

**Measurement of  $F_2$  and  $\sigma_L/\sigma_T$  on Nuclear-Targets  
in the Nucleon Resonance Region – PR 03-110  
First Stage of a Program to Investigate  
Quark Hadron Duality in Electron and Neutrino Scattering  
on Nucleons and Nuclei**

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*Focus on the Physics Motivation, see proposal for a Detailed Run Plan*

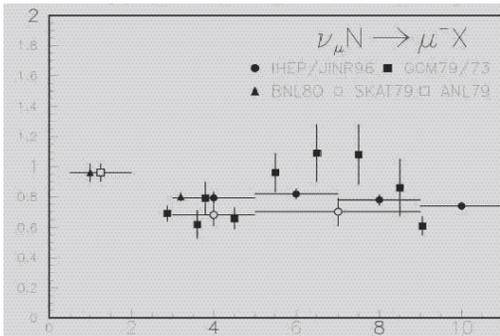
## Neutrino cross sections at low energy

- Many dedicated neutrino oscillation experiments (K2K, MINOS, CNGS, MiniBooNE, and JHF) are in the few GeV region.
- ✓ **Neutrino cross section models at low energy are *crucial* for precise next generation neutrino oscillation experiments.**
- The high energy region of neutrino-nucleon scatterings (30-300 GeV) is well understood at the few percent level in terms of the quark-parton model (PDFs) constrained by data from a series of  $e/\mu/$  DIS and collider experiments. In addition, nuclear effects have been measured at high  $Q^2$ .
- *However*, neutrino cross sections in the low energy region are poorly understood. ( especially the **resonance and low  $Q^2$  DIS** contributions). Aim to know them to the 2 % level.
- **\* *Renewed Interest of the High Energy Physics community in joining the Medium Energy community in understanding QCD/ Nucleon/ Nuclear Structure at Low Energies.***

# $\nu_\mu$ Charged Current Process is of Interest

Charged - Current: both differential cross sections and final states

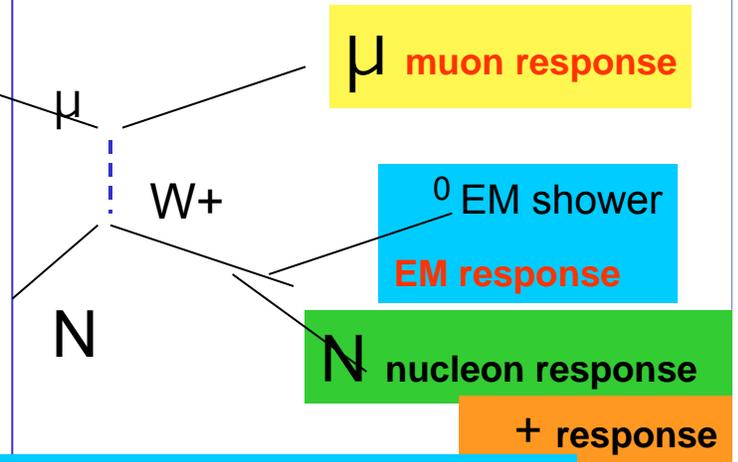
- **Neutrino mass  $\Delta M^2$  and Mixing Angle:** charged current cross sections and final States are **needed:** **The level of neutrino charged current cross sections versus energy provide the baseline against which one measures  $\Delta M^2$  at the oscillation maximum and mixing angles (aim to study CP viol.)**



$T/E_\nu$   
Poor neutrino data

$E_\nu$  Low energy current flux errors 10% to 20%

- **Measurement of the neutrino energy in a detector depends on the composition of the final states (different response to charged and neutral pions, muons and final state protons (e.g. Cerenkov threshold, non compensating calorimeters etc)).**



PR 03-110 helps pin down cross sections -aim 2% and Study CP Violation

# $\nu_\mu$ Neutral Current Process is of Interest

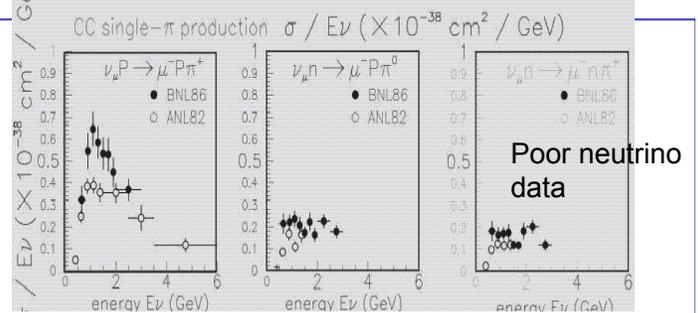
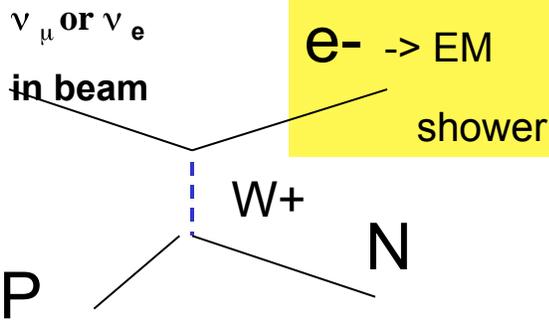
Neutral - Current both differential cross sections and final states

$E_\nu$  Low energy current flux errors 10% to 20%

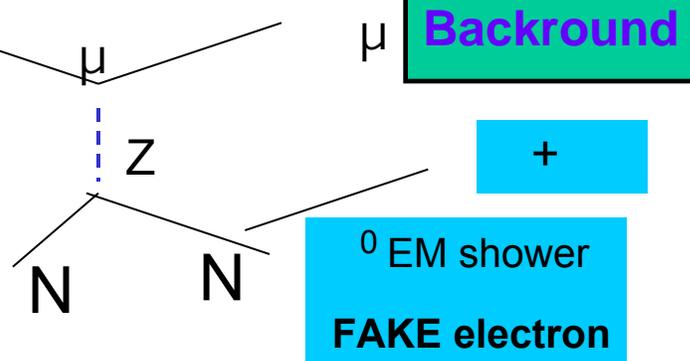
What do muon neutrinos oscillate to?

• **SIGNAL**  $\nu_\mu \rightarrow \nu_e$

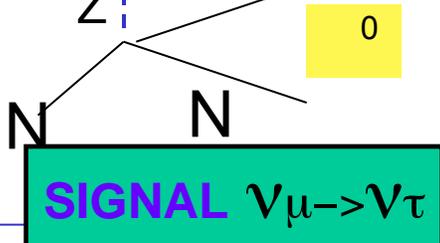
transition ~ 0.1% oscillations probability of  $\nu_\mu \rightarrow \nu_e$ .



• **Background:** Electrons from misidentified  $\pi^0$  in NC events without a muon from higher energy neutrinos are a background



Can observe  $\nu_\tau$  events below  $\tau$  threshold Vs. sterile  $\nu$



PR 03-110 pins down cross sections -aim 2%-Study CP Violation

## Currently - Low Energy Neutrino Data worse than where electron scattering was in the 1960's

- In the 1960's: Electron scattering data was poor. We measured the momentum sum rule, but we *never thought* that we will investigate the  $Q^2$  dependence of many QCD sum rules (logarithmically varying with  $Q^2$ ). A few examples include.
- (1) The Bjorken Sum rule in Polarized lepton scattering
- (2) The Gross-Llewellyn-Smith Sum (GLS) sum rule in neutrino scattering
- (3) The Gottfried Sum Rule (proton-neutron) in electron/muon DIS scattering

In 2002:

- (1)  $Q^2$  dependence of Bjorken and GLS rules has been used to extract  $\alpha_s(Q^2)$
- (2) Gottfried Sum is used to extract  $(\bar{d}-\bar{u})$

*In a few years, next generation neutrino beams will have fluxes known to 2%. Aim at testing current-algebra (exact sum rules) like the Adler Sum rule. However, input from electron scattering experiments is crucial.*

**Motivation of next generation neutrino experiments is neutrino oscillations.**

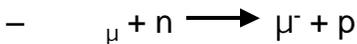
**Need these cross sections to 2 % to get precise neutrino mixing angles**

