High Energy Particle Acceleration

A Supernova shocks show power law spectra for relativistic electrons incurring synchrotron emission: for particle energy distribution

$n(E) \propto E^{-p}$, E = particle energy

$S_\nu \propto B^{\frac{(1+p)}{2}} \nu^{-(p-1)/2}$ ν = emission frequency

$\nu_c \simeq \frac{3e\gamma^2 B}{2mc}$ p = power law index

characteristic emission freq. for electron with $E = \gamma mc^2$ (see Rybicki & Lightman)

But in addition to spectrum, total flux from high energy particles reveals that energy in relativistic particles is a "double digit" percentage of the outflow energy at all times. But this requires an acceleration process because even if only adiabatic losses operated for the relativistic particles then the decay in energy would be too significant →

Adiabatic expansion implies

that

$$d(\mathcal{E}V) = -P\,dV$$

volume

energy density, volume, pressure

for simple relativistic gas $P = \frac{1}{3}\mathcal{E}$

so $d(\mathcal{E}V) = -\frac{1}{3}\mathcal{E}\,dV$

$V\,d\mathcal{E} + \mathcal{E}\,dV = -\frac{1}{3}\mathcal{E}\,dV \Rightarrow \mathcal{E} \propto V^{-4/3}$

$\Rightarrow \mathcal{E} \propto r^{-4}$. Total energy of electrons is $E = \mathcal{E}V \propto r^{-1} \Rightarrow E = E_0\left(\frac{r_0}{r}\right)$.

Thus the total energy in relativistic electrons would rapidly decay during the expansion if there were no in-situ acceleration during this expansion.

Cosmic Ray Acceleration – Application of magnetic mirroring

1912 · Balloon flights measured ionizing effect of cosmic rays (Hess 1912).

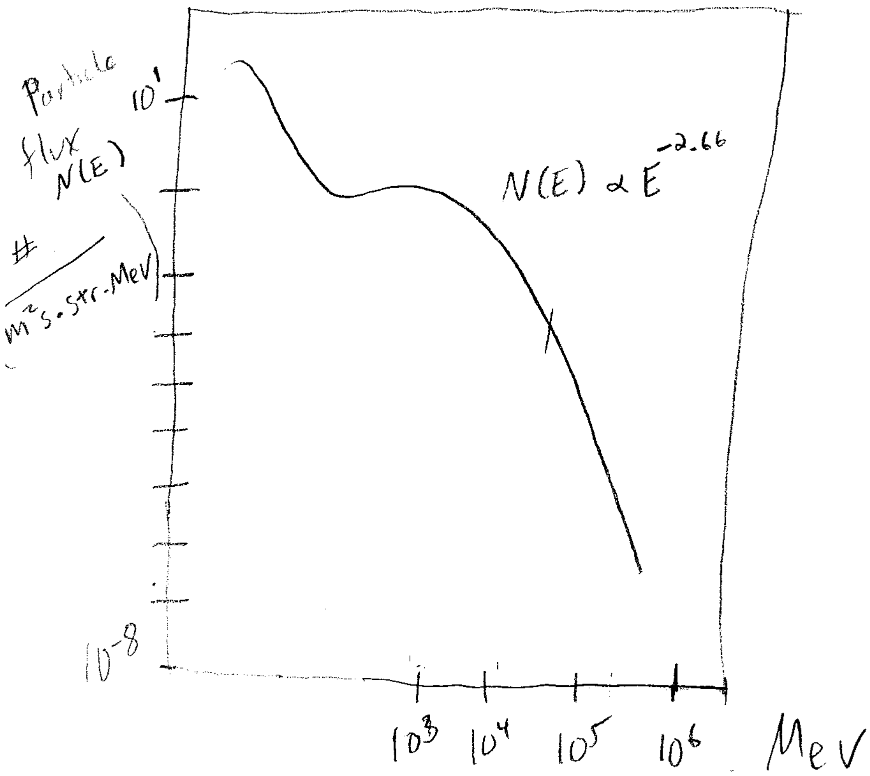
poorly named, cosmic rays are really particles, mainly light nuclei & electrons.

Generally, evidence suggests that cosmic rays are mainly a galactic phenomena, as unless they are extremely high energy, the particles would be gyrotrapped in their host galaxy or by the IGM B-field.

We see evidence from radio sources for particles with energies as high as 10^{20} eV, 10^{11} times the rest energy of a proton.

Cosmic rays originally studied by particle physicists before there were accelerators. Before 1949, astrophysicists paid little attention. But in 1949 →

Fermi developed theory of particle acceleration:
Observations of cosmic rays showed clear power law spectrum

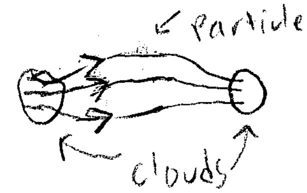


The question is how to get the power-law spectrum?

Fermi knew interstellar clouds had B-fields and thus field lines would converge inside the clouds & diverge outside the clouds thus acting as magnetic mirrors.
But the clouds are moving and this allows energy to be transferred to particles

Consider clouds moving with velocity U.
Consider the 1-D problem in which half of the clouds move in one direction with velocity U and half with $-U$.

