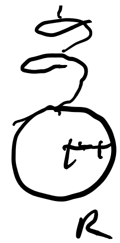
Magnetic Breaking and magneto centrifugal launch

Consider collapse of star forming region due to "Jeans Instability."

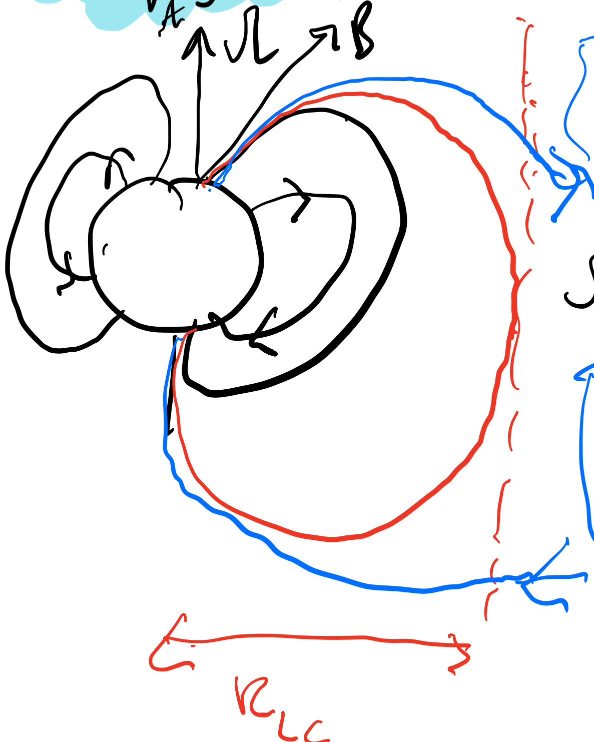
Collapse preserves angular momentum so as Ωr^2 is conserved and r decreases then Ω increases. This results in ϕ velocity of collapsing cloud being larger than ambient plasma. Strong B-field lines connecting the cloud to ambient plasma resist variation in ϕ velocity and act to "break" the rapidly spinning cloud. this is magnetic breaking

Suppose plasma up to distance $r \leq a$ is rotating at the ϕ velocity of the collapsing cloud Ω, and that the ϕ velocity at $r > a$ is much less.