## Motivation, the Short Story

- Similar to electron scattering experiments needing good models of the cross sections at all  $Q^2$  to do radiative corrections, neutrino experiments need good models of cross sections and final states to extract cross sections
- However, neutrino Monte Carlo models *must* be based on understanding of the physics, and checked by data
- A collaborative program between the high and medium energy communities to develop reliable global models linking electron and neutrino scattering measurements covering a wide range of kinematics
- 
- Nuclear data necessary for comparison with neutrino measurements for global modeling efforts
- No L/T separated structure function measurements exist on nuclei in the resonance region
- In the resonance region, nuclear effects may be large, different from the DIS region, and  $Q^2$  dependent.
- *Will reduce large, model-dependent uncertainties in neutrino oscillation measurements - Of interest to the neutrino oscillations community*
- *Further tests of duality, QCD, and Current Algebra sum rules.*
- *----> Of interest to the medium energy physics community*

# Motivation, the Long Story: Neutrino Cross Sections at Low Energy

## □ Quasi-Elastic / Elastic ( $W=M$ )



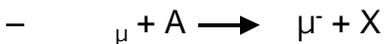
- Input from both electron and neutrino experiments and described by form factors, need axial form factor and nuclear corrections

## □ Resonance (low $Q^2$ , $W < 2$ )



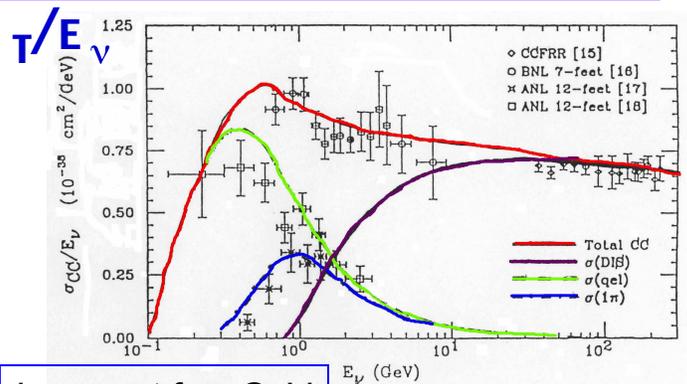
- Can be well measured in electron scattering, but poorly measured in neutrino scattering (fits by Rein and Seghal). Need R, axial form factors and nuclear corrections

## □ Deep Inelastic (DIS)



- well measured in high energy experiments and well described by quark-parton model, *but doesn't work well at low  $Q^2$* . Need low  $Q^2$  structure functions, R, axial structure funct. and nuclear corrections

Arie Bodek, Univ. of



## Issues at few GeV

- Resonance scattering and low  $Q^2$  DIS contribution meet, (How to avoid double counting?).
- Challenge: to describe all these three processes at all neutrino (and electron/muon) energies. See if model satisfies all known sum rules from  $Q^2=0$  to very high  $Q^2$
- (Need to understand duality, QCD, low  $Q^2$  sum rules, transition between DIS and resonance)

Start with: Quasielastic: C.H. Llewellyn Smith (SLAC).Phys.Rept.3:261,1972

$$\frac{d\sigma^{\nu, \bar{\nu}}}{dq^2} = \frac{M^2 G_F^2 \cos^2 \theta_c}{8\pi E_p^2} \left[ A(q^2) \mp \frac{(s-u)B(q^2)}{M^2} + \frac{C(q^2)(s-u)^2}{M^4} \right]$$

Updated recently  
By Bodek, Budd and  
Arrington 2003

$$A(q^2) = \frac{m^2}{4M^2} \left[ \left(4 - \frac{q^2}{M^2}\right) |F_A|^2 - \left(4 + \frac{q^2}{M^2}\right) |F_V^1|^2 - \frac{q^2}{M^2} |\xi F_V^2|^2 \left(1 + \frac{q^2}{4M^2}\right) - \frac{4q^2 \text{Re} F_V^1 \xi F_V^2}{M^2} \right]$$

Axial

$$B(q^2) = \frac{q^2}{M^2} \text{Re} F_A^* (F_V^1 + \xi F_V^2)$$

Vector

$$C = \frac{1}{4} \left( |F_A|^2 + |F_V^1|^2 - \frac{q^2}{M^2} \left| \frac{\xi F_V^2}{2} \right|^2 \right)$$

We have not shown terms in  $(m_l/M)^2$ , and  $F_F(q^2)$  is multiplied by  $(m_l/M)^2$ . (Note,  $F_F(q^2)$  is included in the calculations.) The formulas for  $F_V^1(q^2)$  and  $\xi F_V^2(q^2)$  are

$$F_V^1(q^2) = \frac{G_E^V(q^2) - \frac{q^2}{4M^2} G_M^V(q^2)}{1 - \frac{q^2}{4M^2}}, \quad \xi F_V^2(q^2) = \frac{G_M^V(q^2) - G_E^V(q^2)}{1 - q^2/4M^2}$$

Vector form factors  
From electron  
scattering  
Via CVC

We use the CVC to determine  $G_E^V(q^2)$  and  $G_M^V(q^2)$  from the electron scattering form factors  $G_E^p(q^2)$ ,  $G_E^n(q^2)$ ,  $G_M^p(q^2)$ , and  $G_M^n(q^2)$ .

$$G_E^V(q^2) = G_E^p(q^2) - G_E^n(q^2), \quad G_M^V(q^2) = G_M^p(q^2) - G_M^n(q^2)$$

Vector

Many of the neutrino experiment have assumed the form factors are the dipole approximation,

$$G_D(q^2) = \frac{1}{(1 - q^2/M_V^2)^2}, \quad M_V^2 = 0.71 \text{ GeV}^2$$

$$G_E^p = G_D(q^2), \quad G_E^n = 0, \quad G_M^p = \mu_p G_D(q^2), \quad G_M^n = \mu_n G_D(q^2)$$

Neutrino experiments use  
Dipole form factors with  
Gen=0 -Because this is  
what was put in the LS  
paper (not exactly  
correct)

The axial form factor is given by

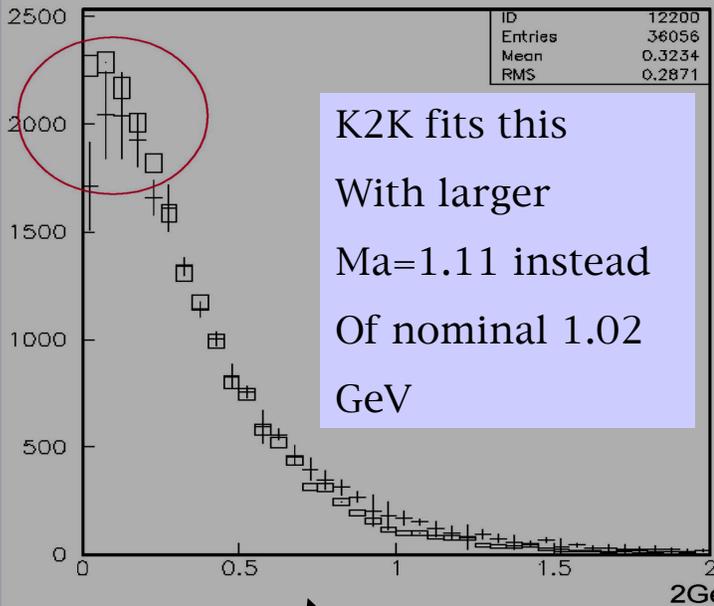
$$F_A(q^2) = \frac{g_A}{(1 - \frac{q^2}{M_A^2})^2}$$

Axial form factor from  
Neutrino experiments

NuInt02: Example- systematic errors that happen when one is not familiar with the latest input from electron scattering.