There can be both head-on (particle and cloud with initially opposite directions of velocity) and catch-up (particle catching up to cloud; initially same direction of velocities) interactions between cloud and particle.

[

Clouds are regions of higher density material, and thus compressed field and as they move along field lines, the compress the field & produce magnetic mirror at their location]

The probability for interaction between particle and cloud is proportional to the collision frequency which is $n \sigma V_{rel}$

n: number density; σ: area of cloud; V_{rel}: relative velocity between cloud and particle

In the frame of the cloud, the cloud sees a head on interaction with a particle as having oncoming velocity $u + U$, and a catch up interaction as particle having velocity $u - U$. Thus probability for head on and catch up collisions are prop to $\frac{u+U}{u}$ and $\frac{u-U}{u}$ respectively, with $u \gg U$ →

now in the cloud frame, the particle incoming with speed $u+U$ is seen to recede with speed $u+U$ (elastic interaction) but in lab frame, the particle is seen to recede with speed $u+2U$. Thus

the energy gain for the particle is then

$$\Delta E_+ = \frac{1}{2}m(u+2U)^2 - \frac{1}{2}mu^2 = 2mU(u+U) \qquad (235)$$

similarly, the energy loss in a catch-up interaction is then

$$\Delta E_- = \frac{1}{2}mu^2 - \frac{1}{2}m(u-2U)^2 = -2mU(u-U) \qquad (236)$$

Then, recalling that the probability for head on interactions is $\propto U+u$ and catch up is proportional to $u-U$, we have for the average energy gain:

$$\Delta E_{ave} = \underbrace{\frac{1}{2}\Delta E_+ \frac{u+U}{u}}_{\text{for head-on interactions}} + \underbrace{\frac{1}{2}\Delta E_- \frac{u-U}{u}}_{\text{for catch up interactions}} = 4mU^2 \qquad (237)$$

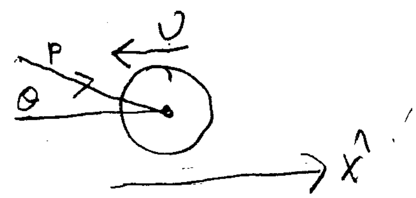
↳ this is the first step to understanding Fermi acceleration... →

Fermi Acc. (Continued)

Note however that (237) is non-relativistic. But cosmic ray spectrum is relativistic. Thus we need to revisit energy gain calc for relativistic particles, and we lose nothing by also allowing clouds to be relativistic:

Consider then the cloud frame again.

The energy of a particle in this frame is



$$E' = \gamma_v(E + \overset{\text{particle momentum}}{p}\,v\cos\theta) \qquad (238)$$

$\gamma_v = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$ is cloud Lorentz factor

the x' momentum component of the particle in the cloud frame is

$$p'_x = p'\cos\theta' = \gamma_v\left(p_x + \frac{vE}{c^2}\right) = \gamma\left(p\cos\theta + \frac{vE}{c^2}\right) \qquad (239)$$

In the interaction, the particle's energy is conserved in the cloud frame so $E'_{before} = E'_{after}$ and momentum is reversed after the collision: $p_x' \to -p_x'$

Then transforming the energy after the collision back to the lab frame, we have:

$$E_{after} = \gamma_v \big(E' - \overbrace{(-p_x')}^{p'_{after}}\, v \big) = \gamma_v (E' + p_x' v) \quad (240)$$

sign reverses for transforming back to lab frame

using (238) and (239)

$$\Rightarrow E_{after} = \gamma_v \left(\gamma_v (E + p\cos\theta\, v) + \gamma_v \left(p\cos\theta + \frac{vE}{c^2}\right) v \right)$$

$$= \gamma_v^2 \left(E + 2p\cos\theta\, v + \frac{v^2 E}{c^2} \right)$$

$$= \gamma_v^2 \left(E + E\frac{2u\cos\theta}{c^2} v + v^2 \frac{E}{c^2} \right)$$

$$= \gamma_v^2 E \left(1 + \frac{2u\cos\theta}{c^2} v + \frac{v^2}{c^2} \right) \quad (241)$$

(u = particle velocity

so that $\frac{p_x}{E} = \frac{u\cos\theta}{c^2}$)

Now expand (241) to second order in $\frac{v^2}{c^2}$ ("weakly" relativistic)

$$E_{after} = \frac{E\left(1 + \frac{2u\cos\theta}{c^2}v + \frac{v^2}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}$$

$$\simeq E\left(1 + \frac{2u\cos\theta}{c^2}v + \frac{v^2}{c^2}\right)\left(1 + \frac{v^2}{c^2}\right)$$

$$\simeq E\left(1 + \frac{2u\cos\theta}{c^2}v + \frac{v^2}{c^2}\right) + E\frac{v^2}{c^2}$$

$$\simeq E + E\left(\frac{2u\cos\theta}{c^2}v + 2\frac{v^2}{c^2}\right)$$

$$\Rightarrow \left(E_{after} - E\right) = \Delta E = E\left(\frac{2vu\cos\theta}{c^2} + 2\left(\frac{v}{c}\right)^2\right) \quad (242)$$

now we must average over θ: Here is where the extra probability of head-on interactions enters. Rather than average over θ, lets consider the 1-D problem again such that there are only two possibilities for $\cos\theta$: 1 & -1

then $\longrightarrow$

as in (237) we have

$$\Delta E_{ave} \simeq \frac{1}{2}\Delta E_+ \underbrace{\left(\frac{u+U}{u}\right)}_{\text{probability}} + \frac{1}{2}\Delta E_-\left(\frac{u-U}{u}\right)$$

