + \vec{V}_e) \times B}$$

$$= \boxed{E \simeq -\left(\frac{V_i + V_e}{2}\right) \times \vec{B}}$$

Ohm's law for plasma $\boxed{\vec{E} = -\vec{V} \times \vec{B}}$

$$\boxed{E = -\vec{V}_{flow} \times \vec{B} + \eta J} + \cancel{\alpha \frac{\partial \vec{J}}{\partial t}} + \cancel{A \nabla P_e} + \cancel{\ldots}$$

IDEAL MHD OHM'S LAW

- Adding equations for electron + ion "fluids" gives MHD $J \times B$ force in the total momentum eqn.
- Subtracting equations for e^- and ion fluid momenta $\Rightarrow$ Ohm's Law

drifts: we focused on $\vec{V}_p$ and F.A., and mirroring

collective approaches focus on

$$\vec{V}_{flow} \equiv \frac{\int f(V_p', \vec{x}, t)\, \vec{V}_p'\, d^3V_p'}{\int f(V_p', \vec{x}, t)\, d^3V_p'}$$

and MHD

$\vec{V}_f, \vec{B}, P, T, \rho, \cancel{\vec{E}}$

MHD

Navier stokes equation $+ \vec{J} \times \vec{B}$

- $\partial_t \vec{V} = -\vec{V} \cdot \nabla \vec{V} - \nabla p + \frac{\vec{J} \times \vec{B}}{c}$
- ohm's law : $\boxed{\vec{E} = -\frac{\vec{V} \times \vec{B}}{c}}$
- Maxwell's equations
- Maxwell's equations + OHM's law

$\Rightarrow \boxed{\frac{\partial B}{\partial t} = -\nabla \times E} \Rightarrow \boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{V} \times \vec{B})}$

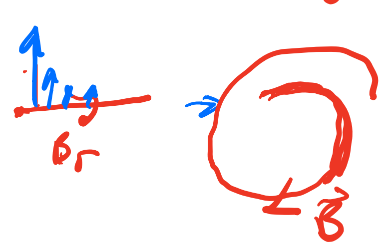
induction equation

- $\nabla \times \vec{B} = \frac{4\pi \vec{J}}{c}$

$$\frac{\partial \vec{B}}{\partial t} = \vec{B} \cdot \nabla \vec{V} - \vec{V} \cdot \nabla \vec{B} +$$

($\nabla \cdot \vec{V} = \nabla \cdot B = 0$ assume)

$$\left.\begin{aligned} \frac{\partial B_\phi}{\partial t} &= [\vec{B} \cdot \nabla \vec{V}]_\phi \\ &= B_r \partial_r V_\phi \end{aligned}\right\}$$



Core rotation and Poynting (corrected) Flux from Magnetized Rotator and role in SN and Pulsars

The presence of strong differential rotation leads to the possible amplification of magnetic fields by shear. The fields can then mediate the extraction of core rotational energy via Poynting Flux.

I mentioned that core contraction and envelope expansion in a SN progenitor would naturally lead to this circumstance of differential rotation. More specifically, consider an initially uniformly rotating star with a uniform density. Any momentum conservation tells us that if the core collapses and envelope expands, the envelope will rotate more slowly than the core. Typically, the envelope then rotates negligibly compared to the core. To estimate the core rotation, consider the initial star to have uniform density of approximately that of the sun ($\sim 1 g/cm^3$). Thus a stellar mass of material would be located at $\simeq 1 R_\odot$ from the core, while the star burns H on the main sequence

Then, upon collapse, conservation of specific angular momentum gives $\Omega_{NS} = \Omega_0 \left(\frac{R_0}{R_{NS}}\right)^2$ (74)

therefore for an initial star on the main sequence rotating at the sun's rotational velocity $\left(\Omega_\odot = \frac{2\pi}{T} \simeq \frac{6}{1\,\text{month}} \simeq 2\times10^{-6}\ \text{rad/sec}\right)$ the core NS rotates at $\Omega_{NS} \simeq 2.5\times10^{3}\ \text{rad/sec}$!

$= \Omega_0 \left(\frac{7\times10^{10}}{10^6}\right)^2$

$\boxed{= 5\times10^9\,\Omega_0}$

this is extremely fast, implying periods of milliseconds! Thus we can expect fast rotating neutron star cores, in general.