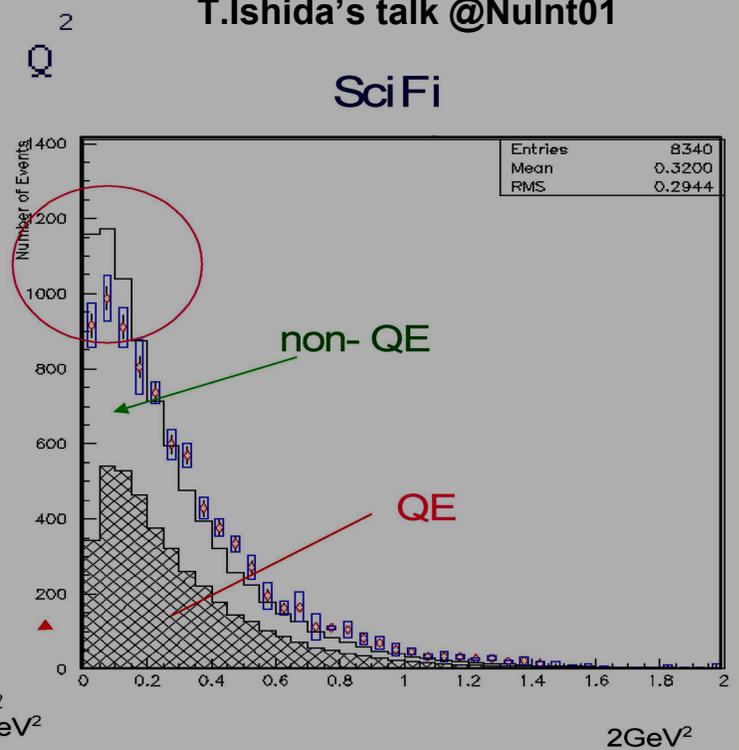
From Ito NuInt02

1kt



T.Ishida's talk @NuInt01

SciFi



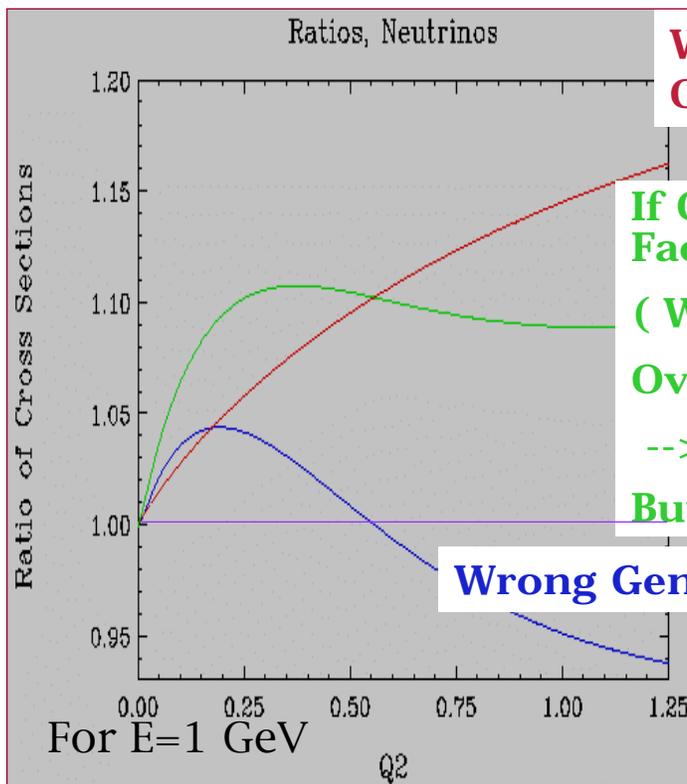
- \* Errors shown here is an energy scale error ( $\pm 5\%$ )
- \* Nuclear binding energy is not taken into account..

- \* Errors shown here is a typical energy scale error ( $\pm 3\%$ ).
- \* Nuclear binding energy  $B = -30\text{MeV}$  (for Oxygen) is taken into account.

K2K experiment thought this was a nuclear effect on  $M_A$

But the true reason - as we is that the Neutrino Community was using Outdated Dipole Form Factors

Effect is really Low  $Q^2$  suppression from non Zero Gen



**Wrong  $M_A=1.1$  (used by K2K)  
Over  $M_A=1.02$  (Ratio)**

**If One Uses Both wrong Form  
Factors (used in K2K MC)  
( Wrong Gen =0 +Wrong  $M_A=1.1$ )  
Over Best Form Factors (Ratio)**

**--> Get right shape**

**But wrong normalization of 10%**

**Wrong Gen / Best Form Factors (Ratio)**

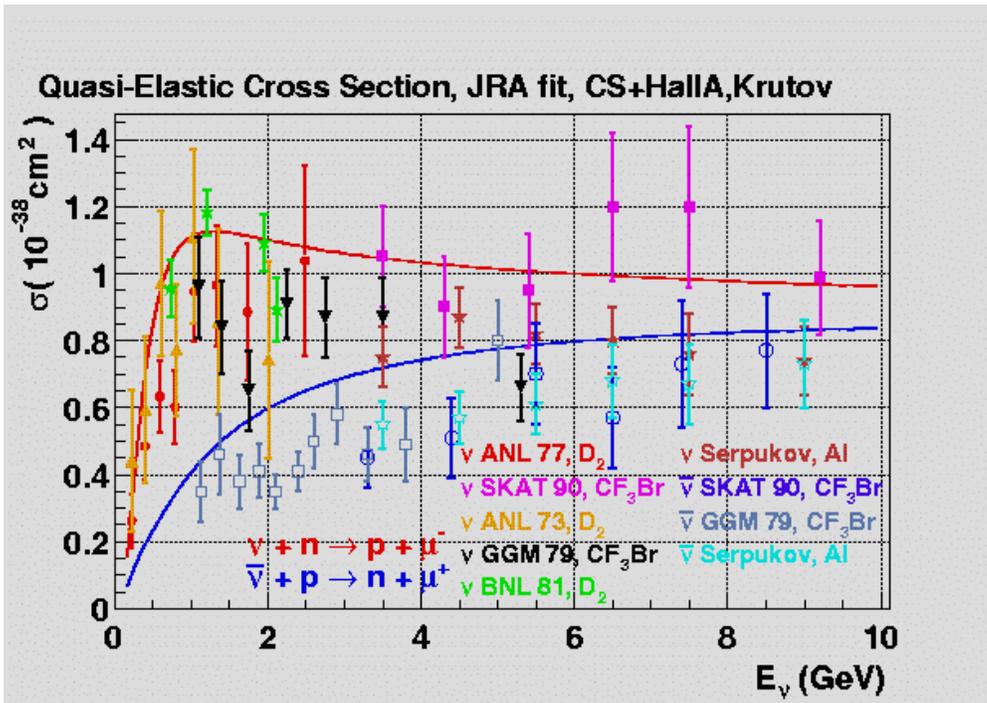
Can fix the  $Q^2$  dependence either way (by changing  $M_A$  or using correct vector form factors). However the overall **cross sections will be 10-15% too high if one chooses wrong**

quasi-elastic neutrinos on Neutrons- ( - Calculated)

quasi-elastic Antineutrinos on Protons - Calculated

From H. Budd -U of Rochester (NuInt02) (with Bodek and Arrington) DATA - FLUX ERRORS ARE 10% to 20%

Even with the most Up to date Form Factors The agreement With data is *not spectacular*



Data mostly on nuclear targets are lower - *Nuclear Effects are important - Next work on nuclear corrections and chose nuclear models that describe electron quasielastic scattering*

Next Generation Neutrino Experiments Need this to 2%

## Next - Resonance Models

e.g. **Current Matrix Elements from a Relativistic Quark Model** - *Phys. Rev. D* 3, 2706–2732(1971) R. P. Feynman, M. Kislinger, and F. Ravndal

Lauritsen Laboratory of Physics, California Institute of Technology, Pasadena, California 91109

