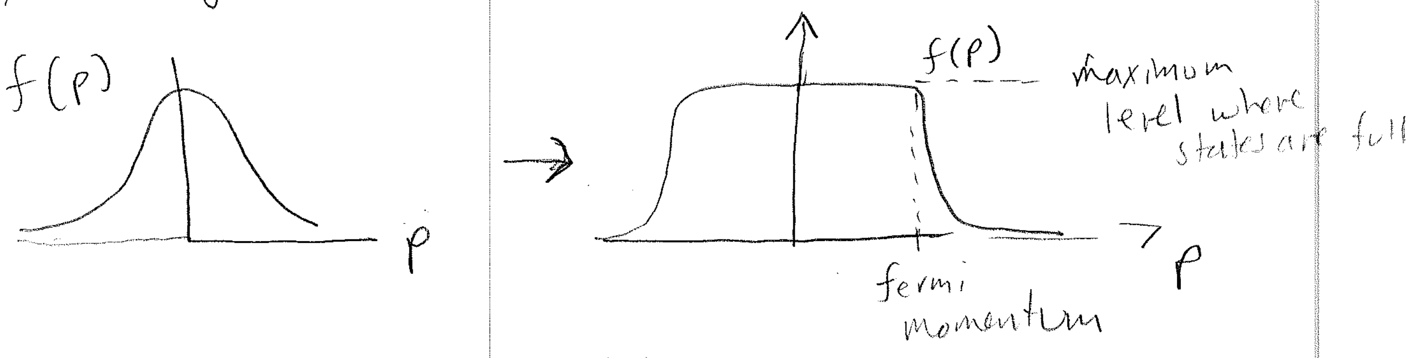
# Quick review of Degeneracy Pressure

At high densities fermi-dirac dist:




- Equation of state no longer $P = nkT$, but the pressure is dominated by quantum exclusion principle.
- Degeneracy is important when density is high enough to squeeze particles to $\Delta x$ limited by $\frac{\hbar}{\Delta p}$.
- Consider electron energy in a proton-electron plasma:

$$E = \frac{3}{2}kT \simeq \frac{\Delta p^2}{2m_e} \quad \text{(non-relativistic)} \qquad (6)$$

$$\Rightarrow \Delta p_e \simeq (3kTm_e)^{1/2} = \frac{\hbar}{\Delta x}$$

$$\Rightarrow \rho_{crit} = \frac{m_p \leftarrow \text{(protons dominate density)}}{(\hbar/\Delta p)^3} = m_p\left(\frac{3m_e kT}{\hbar^2}\right)^{3/2} \qquad (6a)$$

$$(7)$$

$$\rho_{crit} \propto (m_e T)^{3/2}$$

↑ because $m_e$ appears here, electron degeneracy pressure requires lower densities than e.g. neutron degeneracies explaining why white dwarfs re less dense than neutron stars.

→

Equation of state for degenerate matter can be approximated very simply:

Use $E = \frac{1}{2} m_e v^2$ (non-relativistic)

$$= \frac{\Delta p^2}{2m_e} = \frac{\hbar^2}{2m_e \Delta x^2}$$ (using $\Delta p \Delta x \simeq \hbar$)

$$\underset{\substack{\uparrow \\ \text{energy density}}}{\mathcal{E}} \equiv \frac{E}{\Delta x^3} = \frac{\hbar^2}{2m_e \Delta x^5} \underset{\substack{\uparrow \\ \text{from (6a)}}}{=} \frac{\hbar^2}{2m_e}\left(\frac{\rho}{m_p}\right)^{5/3} \qquad (8)$$

but $P = (\gamma - 1)\mathcal{E}$, $\gamma \equiv \frac{C_p}{C_v} = 5/3$ (non-rel monatomic system)

$$\Rightarrow \boxed{P = \frac{2}{3}\mathcal{E} = \frac{\hbar^2}{3m_e}\left(\frac{\rho}{m_p}\right)^{5/3}} \qquad (9)$$

approx. EOS for non-rel. degenerate plasma

For relativistic case,

$$E = \Delta p c = \frac{\hbar c}{\Delta x} =$$

$$\Rightarrow \mathcal{E} = \frac{\hbar c}{\Delta x^4} = \hbar c \left(\frac{\rho}{m_p}\right)^{4/3}$$

$$P = (\gamma - 1)\mathcal{E} = \frac{1}{3}\mathcal{E} = \boxed{\frac{1}{3}\hbar c \left(\frac{\rho}{m_p}\right)^{4/3}} \qquad (10)$$

($\gamma = 4/3$ for relativistic monatomic system)

approx EOS for relativ. plasma

(No T dependences in degeneracy pressure)

For the WD end states of low mass stars, typically $r = 10^9$ cm, $\rho = 10^6$ g
$\Rightarrow$ from e.g. (9) that electron energy is $\simeq$ 100 keV. Thus for kinetic temp at $10^7$K $\simeq$ 1 keV the ions, which are not degenerate play little role in the pressure support

- mean free path of $e^-$ is high in degenerate systems because phase space is so full; hard for $e^-$ to find a "hole"
  $\Rightarrow$ high conductivity.
- photon opacity is low since $e^-$ do not easily move from one state to another.