(compare to Earth core:

Earth core	NS
$\Omega \simeq 7\times10^{-5}\ \frac{\text{rad}}{\text{sec}}$	$\Omega_{NS} \simeq 10^3$ sec
$\rho = 5\ \frac{\text{gm}}{\text{cm}^3}$	$\rho \simeq 10^{15}$ g/cm^3
$r \simeq 1250$ km	$r \simeq 10$ km
liquid	solid
$B \simeq 1$ Gauss	$B \gtrsim 10^{12}$ Gauss)

Now, suppose that one takes typical magnetic fields observed on normal main sequence stars, collapses to NS, and "flux freezes"

The poloidal field does not change but the toroidal field amplifies in the wind up.

The wind up is draining energy from the rotation of the core, relative to the rotation in the envelope. Most of the amplification takes place right near the core-envelope interface. Roughly speaking, the amount of energy the field can acquire is approximately

$$\left(\frac{B_\phi^2}{8\pi}\right)\pi R^2 \Delta R \simeq \underbrace{f M_{NS} R_{NS}^2}_{\text{moment of inertia}} \Omega^2 \qquad (75)$$

energy density of toroidal field amplified

radius of shear layer $\simeq R_{NS}$

thickness of shear layer

roughly then for $R \sim \Delta R \sim R_{NS}$ (75) implies

$$B_\phi^2 \simeq \frac{\pi \Omega^2 M_{NS}}{R_{NS}} \lesssim 6\times10^{16} \left(\frac{\Omega}{10^3/s}\right)^2 \left(\frac{M}{M_\odot}\right)\left(\frac{R}{10^6\,cm}\right)^{-1} G \qquad (76)$$

thus, in principle one can get huge fields.

In practice there are complications:

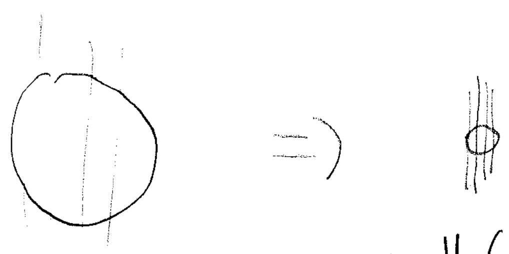
i) turbulent convection leads to enhanced diffusion of field that one is trying to grow

→

down to NS scales: $\Rightarrow$

$$\frac{B_{NS}}{B_0} \simeq \frac{R_0^2}{R_{NS}^2} \simeq \left(\frac{7\times10^{10}}{10^6}\right)^2 = 5\times10^9 \, , \qquad (74a)$$

↑ same as in (74)

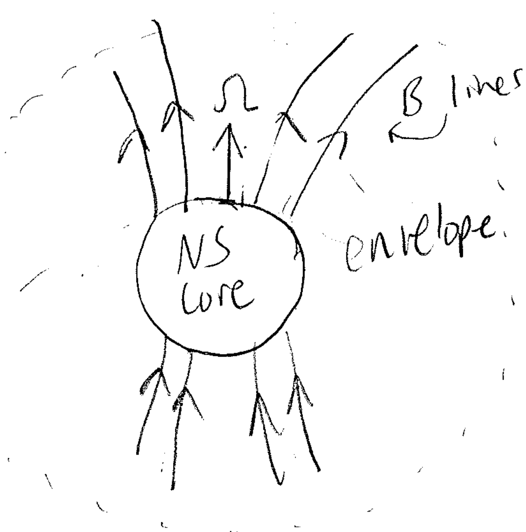


so $B_{NS} \simeq 10^{11}\left(B_0/1\,G\right)$ Gauss

from flux freezing.

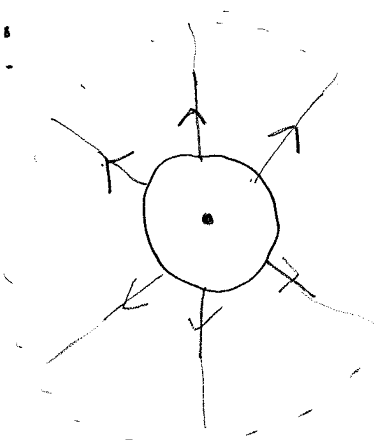
But if this field is threaded between the NS core and envelope, and core is spinning fast:

suppose we start core rotating just at time $t=0$ $t=0$



top view:

$t=0$

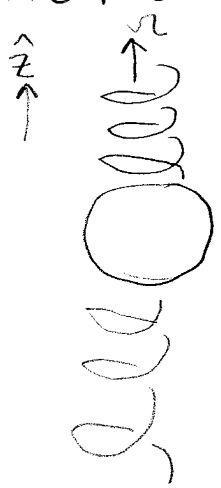


$\rightarrow$ $t>0$ top view

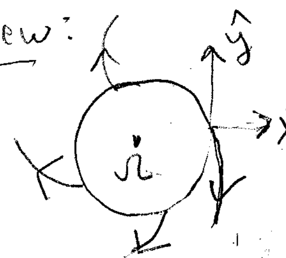
(flux freezing also means low slippage between field and plasma, $\Rightarrow$ field winds up) the turbulent diffusion, or if the latter subsides (e.g. neutrino driven convection in the core subsides) then the core has both a poloidal and toroidal field (∵ poloidal from initial collapse and dynamo; toroidal from winding and dynamo)

we can then ask what a rigid rotator with poloidal and toroidal field can do for us:

consider the Poynting Flux integrated over the $\hat{z}$ direction:



top view:



let $B_r \simeq B_x$
$B_z = B_z$
$B_y \simeq B_\phi$

$$\frac{c}{4\pi}\int (\vec{E}\times\vec{B})_z \, dS_z \simeq \frac{c}{4\pi}\left[(E_x B_y - E_y B_x)_{top} + (E_x B_y - E_y B_x)_{bottom}\right]\pi R^2 \tag{77}$$

Remember from Jackson

$$\partial_t \int \frac{B^2}{8\pi} d^3x = -\frac{c}{4\pi}\int (E\times B)\cdot d\vec{S} - \eta\int J\cdot E\, d^3x$$

$$\vec{E} = -\frac{\vec{V}\times\vec{B}}{c} \qquad V_y = \Omega R, \quad V_x = V_z = 0 :$$

$$\Rightarrow \quad E_x = -\frac{V_y}{c}B_z + \frac{V_z}{c}B_y = -\frac{V_y}{c}B_z = -\frac{\Omega R B_z}{c}$$

$$E_y = -\frac{V_z}{c}B_x + \frac{V_x}{c}B_z = 0$$

negative in north for dipole field

$$\Rightarrow \quad (E_x B_y - E_y B_x)_{north} = (E_x B_y)_{north} = -\frac{\Omega R B_z B_y}{c}$$

positive in south for dipole

$$(E_x B_y - E_y B_x)_{south} = (E_x B_y)_{south} = -\Omega R \left(\frac{+B_z}{c}\right)(-B_y)$$

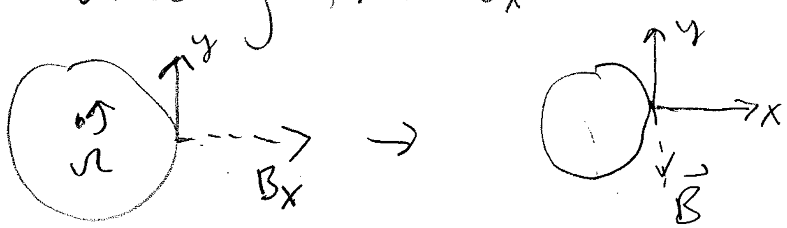
$$= -(E_x B_y)_{north}$$

$$\Rightarrow \quad \frac{c}{4\pi}\int (\vec{E}\times\vec{B})_z \cdot d\vec{S}_{\uparrow z} = \overbrace{\left(\frac{\pi R^2}{4\pi}\right)}^{dS_N} \cancel{\frac{c}{c}} (E_x B_y)_{north} - \overbrace{\left(\frac{-\pi R^2}{4\pi}\right)}^{dS_S} (E_y B_y)_{south}$$

$d\vec{S}$ has opposite sign in north & south hms

$$= -\frac{1}{2}\Omega R^3 (B_z B_y) \qquad (78)$$

note $\quad \frac{B_x}{|B_x|} = -\frac{B_y}{|B_y|}$ which can be seen from the winding of a dipole field: In northern hemisphere, B_x and B_z have same sign, so consider winding the B_x component:



$B_y < 0$ when B_x initially > 0 from winding

$\rightarrow$

thus equation (78)

$\Rightarrow$

$$\frac{c}{4\pi}\int (E\times B)\,d\vec{S} = +\frac{1}{2}\Omega R^3 (|B_z||B_y|)$$

$$\simeq \frac{1}{2}\Omega R^3 |B_z||B_\phi|$$

This quantity has units of luminosity:

$$L_{PF} \simeq 10^{49}\left(\frac{\Omega}{10^3}\right)\left(\frac{R}{10^6\,cm}\right)\left(\frac{B_z}{10^{13}}\right)\left(\frac{B_\phi}{10^{15}}\right)\ erg/s \quad (79)$$

This is important because the energy that this poynting flux can deliver is $\simeq L_{PF}\tau_{spin\ down}$, where $\tau_{spindown}$ is the time scale for the core to spin down by extracting its poynting flux:

[Exercise: (1) calculate the spin down time for a NS core using L_{PF} from (79).]