Received 17 December 1970      referred to as the *FKR Model*

### Abstract

**A relativistic equation to represent the symmetric quark model of hadrons with harmonic interaction** is used to define and **calculate matrix elements of vector and axial-vector currents**. **Elements between states with large mass differences are too big compared to experiment, so a factor whose functional form involves one arbitrary constant is introduced to compensate this**. The vector elements are compared with experiments on photoelectric meson production,  $Kl3$  decay, and  $\omega \rightarrow \pi \gamma$ . Pseudoscalar-meson decay widths of hadrons are calculated supposing the amplitude is proportional (with one new scale constant) to the divergence of the axial-vector current matrix elements. Starting only from these two constants, the slope of the Regge trajectories, and the masses of the particles, **75 matrix elements are calculated, of which more than 3 / 4 agree with the experimental values within 40%**. **The problems of extending this calculational scheme to a viable physical theory are discussed.**

**Improvements on parameters within this Resonance Model:**

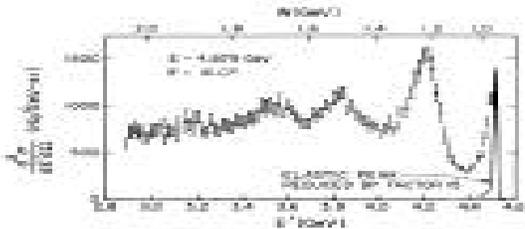
**D. Rein and L. M. Sehgal, *Annals Phys.* 133, 79 (1981) ;D. Rein, *Z. Phys. C.* 35, 43 (1987)**

**These are coded in MC generators - but there are also other proposed recently.**

## Resonance Model applied to Photo-production Electroproduction/Neutrino production

**Photoproduction: FKR: Kneis, Moorhouse, Oberlack, Phys. Rev. D9, 2680 (1974)**

**Electroproduction: FKR: F. Ravndal, Phys. Rev. D4, 1466 (1971)**



$$\sigma_T(Q^2) = \frac{4\pi^2\alpha}{K^2} \frac{1}{2} (L_T + P + L_T - P) \frac{\Gamma/2\pi}{(W - M)^2 + \Gamma^2/4} \quad (18)$$

$$\sigma_L(Q^2) = \frac{4\pi^2\alpha}{K^2} \left( \frac{-Q^2}{Q^2 + M^2} \right) L_T + P \frac{\Gamma/2\pi}{(W - M)^2 + \Gamma^2/4}$$

Harry Lee from Argonne has offered to work with Us on modeling of resonance electro-production and neutrino-production. He has done work on the Delta region: **Electroproduction:** Phys. Rev. C63.-55201 (2001) **Neutrino productions :** nucl-th/0303050 (2003)

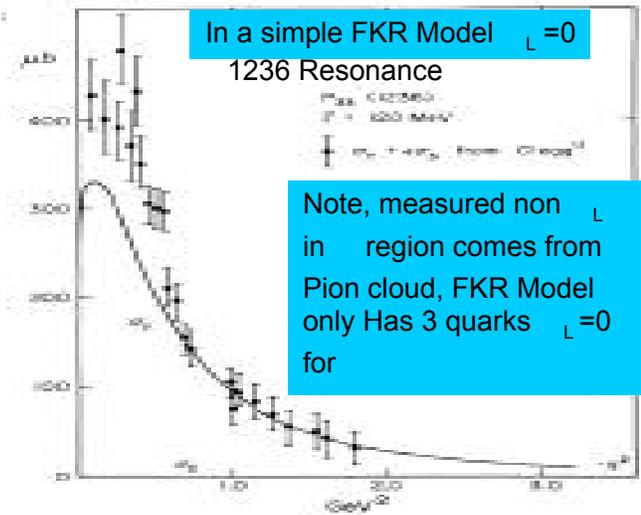
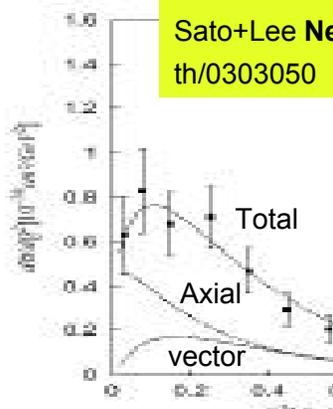


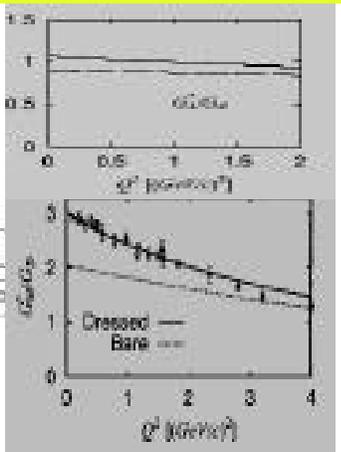
FIG. 3. Resonance cross section at  $W = 1236 \text{ MeV}$  with proton target. Data from Ref. 12.

IV. RESULTS AND DISCUSSION  
**Electroproduction** Region



Sato+Lee **Neutrino** Region nucl-th/0303050 **More sophisticated**

**Neutrino production** Region



# FKR Resonance Model applied to Electroproduction

Photoproduction: Kneis, Moorhouse, Oberlack, Phys. Rev. D9, 2680 (1974)

Electroproduction: F. Ravndal, Phys. Rev. D4, 1466 (1971)

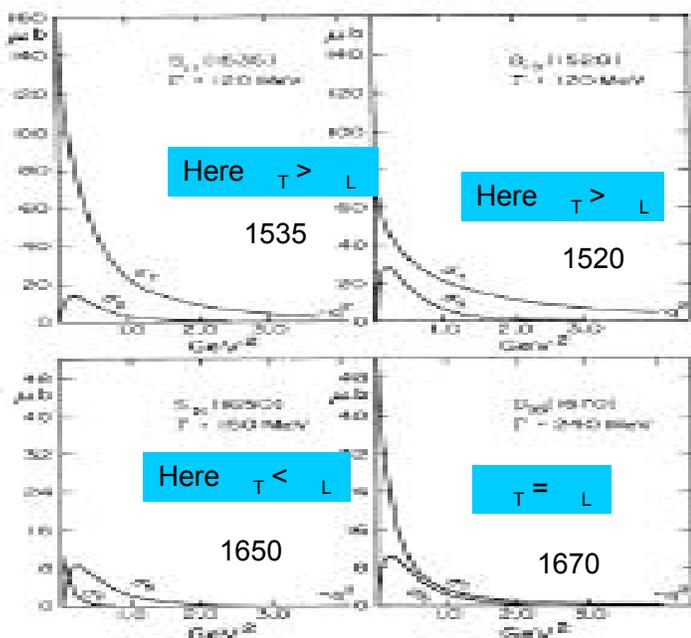


FIG. 4. Transverse ( $\sigma_T$ ) and scalar ( $\sigma_S$ ) cross sections for the  $S_{11}(1535)$ ,  $D_{13}(1520)$ ,  $S_{11}(1650)$ , and  $D_{13}(1670)$  resonances with proton target.