$$B_{r,A}\, B_{\phi,A}\, R_A^3\, \Omega = L_{mag}(R_A)$$



$$\frac{B_*^2\, \Omega^4\, R_*^6}{V_A^3} = L_{mag}(R_*)$$: dipole rotator



$$\boxed{R_{LC}\,(R_A) \simeq \frac{c\;(V_A)}{\Omega}}$$

if $V_A \ll c$ $\Rightarrow$ $\boxed{R_A \ll R_{LC}}$

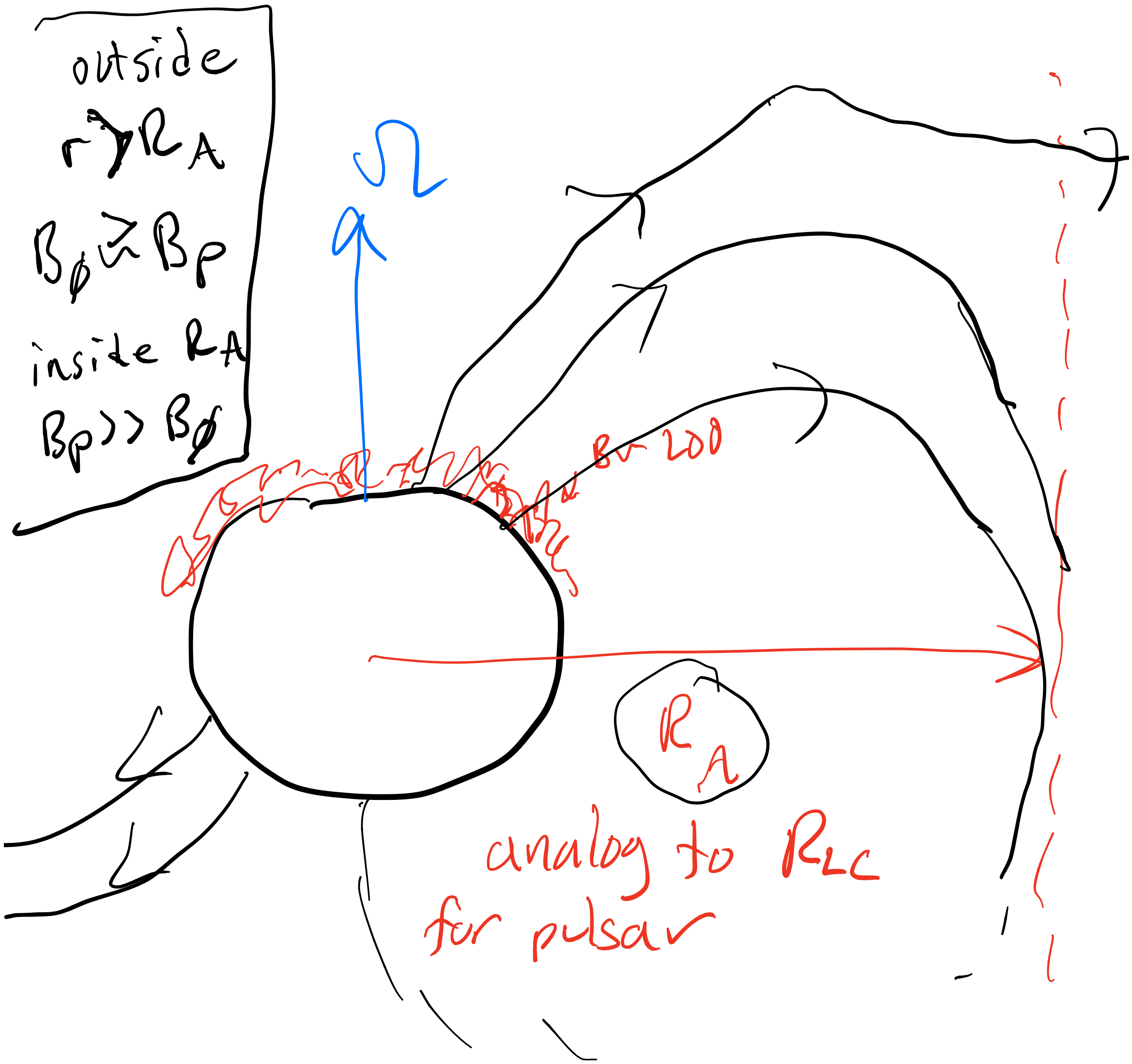
$$\Omega R_{LC} = c$$

at R_{LC} B develops $B_\phi \simeq B_p$; whereas $\forall\; r < R_{LC}$ $B_p \gg B_\phi$

at R_{LC}: $L_{mag} \simeq B_{LC}^2 R_{LC}^3\, \Omega \simeq \dfrac{B_*^2 R_*^6}{R_{LC}^3}\, \Omega$

$$B_{LC}^2 \simeq B_*^2 \left(\frac{R_*}{R_{LC}}\right)^6$$

$$\boxed{\simeq \frac{B_*^2 R_*^6\, \Omega^4}{c^3}}$$


outside
r > R_A
B_φ ≳ B_p
inside R_A
B_p >> B_φ
Ω
B ~ 200
R_A
analog to R_LC
for pulsar

The mag stresses try to spin up the plasma outside $r = a$ to the velocity Ω. Now, magnetic disturbances propagate at the Alfvén speed V_A, so in time δt, plasma between $a + \delta t V_A$ can be spun to $\measuredangle$ velocity Ω. The $\measuredangle$ momentum for this shell per unit mass is $\frac{dL}{dm} = \frac{2}{3}a^2\Omega$, for $\delta t V_A \ll a$, so the total added $\measuredangle$ momentum to the shell is

$$\boxed{\frac{B}{\sqrt{4\pi\rho}} = V_A \gamma_A} \qquad \gamma_A = \left(\frac{1}{1 - V_A^2/c^2}\right)^{1/2}$$