$L_{PF}\,\tau_{spin}$ can then be compared with the Gravitational binding energy of material ABOVE the shear layer where the field is amplified: The two energies can be comparable so the poynting flux may blow off some of the envelope via a bipolar jet!

$\longrightarrow$

"Comparison of Poynting flux from NS to that from Earth's core"

For NS: (SN progenitor core) $L_{mag} \simeq 10^{49}\left(\frac{B_\phi}{10^{15}}\right)\left(\frac{B_z}{10^{13}}\right)\left(\frac{\Omega}{10^3/\mathrm{sec}}\right)\left(\frac{R}{10^6\,\mathrm{cm}}\right)^3$ erg/sec

For Earth core $L_{mag} \simeq 7\times10^{19}\left(\frac{B_\phi}{1G}\right)\left(\frac{B_z}{1G}\right)\left(\frac{\Omega}{7\times10^{-5}/\mathrm{sec}}\right)\left(\frac{R}{10^8\,\mathrm{cm}}\right)^3$ erg/sec

$t_{spin\ down}$ for Earth core:

$$\simeq \frac{\mathcal{E}_{rot}}{L_{mag}} \simeq \frac{M_c\Omega^2R^2}{L_{mag}} \simeq \frac{(3\times10^{27}\,g)(7\times10^{-5}/\mathrm{sec})^2(10^8\,\mathrm{cm})^2}{7\times10^{19}\,\mathrm{erg/sec}} \simeq \underline{9\times10^7\,\mathrm{yr}}$$

M_c

$\mathcal{E}_{rot}$ compared to binding energy for Earth above core:

$$\mathcal{E}_{bind} \simeq 4\pi\int\frac{GM(r)\rho(r)}{r}r^2dr \simeq \frac{GM_cM_{ext}}{r_{ext}},$$

$\rho_{core} \simeq 10^3\,g/cm^3$

$\bar\rho_{av} \simeq 5\,g/cm^3$

$r_{core} \simeq 10^8\,cm$

$r_{ext} \simeq 6\times10^8\,cm$

Note: to get M_{ext} & M_c from data at left:

$$M_{ext} = M_\oplus - \frac{4}{3}\pi\rho_{core}r_{core}^3$$

$$\simeq 6\times10^{27}g - 4/3\,\pi\,10^3(10^{24})$$

$$\simeq 2.1\times10^{27}g$$

$$M_c = M_\oplus - M_{ext} \simeq 3.9\times10^{27}g.$$

$$\Rightarrow \frac{\mathcal{E}_{bind}}{\mathcal{E}_{rot}} \simeq \frac{1.2\times10^{39}\,\mathrm{erg}}{(3\times10^{27})(7\times10^{-5})^2(10^8\,\mathrm{cm})^2} \simeq \boxed{8163} \gg 1.$$

for <u>Earth</u> binding energy $\gg$ rotational energy point is that rotational energy is small compared to binding energy so the poynting flux could never unbind the star since P.F. draws energy from rotation. the P.F. could never unbind the Earth. On the other hand, the B-field could spin down the core, or rather, lock the core in uniform rotation with the outer core & mantle after $\simeq 10^8$ yr.

Recall that by contrast, the NS core in SN can spin down on time scales of order $\simeq$ 10 - 100 sec, and the rotational energy can approach the binding energy, at least within an order of magnitude which explains why rotation is energetically important for unbinding SN but not the Earth.

Note also the general lesson that even if a rotator can transport $\measuredangle$ momentum outward and reach a rotation profile $\Omega \propto \frac{1}{r^{|q|}}$, where $|q|$ is some index, the $\frac{\measuredangle \text{ momentum}}{\text{mass}}$ goes as $r^2\Omega \propto r^{2-|q|}$ where as $\frac{\text{rotational energy}}{\text{mass}}$ goes as $r^2\Omega^2 \propto r^{2-2|q|}$.