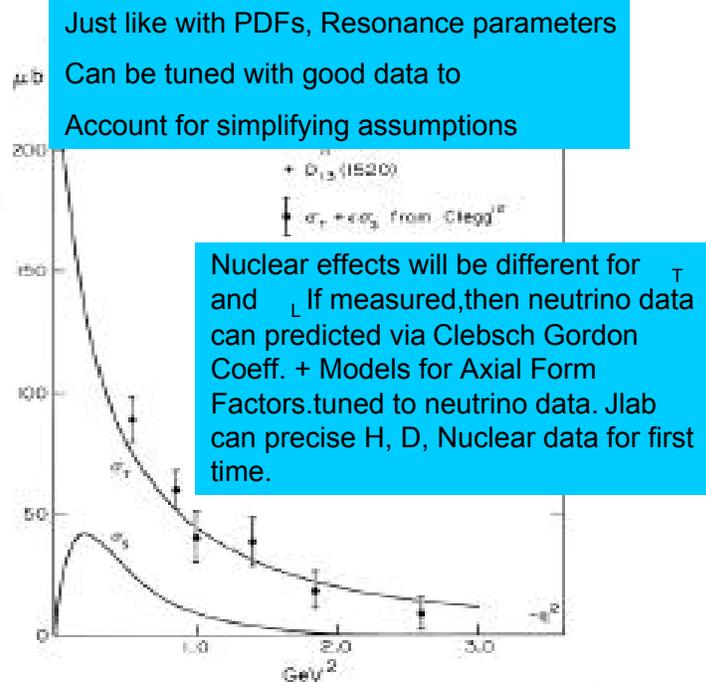
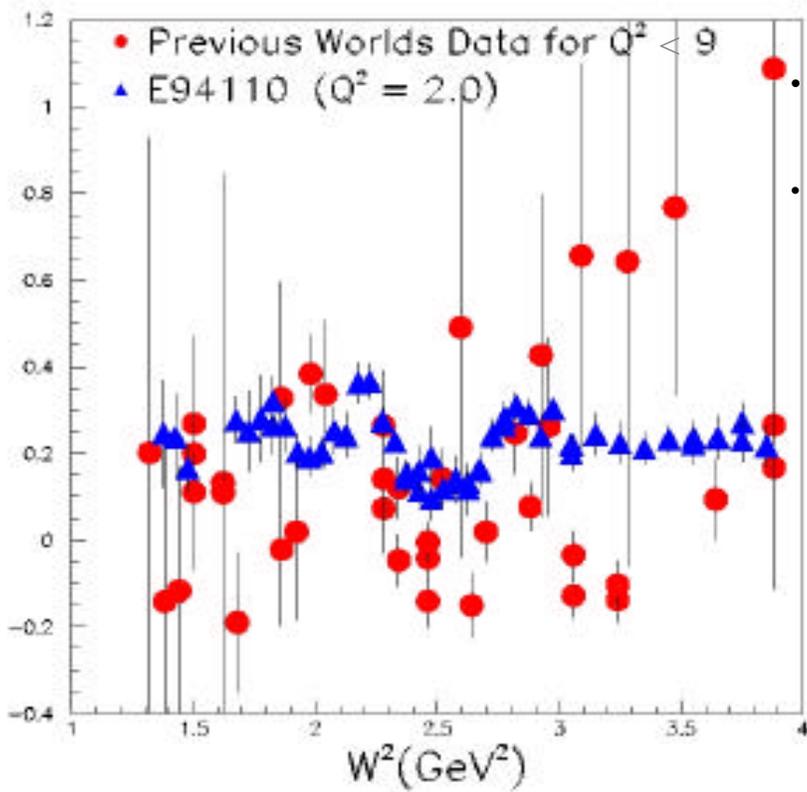


FIG. 4. Total transverse ( $\sigma_T$ ) and scalar ( $\sigma_S$ ) cross sections at the second resonance peak  $W = 1525 \text{ MeV}$  with proton target. Data from Ref. 12.

Compare to what one has done for Hydrogen in E94-110 F2, FL, F1?

$$R = \frac{L}{T}$$

- Now able to study the  $Q^2$  dependence of individual resonance regions for F2, FL, F1!
- Clear resonant behaviour can be observed!



Now able to extract  $F_2$ ,  $F_1$ ,  $F_L$  and study duality! with high precision .

# Correct for Nuclear Effects measured in e/ $\mu$ expt.

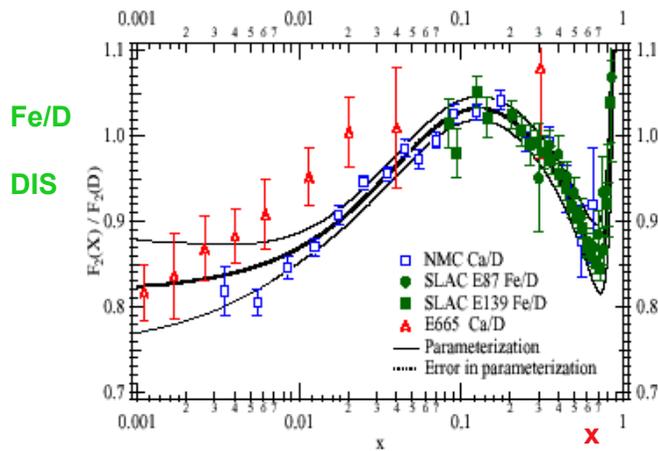
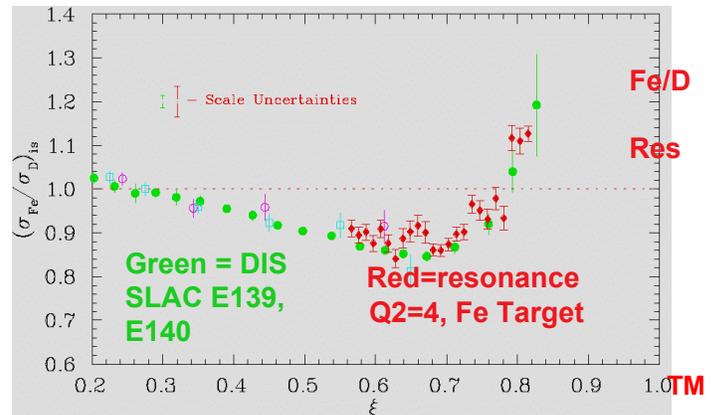


Figure 5. The ratio of  $F_2$  data for heavy nuclear targets and deuterium as measured in charged lepton scattering experiments (SLAC, NMC, E665). The band shows the uncertainty of the parameterized curve from the statistical and systematic errors in the experimental data [16].

$$TM = [Q^2] / [Mv (1+(1+Q^2/v^2)^{1/2})]$$



Comparison of Fe/D  $F_2$  data in resonance region (JLAB) versus DIS SLAC/NMC data in  $TM$  (However, what happens at low  $Q^2$ ? Is it versus  $\xi_w$  or other scaling variable. What happens when  $R$  is large at low  $Q^2$  in the resonance region?)

From SLAC E87, E139, E140, and Muon Scattering

(People involved in E139, E140 Bodek, Rock, Bosted are also in E03-110...)

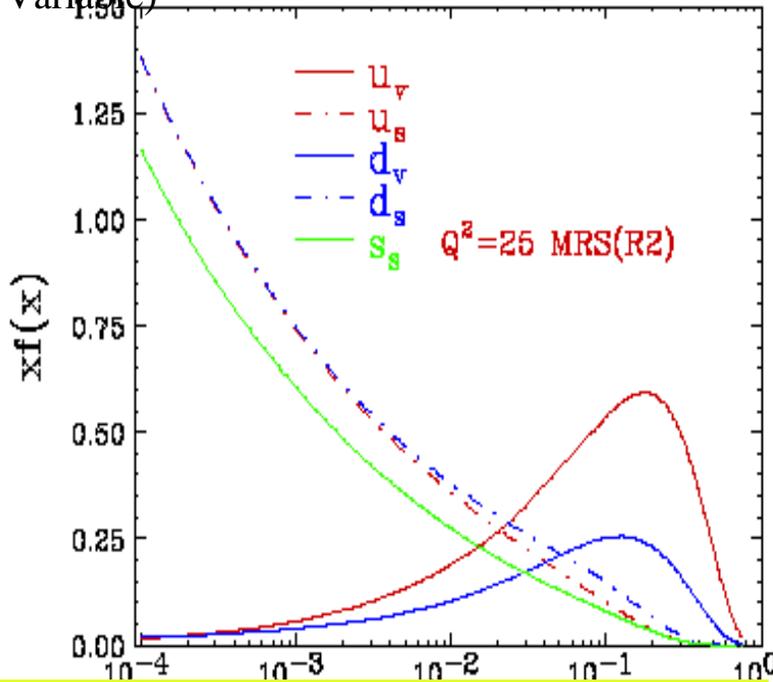
$$w = [Q^2 + B] / [Mv (1+(1+Q^2/v^2)^{1/2}) + A]$$

# How are PDFs Extracted from global fits to High Q<sup>2</sup> Deep Inelastic e/μ/ν Data

Note: additional information on Antiquarks from Drell-Yan and on

MRSR2 PDFs  $xq$  is the probability that a Parton  $q$  carries fractional momentum  $x = Q^2/2M$  in the nucleon ( $x$  is the Bjorken Variable)

Gluons from  $p$ - $p$ bar jets also used.