moment of inertia coefficient for sph. shell

$$\frac{dL}{dm}\,dm = \left(\frac{2}{3}a^2\Omega\right)\underbrace{\left(4\pi a^2 \delta t V_A \rho\right)}_{dm} = \underbrace{\frac{8}{3}\pi a^4 \Omega \rho V_A \delta t}_{dL}$$

amount of mass in time δt that gets spun up to Ω

but this gain must come from the angular momentum of the central object so:

moment of inertia coeff. for solid sphere

$$\frac{dL_{NS}}{dt} = \frac{2}{5} M a^2 \frac{\delta\Omega}{\delta t} = -\frac{8\pi}{3} a^4 \rho V_A \Omega$$

or

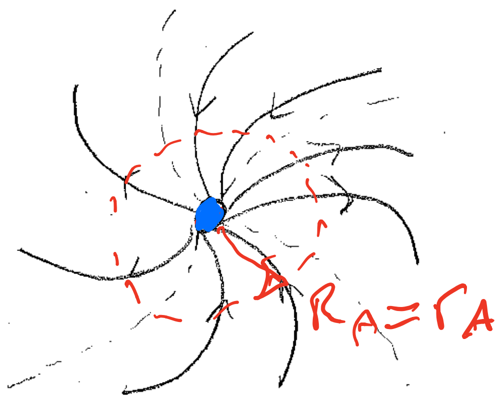
$$\boxed{\frac{d\Omega}{dt} = \frac{\rho V_A \Omega a^2}{M}\frac{20\pi}{3}} = \frac{\rho^{1/2} B \Omega a^2}{M}\frac{10\pi^{1/2}}{3}$$

using $V_A = \frac{B}{(4\pi\rho)^{1/2}}$

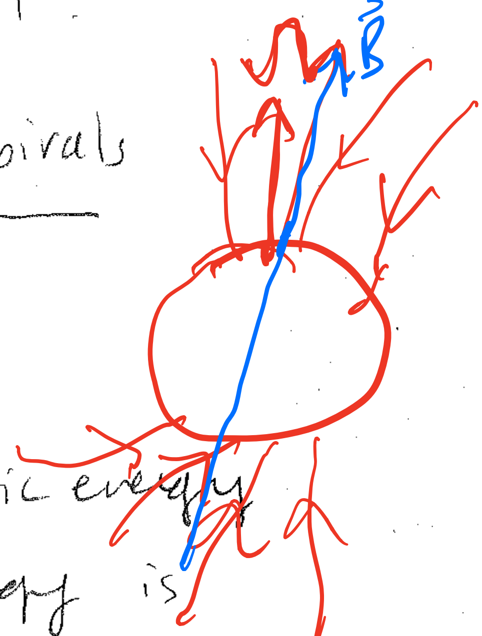
Magnetized Winds

Consider solar corona:

In the lower solar corona, the magnetic energy density dominates both the thermal and kinetic energy densities associated with the outward solar wind. Thus, we expect field to establish near rigid rotation. But far out from sun, ∡ rotation is less, so field lines get bent:



parker Spirals



The distance up to which magnetic energy remains larger than kinetic energy is called the Alfvén radius. Plasma rotates like a solid body up to this radius. Now if there were no B-field, then the amount of ∡ momentum carried away by the solar wind would be

$r_A > R_\odot$

$\boxed{\Omega_\odot R_\odot^2}$ per unit mass. But B-field makes plasma rotate at $\Omega_\odot$ out to r_A

$\Rightarrow$ B-field enables $\boxed{\Omega_\odot r_A^2}$ of $\measuredangle$ momentum per unit mass to be taken away by the solar wind. It is estimated for the sun that $r_A \approx 10 R_\odot$, thus B-field enables 100 times larger $\measuredangle$ momentum per unit mass to be extracted compared to that which is at the solar surface.

The actual mass lost from the solar wind is less significant than the $\measuredangle$ momentum lost.