Thus for typical $|q|$ (e.g. keplerian, $q = 3/2$) $\measuredangle$ mom. increases out and but rotational energy increases on smaller scales!

Recent perspectives on SN explosions

(see papers by Wheeler and Burrows et al. handed out)

- Evidence for asymmetry:
  - Crab Nebula: optical/X-ray jet-torus structure (Chandra X-ray obs)

crab pulsar is $\approx$ 1000 years old (SN in 1054 AD)

- But crab nebula asymmetries are on large (10^{17} cm) scale with respect to explosion scale (10^{6} cm) so connection requires more evidence
- spectropolarimetry has been important (e.g. Wang 2003) as it measures polarization as a function of frequency and can probe generic asymmetries as well as asymmetries in composition
- spectropolarimetry shows that core collapse SN are all polarized and therefore asymmetric

(• link between SN asymmetry and the inferred collimated nature of Gamma-ray bursts has been very important - more later) → (⇒ 2 to 1 axis ratio Höflich 1991)

- polarization is at the 1% level for type II SN but type Ia are substantially less polarized
- polarization increases with time as photosphere recedes into the envelope ejecta
- polarization is higher for thinner envelopes

⇒ POLARIZATION IS ASSOCIATED WITH CORE!

- While most SN show preference for consistent bipolar structure as function of time and wavelength, there are exceptions, suggesting either weak bipolarity, or rapid bipolar wander. SN 1993J, SN 1996cb, SN 2002ap (showed high photospheric velocities in early phases with H, O, Ca axes oriented differently!

Are SN exploded by Jets?

- Numerical simulations in which jets are put in by hand, can explode the star (e.g. Khokhlov et al 1999).
- Few simulations which actually "self-generate" the jet however.
- There should be tendency for iron peak elements to be ejected along the jet axis, if the jet drives from the very core. This is seen in SN 1987A.
- However in remnant of Casseopeia A (SN remnant in our Galaxy from SN in 17th century) Si seems to show this behavior, not Fe. One would expect both to exhibit the jet distribution.
- Another issue is that the compact core (pulsar) is moving 330 km/s $\perp$ to the "jet"! This is hard to explain if the jet represents the explosion as one might expect "kick" to be imparted parallel to the dominant outflow axis →

this latter point has been cited by Burrows to argue that the SN is NOT along the "jet" axis but primarily in the equatorial plane! more on this below: it is related to what influence rotation has on neutrino driven SN.

The observed asymmetry in SN lead us to 1 fairly definitive conclusion: that ROTATION IS IMPORTANT FOR DRIVING CORE COLLAPSE SN.
However, we cannot yet say that magnetic fields are important from direct evidence. Their role is plausible but rotation may instead influence the way neutrinos drive SN.

Our previous discussion of B-fields in SN should make it clear that any poynting flux expected would be along the axis of rotation. Thus, if a SN is driven by a jet and the jet driven by B-fields, the explosion should be primarily along the axis of rotation.

• THIS NEED NOT BE TRUE IF ROTATION FACILITATES A NEUTRINO DRIVEN SN!

Burrows perspective on rotation

Like Wheeler, Burrows agrees that rotation must be important but questions the dominant role of magnetic fields

Issues

- although there has been an association of GRB with SN of Type Ic (these are SN that don't show much Si in their spectra and don't have much H in spectra, but are less luminous than type Ia and likely are core collapse SN. Progenitors are likely stars that have lost much of their hydrogen envelope in a wind e.g: Wolf-Rayet stars), the GRB rate in the universe is 1-2 per day, (about 10^{-6}/Galaxy/yr), but SN rate is one per second.
- Even with beaming angle of GRB of 5° ... (bulk Lorentz factors of ≃ few hundred) $\frac{GRB}{SNe}$ ratio is 0.0005. This is several orders of magnitude lower than fraction of SN that are type Ic.
- Also, canonical type Ic SN do not show ^{56}Ni abundances and high velocities characteristic of GRB.

⇒ GRB are only occuring for a very special class of SN, not generic. What determines whether one gets a GRB?