For data on nuclei, need nuclear Corrections. Discuss Model for DIS at all Q<sup>2</sup> later

$$u_v + d_v \text{ from } F_2^v \quad x(u + \bar{u}) + x(d + \bar{d})$$

Valence, Sea

$$\text{Strange dist. } xF_3^v \quad x(u - \bar{u}) + x(d - \bar{d})$$

$$u + \bar{u} \text{ from } {}^uF_2^p \quad \frac{4}{9} x(u + \bar{u}) + \frac{1}{9} x(d + \bar{d})$$

$$d + \bar{d} \text{ from } {}^uF_2^n \quad \frac{1}{9} x(u + \bar{u}) + \frac{4}{9} x(d + \bar{d})$$

nuclear effects

typically ignored

$${}^uF_2^n = 2 \frac{{}^uF_2^d}{{}^uF_2^p} - 1$$

$$l/u \text{ from } p\bar{p}W \text{ Asymmetry } \frac{d/u(x_1) - d/u(x_2)}{d/u(x_1) + d/u(x_2)}$$

At high  $x$ , deuteron binding effects introduce an uncertainty in the  $d$  distribution extracted from  $F_2^d$  data (but not from the  $W$  asymmetry data).  $X=Q^2/2M\nu$  Fraction momentum of quark

## Duality, QCD Sum Rules, and Current Algebra Sum Rules.

Local duality and Global duality appears to work for  $Q^2 > 1.5 \text{ GeV}^2$  in electron scattering: This is basically a consequence of the fact that if target mass effects are included, higher twists are small and **QCD sum rules are approximately true for  $Q^2 > 1.5 \text{ GeV}^2$ .**

(e.g. **momentum sum rule** - quarks carry about 1/2 of the proton momentum)  $F_2^{eP}$ ,  $F_2^{eN}$  are related to PDFs weighted by quark charges).

At high  $Q^2$ , duality also seems to work for nuclear corrections.

What happens at low  $Q^2$  ?

Adler Sum rule **EXACT** all the way down to  $Q^2=0$  includes  $W_2$  quasi-elastic  
 S. Adler, Phys. Rev. 143, 1144 (1966) Exact Sum rules from  
 Current Algebra. Sum Rule for  $W_2$  **DIS LIMIT is just  $U_v - D_v = 1$**

1.  $\beta^- = W_2$  (Anti-neutrino -Proton)
2.  $\beta^+ = W_2$  (Neutrino-Proton)  $q_0 = \nu$

The vector current part of the original sum rule of Adler for neutrino scattering can be written

$$\int_0^\infty dq_0 [\beta^{(-)}(q_0, q^2) - \beta^{(+)}(q_0, q^2)] = 1. \quad (18)$$

If we explicitly separate out the nucleon Born term in Adler is a number sum rule at **high  $Q^2$**   
 Eq. (18), we have **DIS LIMIT is just  $U_v - D_v$ .**

Elastic Vector = 1  $Q^2=0$

Elastic Vector = 0 high  $Q^2$

$$[F_1^V(q^2)]^2 + q^2 \left( \frac{\mu^V}{2M_N} \right) [F_2^V(q^2)]^2 + \int_{M_\pi + (q^2 + M_\pi^2)^{1/2}}^\infty dq_0 [\beta^{(-)}(q_0, q^2) - \beta^{(+)}(q_0, q^2)] = 1,$$

Vector Part of  $W_2$ , 0 at  $Q^2=0$ , 1 at high  $Q^2$ -  
**Inelastic**

[see Bodek and Yang hep-ex/0203009  
 and references therein

at fixed  $q^2 = Q^2$

$$g_A(q^2)^2 + \int_{M_\pi + (q^2 + M_\pi^2)^{1/2}}^\infty dq_0 [\beta^{(-)}(q_0, q^2) - \beta^{(+)}(q_0, q^2)] = 1,$$

Elastic  $g_A = (-1.267)^2$   $Q^2=0$

Elastic  $g_A = 0$  high  $Q^2$

**Axial  $W_2 = \text{non zero at } Q^2=0$**

**Axial  $W_2 = 1$  at high  $Q^2$ ,  
 Inelastic**

$$\int_0^\infty dq_0 [\beta^{(-)}(q_0, q^2) - \beta^{(+)}(q_0, q^2)] = 1 \text{ is}$$

$$\int_0^1 \frac{[F_2^-(\xi) - F_2^+(\xi)]}{\xi} d\xi = \int_0^1 [U_v(\xi) - D_v(\xi)] d\xi = 2 - 1$$

$F_2^- = F_2$  (Anti-neutrino -Proton) =  $\nu W_2$

$F_2^+ = F_2$  (Neutrino-Proton) =  $\nu W_2$

we use:  $d(q_0) = d(\nu) = (\nu) d\xi / \xi$

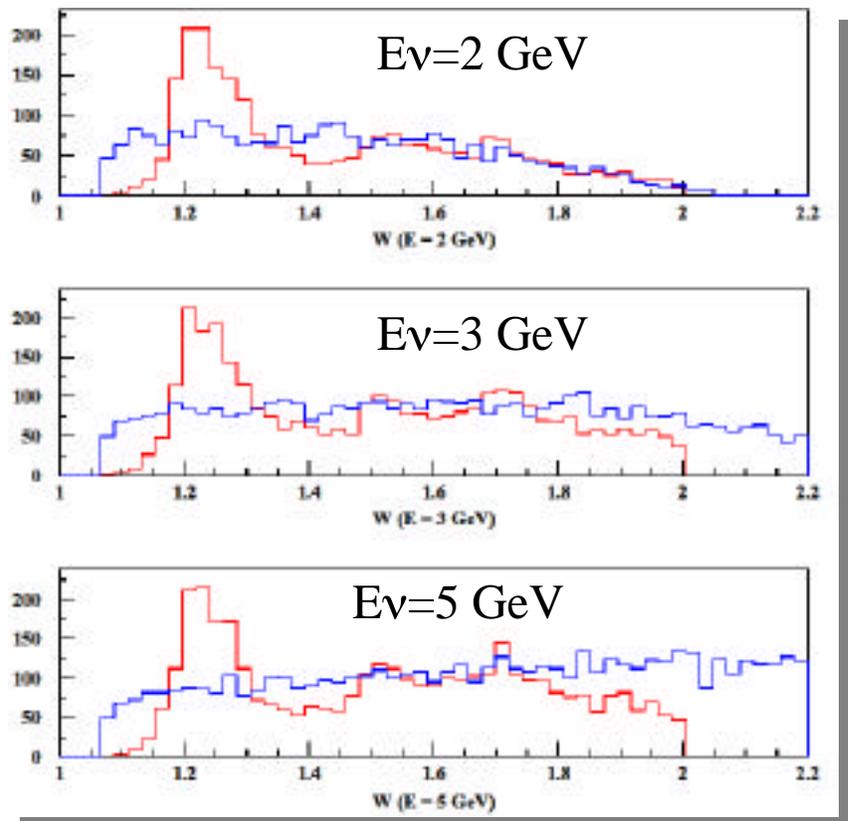
+ Similar sum rules for  $W_1$ ,  $W_3$ , and strangeness changing structure functions

From: D. Casper, UC Irvine K2K NUANCE MC 2003 **W, Final Hadronic Mass Comparison on Water -success**

----- Bodek/Yang  
modified  $\xi_w$  scaling +  
GRV98 PDFs  
2003. Model from fits to  
electron data based on  
duality and violation of  
duality at low  $Q^2$  -  
motivated by Adler sum  
rule (see backup slides)

----- D. Rein and L. M.  
Sehgal, Annals Phys.  
133, 79 (1981)  
Resonance +Non  
Resonance model