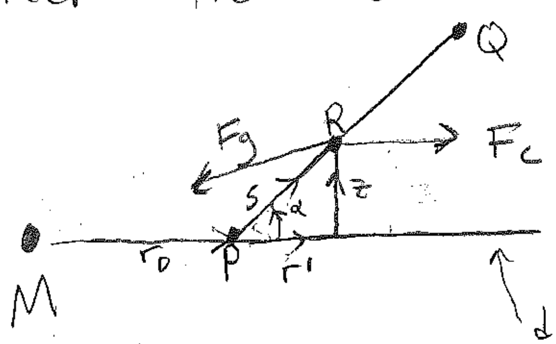
Jets

Leading model for jet launching in AGN, YSOs, microquasars, and Gamma-ray bursts is magnetic launching mechanism $\rightarrow$ magnetic fields threading an accretion disk provide the lever arm to extract $\measuredangle$ momentum and to launch flow. Basically the source of energy is the rotational energy density of the accreting material, or equivalently the accretion luminosity $\frac{GM\dot{M}}{2r} = \frac{1}{2}\dot{M}V_\phi^2 = \frac{1}{2}\dot{M}V_k^2$ (195)

keplerian speed

The idea is that this gets converted into outflow. ~~It~~ is likely that the fields themselves are also produced by a dynamo whose energy source is the turbulence driven by the shear. But one can understand the "magneto centrifugal launch mechanism" (MCL) starting from the field threading the disk as an initial condition:

Consider the figure: line $\overline{QP}$ represents field line at $\measuredangle\ \alpha$ from disk plane. F_g is gravity force. F_c is centrifugal force.



The material at P moves with Keplerian ∡ velocity $\Omega_0 = \left(\frac{GM}{r_0^3}\right)^{1/2}$.

For strong field line the plasma on field line rotates with the ∡ velocity of its anchoring point, that is, the line PQ rotates with ∡ velocity Ω_0.

Use cylindrical coords and now focus on point R.

Forces acting there are centrifugal force $F_c = \Omega_0^2 r$ directed away from the rotation axis and F_g pointed toward M.

There is no magnetic force along a field line ($(J \times B) \cdot B = 0$ and in rotating frame $V \cdot B = 0$ ← from mag. force)

Then if the resultant of F_c and F_g has a component along RQ, then plasma can be launched along $\vec{B}$. To find the force, write down gravitational potential:

$$\phi(r, z) = \underbrace{-\frac{GM}{r_0}\left[\frac{1}{2}\left(\frac{r}{r_0}\right)^2\right.}_{-\frac{1}{2}\Omega_0^2 r^2 \text{ = centrifugal potential}} + \underbrace{\left.\frac{r_0}{(r^2+z^2)^{1/2}}\right]}_{\text{gravity potential term}}$$

→

(147)

now let $r = r_0 + r'$ and expand assuming

$|r'|, |z| < |r_0|$. keeping terms of order

$(r_0 + r')$

$\frac{z^2}{r_0^2}$ & $\frac{r'^2}{r_0^2}$:

$$\phi(r,z) = -\frac{GM}{r_0}\left(\frac{1}{2r_0^2}(r'^2 + 2r'r_0 + r_0^2) + \frac{r_0}{(r^2+z^2)^{1/2}}\right)$$

$$= -\frac{GM}{r_0}\left(\frac{r'^2}{2r_0^2} + \frac{r'}{r} + \frac{1}{2} + \frac{r_0}{(r_0^2+z^2)^{1/2}} - \frac{r_0^2 r'}{(r_0^2+z^2)^{3/2}}\right.$$

$$\left. - \frac{r_0 r'^2}{(r_0^2+z^2)^{3/2}} + \frac{3r_0^3(r'^2/2)}{(r_0^2+z^2)^{5/2}}\right) \quad (*)$$

[where I have used $r_0 \frac{d}{dr}\left[\frac{-r}{(r^2+z^2)^{3/2}}\right] = \frac{-r_0}{(r^2+z^2)^{3/2}} + \frac{3r^2 r_0}{(r^2+z^2)^{5/2}}$