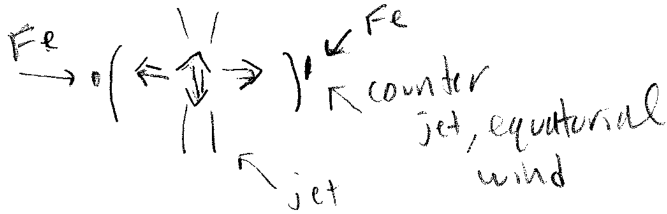
→

- One possibility is mass: MacFadyen & Woosley 1999 collapsar model of GRB requires initial stellar mass > $80 M_\odot$; and very rapid core rotation. The mass function (= distribution function of stars of mass M) places about 1-2% of stars in this category which is acceptable.
- Thus GRB are special. It is plausible that MHD jets play a key role in the GRB both in the driving and collimation, but given that normal SN are so different from GRB maybe B-fields are only dominant in GRB, not SN.
- GRB energies and SN energies are comparable so their energetics are less of a distinction than the "quality" of their emission.
- Burrows points out that asymmetries observed via polarization don't necessarily explicitly require rotation if neutrino driven convective instabilities are present. (Large scale "Rayleigh-Taylor" fingering) However, there seems to be association of polarization axis with axes of rings in ejecta, so rotation axis seems to be present. (SN 1987A) => core rotation.
- SN1987A ejecta is bipolar but which axis is the supernova?
- Burrows favors that rotation is important but it affects the way neutrinos drive the explosion with MHD possibly driving a subdominant secondary outflow that would dominate only for GRB.

→

Cas A

- exibits jet collimation

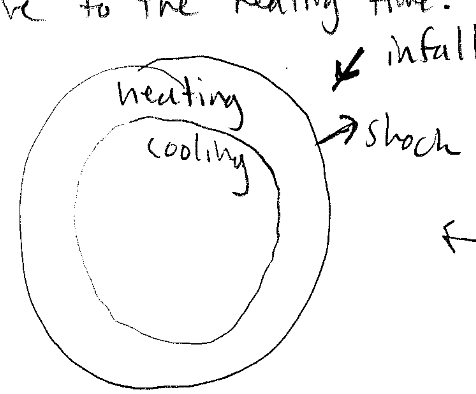


more momentum in equatorial wind than in jet, and Fe shows up more prominently in the equatorial wind. SN arguably along the equator, which also explains "kick" issue described earlier.

Neutrino Explosion & Rotation

Bounce of shock never leads to direct explosion in 1-D, 2-D, or 3-D simulations because shock stalls.

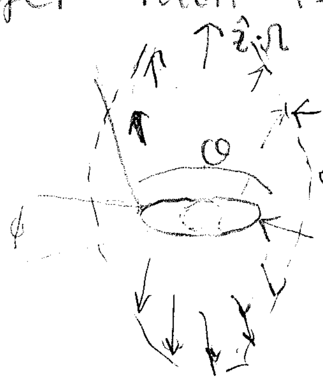
- Delayed neutrino mechanism where shock stall is revived by local increase in heating and neutrino deposition can work in 2-D & 3-D by increasing the infall time relative to the heating time:



material must remain in heating region behind shock long enough before advecting into cooling region

Key result: if the heating time τ_H is small compared to the accretion or advection time through to the cooling region, then explosion results (Thompson et al. 2005).

- rotation helps because it provides centrifugal support to material that would otherwise fall in on a free fall time.
- accretion is slower than pure free fall and this helps make the ratio $\frac{\tau_{acc}}{\tau_H} > 1$, favoring explosion along the equator.
- On the other hand, the neutrino flux is enhanced along the poles because there is a cleaner path to the high temperature core there, and angle subtended by the core at large distances is larger in the poleward direction (see below)
- Interestingly, rotation makes the photosphere oblate, but the neutrino energy deposition at larger radii is prolate



- the large distance neutrino energy density prolateness results because of the large ∡ subtended by the core θ compared to angle ϕ subtended by the neutrino emitting region at the equator. →

- Remarkably, it remains ambiguous as to whether the dominant gain from neutrino driven models is along the equator (due to $\frac{\tau_h}{\tau_{acc}} \ll 1$ there) or along the poles, where the neutrino flux is enhanced.
- these effects depend on the rotation rate and the differential rotation profile.
- another non-magnetic effect which is important is dissipation of the free energy in differential rotation via hydrodynamic viscosity. (Thompson et al 2005; Blackman, Nordhaus, Thomas 2005)
- if the timescale to extract the free energy in differential rotation via turbulent dissipation is short compared to dynamo magnetic field amplification time, then extra heating occurs locally which can compete against the accretion (infall) into the cooling region (Page 84) and favorably lower $\tau_h < \tau_{acc}$.