Know how to match **resonance**+continuum models



----- Bodek/Yang  
 modified  $\xi_w$   
 scaling + GRV98  
 PDFs 2003

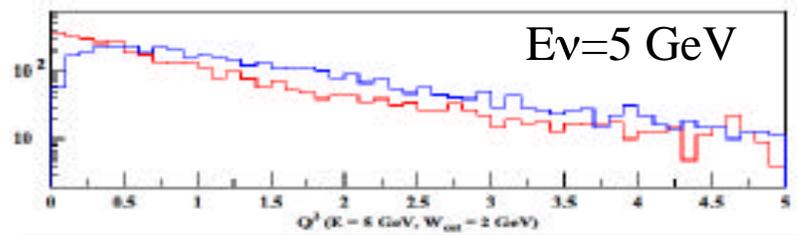
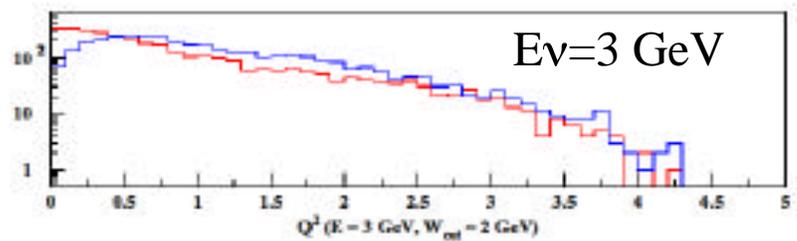
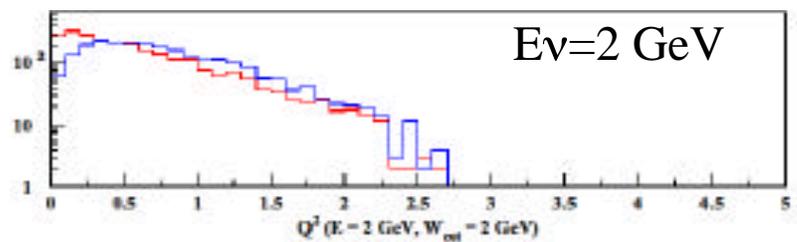
First assume  $V=A$   
 with  $V=0$  at  $Q^2=0$

----- D. Rein and L. M.  
 Sehgal, Annals Phys.  
 133, 79 (1981)  
 Resonance +Non  
 Resonance model

Vector not equal Axial  
 At Very low  $Q^2$

$$G^2_a=1.27^2 \quad G^2_v=1.0$$

## $Q^2$ Comparison on Water Needs work on Axial $W_2$



**NEED to also satisfy Alder sum rule for Axial part -deviation from  
 Duality at low  $Q^2$  different for vector and axial in resonance region.**

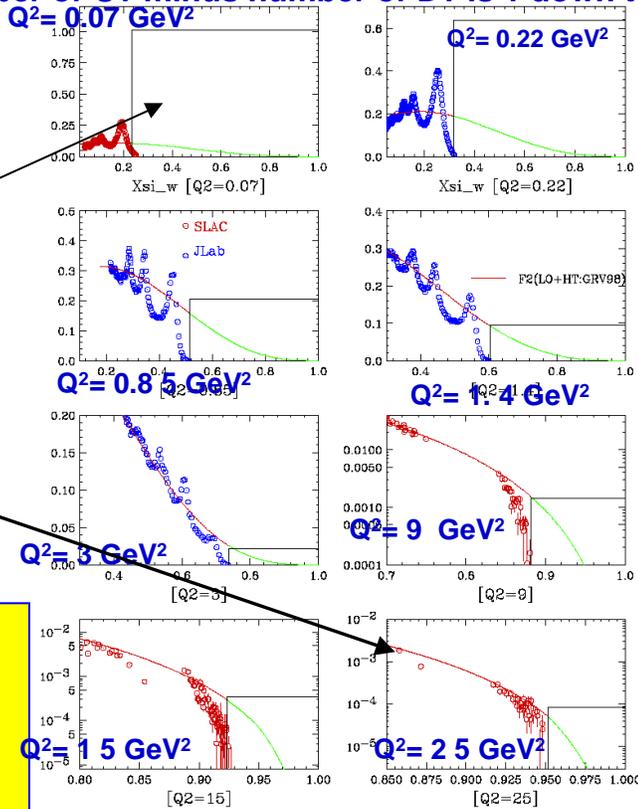
# When does duality break down

Momentum Sum Rule has QCD+non- Perturbative Corrections (breaks down at  $Q^2=0$ ) but ADLER sum rule is EXACT (number of  $U_v$  minus number of  $D_v$  is 1 down to  $Q^2=0$ ).

Elastic peak

Int F2P Elastic	Q2	Int Inelastic
1.0000000	0	0
0.7775128	0.07	
0.4340529	0.25	
0.0996406	0.85	
0.0376200	1.4	
0.0055372	3	
0.0001683	9	
0.0000271	15	
0.0000040	25	0.17

DIS high Q2  
Integral F2p



- In proton :
- QPM Integral of F2p =
- $0.17*(1/3)^2 + 0.34*(2/3)^2 = 0.17$  (In neutron=0.11)
- Where we use the fact that
- 50% carried by gluon
- 34% u and 17% d quarks

Adler sum rule (valid to  $Q^2=0$ ) is the integral  
Of the difference of  $F_2/x$  for Antineutrinos  
and Neutrinos on protons (including elastic)

# Tests of Local Duality at high x, high Q<sup>2</sup> vs. Q<sup>2</sup>=0 Electron Scattering Case

- INELASTIC High Q<sup>2</sup> x-->1.
- QCD at High Q<sup>2</sup> Note d refers to d quark in the proton, which is the same as u in the neutron. d/u=0.2; x=1.
- F<sub>2</sub>(e-P) = (4/9)u+(1/9)d = (4/9+1/45) u = (21/45) u
- F<sub>2</sub>(e-N) = (4/9)d+(1/9)u = (4/45+5/45) u = (9/45) u
- DIS LIMIT High Q<sup>2</sup>
- F<sub>2</sub>(e-N) / F<sub>2</sub>(e-P) = 9/21=0.43
- Elastic/quasielastic +resonance at high Q<sup>2</sup> dominated by magnetic form factors which have a dipole form factor times the magnetic moment
- F<sub>2</sub>(e-P) = A G<sup>2</sup><sub>MP</sub>(el) +BG<sup>2</sup><sub>MP</sub> (res c=+1)
- F<sub>2</sub>(e-N) = AG<sup>2</sup><sub>MN</sub> (el) +BG<sup>2</sup><sub>MN</sub> (res c=0)
- TAKE ELASTIC TERM ONLY
- F<sub>2</sub>(e-N) / F<sub>2</sub>(e-P) (elastic High Q<sup>2</sup>) = μ<sup>2</sup>(N) / β(P) = (1.913/2.79) = 0.47

Different at low Q<sup>2</sup>, where Gep,Gen dominate.

Close if we just take the elastic/quasielastic x=1 term.

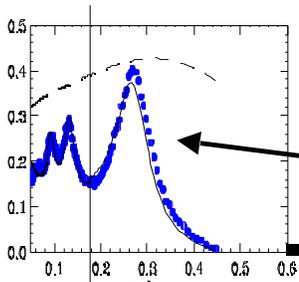
Q<sup>2</sup> = 0 ElasticLimit

Gen/Gep (Q<sup>2</sup>=0) = 0 Since Gen=0.