Now use also $\frac{r_0}{(r_0^2+z^2)^{1/2}} \cong 1 - \frac{1}{2}\frac{z^2}{r_0^2}$, and then (*) gives

$$(*) \cong -\frac{GM}{r_0}\left(\frac{r'^2}{2r_0^2} + \cancel{\frac{r'}{r_0}} + \frac{1}{2} - \cancel{\frac{r'}{r_0}} - \frac{r'^2}{r_0^2} + \frac{3r'^2}{r_0^2} + 1 - \frac{z^2}{2r_0^2}\right)$$

$$\cong -\frac{GM}{r_0}\left(\frac{3}{2} - \frac{1}{2}\frac{z^2}{r_0^2} + \frac{3}{2}\frac{r'^2}{r_0^2}\right)$$

Now let s be distance measured from P along field line so $r' = s\cos\alpha$, $z = s\sin\alpha$

then $$\phi = -\frac{GM}{2r_0^3}\left(3r_0^2 - s^2\sin^2\alpha + 3s^2\cos^2\alpha\right)$$

so to get force along field line we take the negative derivative along s — since $\vec{F}_s = -\nabla_s\phi$. Then, when $\vec{F}_s > 0$ we have an outward force:

$$-\frac{\partial\phi}{\partial s} = \frac{GM}{r_0^3}\left(-s\sin^2\alpha + 3s\cos^2\alpha\right)$$

$$= -\frac{GMs}{r_0^3}\left(\sin^2\alpha - 3\cos^2\alpha\right)$$

which is > 0 when

$$3\cos^2\alpha > \sin^2\alpha$$

or $\tan^2\alpha < 3$ or

$$\boxed{\alpha < 60^\circ}$$

Result a f— (Blandford & Payne '82)

thus this is a magnetocentrifugal launch mechanism & would remove ∡ momentum much like the solar wind. Rigid rotation would occur out to some Alfvén distance, after which mag energy falls below kinetic energy.

Beyond Alfvén distance, the field lines rotate with $\Omega < \Omega_0$, and field lines get twisted, this produces toroidal pinch which can' collimate the flow.

There are many subtleties, but this paper Blandford & Payne '82 has had a big influence on jet launching & collimation physics. <u>One key subtlety though is</u> generating the field that gives the outflow. Though usually assumed to be accreted with the flow this is probably <u>wrong</u>: →

within a turbulent accretion disk, the field actually diffuses faster than it accretes!

Take induction equation

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \lambda \nabla^2 B$$

for mean large scale field:

$$\frac{\partial \bar{B}}{\partial t} = \nabla \times \langle v \times b \rangle + \nabla \times \bar{V} \times \bar{B} + \underbrace{\cancel{\lambda \nabla^2 \bar{B}}}_{\text{ignore}}$$

assume $\bar{V} = \bar{V}_r$, $\bar{B} = \bar{B}_z$,

$$\frac{\partial \bar{B}_z}{\partial t} = \underbrace{\nabla \times \langle v \times b \rangle}_{\beta \nabla^2 \bar{B}_z} + \underbrace{- \bar{V}_r \partial_r \bar{B}_z - \bar{B}_z \partial_r \bar{V}_r}_{\mathcal{O}\left(\frac{V_r}{r} B_z\right)} \quad \text{I}$$

without any helicity this term acts as turbulent diffusion.

But $\beta \simeq \alpha_{ss} c_s H$ (in Shakura-Sunayev disks)

so $\beta \nabla^2 \bar{B}_z \simeq \frac{\beta}{H^2} \bar{B}_z \simeq \mathcal{O}\left(\alpha_{ss} \frac{c_s}{H} B_z\right)$ II

→

comparing I & II

$$\Rightarrow \frac{I}{II} = \frac{V_r H}{\alpha_{ss} c_s R}$$

but in acc disk theory, $V_r = \alpha_{ss} \frac{H}{R} c_s$

$$\Rightarrow \frac{I}{II} = \frac{H^2}{R^2},$$ so the field diffusion wins over the field advection by factor $\frac{R^2}{H^2}$!

$\Rightarrow$ this means large scale field must be generated in-situ by a dynamo!