(probability assuming $\frac{uU}{c^2} \ll 1$)

using 242

$$\Delta E_+ = E\left(\frac{2Uu}{c^2} + 2\left(\frac{U}{c}\right)^2\right)$$

$$\Delta E_- = E\left(-\frac{2Uu}{c^2} + 2\left(\frac{U}{c}\right)^2\right)$$

$$\Rightarrow \Delta E_{ave} \simeq E\left(\frac{Uu}{c^2} + \frac{U^2}{c^2}\right)\left(1+\frac{U}{u}\right) + E\left(-\frac{Uu}{c^2} + \frac{U^2}{c^2}\right)\left(1-\frac{U}{u}\right)$$

$$\simeq \cancel{E\frac{Uu}{c^2}} + \frac{EU^2}{c^2} + \frac{EU^2}{c^2} + \cancel{\frac{EU^3}{uc^2}} - \cancel{\frac{EUu}{c^2}} + \frac{EU^2}{c^2} + \frac{EU^2}{c^2} - \cancel{\frac{EU^3}{uc^2}}$$

$$\simeq 4E\frac{U^2}{c^2}. \qquad (243)$$

The key point here is that the average energy gain is proportional to E

→

thus $\Rightarrow$

$$\frac{\Delta E_{ave}}{E} = \frac{dE}{E} = \frac{4U^2}{c^2}.$$

Now if the mean free path between clouds along field line is L, then time between cloud particle interactions is $\simeq L/u$.
Thus typical rate of energy increase from (243) is then $\frac{\Delta E}{\Delta t} \simeq \frac{4EU^2}{c^2}\frac{u}{L}$ or

$$\frac{dE}{dt} = 4E\frac{U^2}{c^2}\frac{u}{L} \qquad (244b)$$

key simplification

for $u = c$, we then have

$$\frac{dE}{dt} \simeq 4E\frac{U^2}{cL} = \alpha E, \qquad (245)$$

where α is constant. Thus

$$E = E_0 e^{\alpha t} \qquad (246)$$

$$t = \frac{1}{\alpha}\ln\left(\frac{E}{E_0}\right) \qquad (247)$$

is the time to reach energy E by acceleration.

If t_c is the mean confinement time then the probability that confinement time is between t and $t+dt$ is (e.g. Reif or Kittel Stat mech)

$$dN = \frac{dN}{dt}(t)\,dt = \frac{\exp(-t/t_c)}{t_c}\,dt \qquad (248)$$

Analagous to escape of photons: remember probability for photon escape is given by $e^{-\tau}$ (from AST 461) so that probability for confinement is given by $1-e^{-\tau}$.

Then $dN(\tau) \simeq e^{-\tau}\,d\tau$ so that

$$\frac{dN}{d\tau} = e^{-\tau} \quad \text{as in (248)}$$

Now using (245) and (247) in (248)

$$\Rightarrow \frac{dN}{dt}dt = \frac{dN}{dE}dE = \frac{\exp\left(-\frac{1}{2}\frac{\ln E/E_0}{t_c}\right)}{t_c}\frac{dE}{dE} \qquad (249)$$

$$\frac{dN}{dt}\frac{dt}{dE}dE = \frac{dN(t(E))}{dt}\frac{dE}{dE}$$

So that (249) $\Rightarrow$ $\frac{dN}{dE} = \frac{\mathrm{Exp}\left(\ln\left(\frac{E}{E_0}\right)^{-1/\alpha t_c}\right)}{\alpha E t_c}$

$$\Rightarrow \frac{dN}{dE} \propto E^{-(1+\frac{1}{\alpha t_c})} = E^{-s} \qquad (250)$$

So that we have a power law distribution of particle energies, as desired for cosmic rays. <u>BUT</u> there are problems with the theory:

1. not easy to determine t_c and α can vary widely, thus not clear why $2 < s < 3$ in so many sources.
2. Note that acceleration process is <u>second order</u> in $\frac{v^2}{c^2}$ from (243). A first order process would be better, namely one for which a much more predominant number of head-on interactions occur.

<u>Shock - Fermi Acceleration</u> is a way of getting <u>first order Fermi acceleration</u> and $\rightarrow$

Solves both problems ① & ② above.
There the index s emerges to be a simple direct function of the shock compression ratio. This compression ratio is always $= 4 = \frac{\rho_2}{\rho_1}$ for strong shocks as we discussed earlier in the course, thus the power-law index s for the energy spectrum is more easily explained to have a generic value as observed in astrophysical sources (Supernovae, AGN etc).