$\Rightarrow$ temperature gradients are smoothed out, in WD interior

- WD surface is less dominated by degeneracy

$\Rightarrow$ WD is "metal ball surrounded by insulator"

- WD cooling (Mestel 1952) $L \propto t^{-1.5}$ $(10^5\,\mathrm{yr} < t < 10^{9.5}\,\mathrm{yr})$
  using simple polytrope model for star $(P \propto \rho^{1+1/n}, n=3)$
- phase transitions and partial ion support slow cooling at late times.

$$\frac{3}{2}\frac{k\rho}{m_H \mu}\frac{dT}{dt} \simeq \frac{1}{r^2}\frac{\partial}{\partial r}\left\{\alpha \rho T r^2\right\}$$

# High Mass stars $M \gtrsim 8 M_\odot$



- high mass star: "rich person who spends a lot"
  low mass star "poor person who is thrifty"
- core hydrogen burning, & helium core contraction as star moves off of main sequence proceeds as for low mass stars but core is convective because CNO cycle dominates fusion with stronger temperature sensitivity and thus stronger gradients in the core
- As H burns, dumping helium in core, He is mixed during convection => more H must be exhausted to run out of fuel in core and induce contraction.
- eventually core becomes 99% He. Deep in core little H-burning => gravit. contraction => more uniform heat release in core => core is no longer convective => T increases toward core
- H-burning shell has difficulty supporting outer layers => increased core contraction. (note: H-burning still supplies most of luminosity)
- contraction => H-shell burning more efficiently => expansion!! of star => upward but primarily rightward motion on H-R diagram
- Eventually core ignites He => heat in core => expansion of H-burning shell => reduction in temp of H-burning shell => lower supply of heat => envelope contracts => leftward movement on H-R diagram

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- Because photons must take long path to escape through massive envelope, the outer visible regions of the star don't see the internal change in luminosity much, only the change in surface from which the photons escape. This is why for high mass stars, motion is mostly horizontal on H-R diagram.
- Because He burns under nondegenerate conditions and so does Carbon, there is no "flash" to halt the contraction as for low mass stars. during sucessive phases of core contraction, because the degeneracy pressure cannot stop contraction because the core mass is too large; Extremely high temperatures accompany the high core densities. During this process, the core continuously contracts, until silicon burning which takes place under highly degenerate conditions kicks and leaves an iron core at the center.
- But Fe does not release energy by fusion so the core needs to find other ways to release energy if it is to continue fighting gravity →

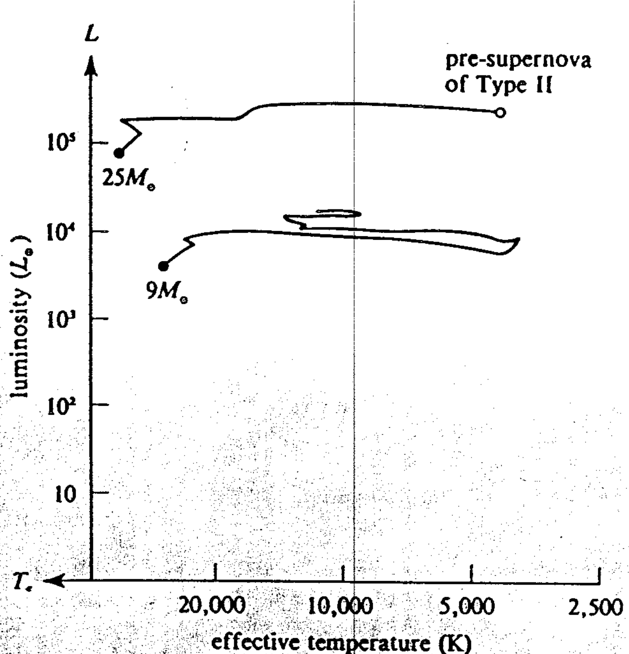


**Figure 8.11.** Evolution of a high-mass and a very-high-mass star in the Hertzsprung-Russell diagram. (Adapted from Icko Iben and from Weaver and Woosley, *Ninth Texas Symp. Rel. Ap., Ann. N.Y. Acad. Sci.*, 336, 1980, 335.)

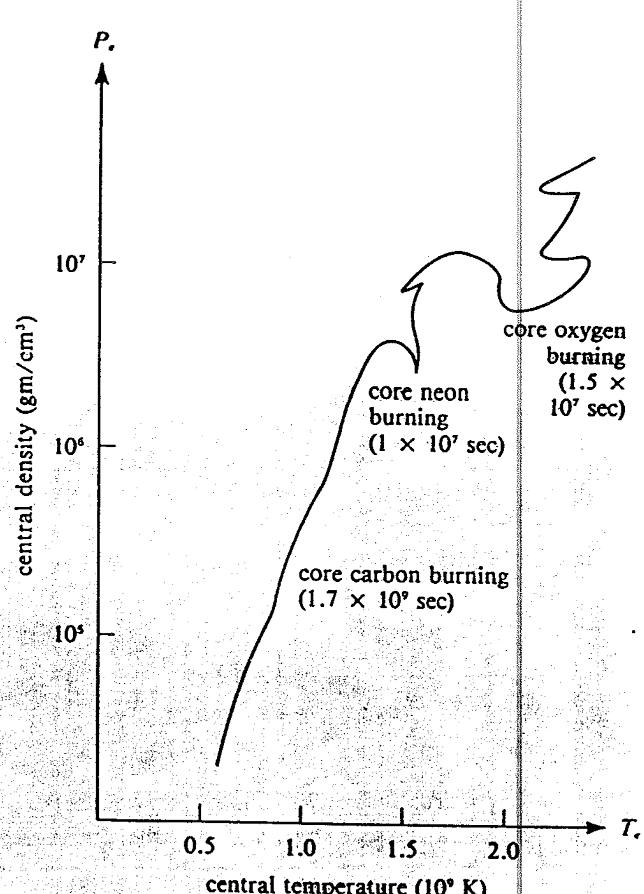


**Figure 8.12.** Evolutionary history of center of a $25M_\odot$ star in late stages. The central density and temperature increase to higher and higher values on timescales less than a year as the iron catastrophe is approached. (Adapted from Weaver and Woosley, *Ninth Texas Symp. Rel. Ap., Ann. N.Y. Acad. Sci.*, 336, 1980, 335.)

# Aspects of Supernovae

As the cores of massive stars ($M > 8 M_\odot$) approach their end states, fusion becomes less and less efficient and the burning proceeds more rapidly. (For 20 $M_\odot$ star MS lifetime is $10^7$ yr, He buring $\simeq 10^6$ yrs C burning lasts 300 yrs. O-burning takes 200 days. Si-burning takes 2-days.

At the high temperatures in the core $\simeq 10^{10}$ K, the photons have enough energy to disassociate heavy nuclei. Most important are

$$^{56}_{26}Fe + \gamma \rightarrow 13\,^{4}_{2}He + 4n$$

$$^{4}_{2}He + \gamma \rightarrow 2p^+ + 2n$$

These are endothermic reactions and thus take away needed thermal support in addition to the fact that Fe fusion is prohibited. This combined effect is bad for stability and accelerates the beginning of the end. →

As alluded to earlier under extreme densities and temps ($\rho \simeq 10^{10} \frac{g}{cm^3}$, $T \simeq 10^{10}$ K for $M = 15 M_\odot$)

$$\left\{ \begin{array}{ll} e^+ + e^- \to \nu\bar{\nu} & \text{plasmon} \to \nu + \bar{\nu} \\ \gamma + e^- \to e + \nu\bar{\nu} & \text{Urca process: } (Z,A) + e^- \to (Z-1,A) + \nu \\ p^+ + e^- \to n + \nu_e & \qquad (Z-1,A) \to (Z,A) + e^- + \bar{\nu} \end{array} \right\}$$

The neutrino energy released is huge:

For a $20 M_\odot$ star $L_{neutrino} \simeq 10^{45}$ erg/s, but $L_\gamma = 5 \times 10^{38}$ erg/s in photons.

These neutrinos don't couple efficiently to the outer regions of the star and can't do much to prevent collapse. As this collapse begins, the collapse starts out to be "homologous" but eventially the violation of this homologous collapse plays a fundamental role.

To ~~better~~ understand this, I want to derive the key feature of homologous collapse, and then discuss how it is violated.

During collapse, the momentum equation dominated by the gravitational force

$$\Rightarrow \frac{d^2 r}{dt^2} = -\frac{G M_r}{r^2} \qquad (174) \quad (11)$$

the expanding shell is $\propto \frac{dN}{dt}$, & after the shell is optically thin this is $\propto L$

thus: $L = K_1 N$ $K_1$ = constant

$$\frac{dL}{dt} = K_1 \frac{dN}{dt} = -K_1 \lambda N$$

$$\Rightarrow \quad \frac{d \ln L}{dt} = -\lambda \quad \text{or} \quad \frac{d \log_{10} L}{dt} = \frac{-\lambda}{\ln 10}$$

Thus the slope of the light curve reals the element incurring the dominant decay. It is worth emphasizing that it is indeed radioactive decay that powers the light curves. Note that the heating by the shock leads to explosive nucleosynthesis which produces these radioactive nuclei.