# NEUTRINOS

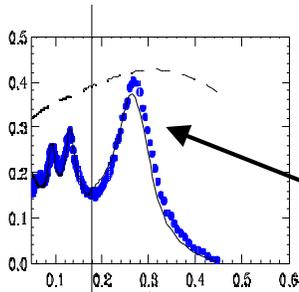
## On nucleons

On neutrons both quasielastic and resonance+DIS production possible. First resonance has different mixtures of  $I=3/2$  and  $I=1/2$  terms. Neutrino and electron induced production are related using Clebsch Gordon Coeff. (Rein Seghal model etc)



1st reson

$X = 1$   
quasielastic



1st reson

$X = 1$   
zero

NEUTRINOS  
On Neutrons

$N=ud$  d

$ud$  u = (P or +)  
Both  
quasi+Res

NEUTRINOS  
On Protons

$P=uu$  d

$uuu$  = ( ++ Res only state)

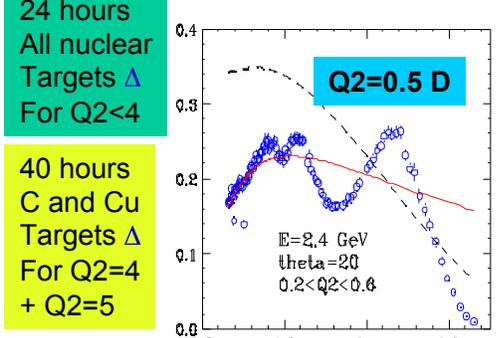
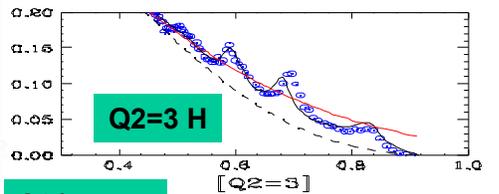
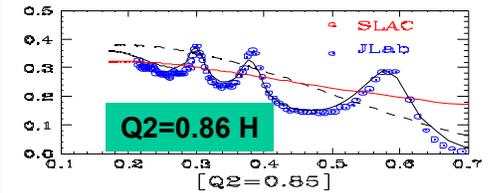
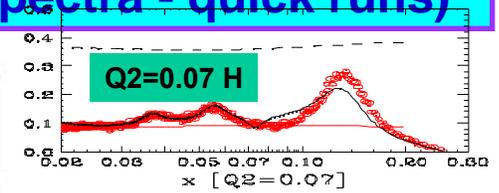
On protons only resonance+  
DIS production possible.

Local Duality at  $x=1$  limit breaks down at all  $Q^2$ , What if we include higher resonances?

And Reverse Case for antineutrinos

# E03-110 Run Plan $\Delta$ Region Part Only - Match E02-109 (and ask to add one or two lower $Q^2$ spectra - quick runs)

$Q^2_{\Delta}$ (GeV/c) <sup>2</sup>	$E$ (GeV)	$\Delta \epsilon_{\Delta}$	$\Delta$ Rate <sub><math>\Delta</math></sub> (Hz)	$\Delta$ D <sub>2</sub> Time (Hours) E02-109	$\Delta$ Nucl. Tgts (hours) E03-110
0.5	1.16	0.54	1 K	0.5	C - 0.00
	1.64	0.78	1 K	0.5	
	4.04	0.97	1 K	0.5	
1.0	1.64	0.53	1 K	0.5	Quartz - 0.06
	2.28	0.77	1 K	0.5	
	4.52	0.95	1 K	0.5	
2.0	2.28	0.43	65	0.5	Cu - 0.06
	3.24	0.73	285	0.5	
	5.64	0.92	1 K	0.5	
3.0	3.24	0.51	16	2	Fe - 0.06
	4.04	0.70	40	1	
	5.64	0.86	172	0.5	
			sub-total	8	24
4.0	3.24	0.23	1	22	C - 0.06
	4.04	0.51	3	8	
	5.64	0.77	53	1	
5.0	4.04	0.29	1	22	Cu - 0.06
	4.52	0.42	3	8	
	5.64	0.66	6	4	
			sub-total	65	40
				Total D $\Delta$ 73 E02-109	Total-Nucl $\Delta$ 64 E03-110



24 hours  
All nuclear  
Targets  $\Delta$   
For Q2<4

40 hours  
C and Cu  
Targets  $\Delta$   
For Q2=4  
+ Q2=5

## DIS+ Resonance: Summary and Plan

- **Modified GRV98LO PDFs with the scaling variable  $\xi w$  and a crude  $K(Q^2)$  factor describe vector SLAC/BCDMS/NMC/HERA DIS data in continuum region (use duality based modeling to match resonance to continuum for electroproduction)**
- **Predictions in reasonable agreement with vector continuum region (down to  $Q^2 = 0$ ), photo-production data, and with high-energy neutrino data on iron (but not in the  $\Delta$  and Second Resonance Region)**
- **We Know how to match Resonance and DIS Models using duality -> Just use DIS model which satisfies duality above a certain  $W$  (e.g.  $W=1.8$ ).**
- **This model should also describe continuum low energy neutrino Vector cross sections reasonably well.**
- **Need to add modeling of quasielastic vector and axial form factors.**
- **Resonance region especially in the  $\Delta$  region for F2 and R for Neutrons and Protons and neutrinos needs to be measured and modeled -> E02-109**
- **Axial contribution F2, and R in neutrino scattering needs to be measured/ modeled.**
- **Nuclear Corrections in resonance region to Vector F2 and R need to be measured in electron scattering - P03-110**
- **When done -Check that the models (a) satisfy current algebra and QCD sum rules and (b) describe neutrino data on same nuclear targets, (c) Describe low statistics Neutrino Data on H,D. (d) Precise neutrino data with C target in a few year (e) H and D possible in 10 years (Second phase of MINERvA) *much more difficult technologically.***

## Run Plan Including All Resonance

	D- Time Required (Hours) E02-109	Nucl. Tgts. (Hours) E03-110
Data acquisition (Deuterium $\Delta$ )/+Nucl. Tgts.	73	64
Data acquisition (Deuterium $W^2 > \Delta$ )/+Nucl. Tgts.	40	38
Data acquisition (Dummy)	60	
Data acquisition (hydrogen elastics)	24	
Data acquisition (hydrogen resonance region)	15	
Data acquisition (additional positrons)	22	
D Angle changes (12)/+Nucl. Tgt Changes	3	10
Spectrometer momentum changes (60)	15	
Major beam energy changes (1)	8	
Minor beam energy changes (5)	20	
D Checkout /+ Nucl. Rad correction Tests	24	10
<b>Total</b>	<b>305</b>	<b>120</b>
	E02-109	E03-110

**13 days**  
**E02-109**

**5 days**  
**E03-110**

# Backup Slides Duality, Sum rules and Neutrino data at low Energy

**Outline of a Program in Investigating Nucleon and Nuclear Structure at all  $Q^2$  -  
Starting with PR 03-110 (Details follow up in this talk and backup slides)**

1. Update Vector Form Factors and Rvector of the large number of **resonances** in the Nucleon, e.g. within *Rein-Seghal-Feynman Quark Oscillator model* (and other resonance models) by fitting all F2 and R Electron Resonance data **E94-110 (H)** , **E02-109 (D)** (+ SLAC + photoproduction+ and other data)  
*\* [propose to run PR 03-110 on nuclear targets at the same time as E02-109 (D)]*
1. Improve on **Inelastic Continuum** modeling of Vector F2 and R (e.g. using a formalism like Bodek/Yang) using Jlab, SLAC, H and D data, photoproduction and HERA data.
2. Within these models, convert EM Vector Form Factor to **Weak Vector Form Factors** - use the Various isospin rules  $I=1/2$  and  $I=3/2$  of elastic, resonance and inelastic Form Factors fits to H and D data **E94-110, E02-109**
3. Investigate if the Model predictions for Vector Scattering in neutrino reactions satisfy QCD sum rules and duality at high  $Q^2$  and **Adler Vector Sum rules at ALL  $Q^2$ .**
4. Investigate if the Models predictions for Axial scattering in neutrino reactions satisfy QCD sum rules and duality at high  $Q^2$  and **Adler Axial Sum rules at ALL  $Q^2$ .**

1. Apply nuclear corrections for DIS and resonance region to predict **Neutrino and Antineutrino data on nuclei from PR 03-110** - Requires 5 days of running - Also use E99-118 and SLAC E140 and other for DIS A dependence.
2. Compare predictions to existing **low statistics neutrino data** and to **new precise neutrino data** to become available in a couple of years (MINERvA, and JHF- Japan) - Do the predictions from models (which satisfy all sum rules and duality) model the neutrino and antineutrino data well?
3. In parallel - **Final states in nuclear targets** to be investigated in a collaboration with **Hall B experiments** in electron experiments and in **new neutrino experiments**.

#### Things can be learned from electron scattering

- Nucleon + Resonance Vector Form Factors, Vector Continuum  $F_2$  at all  $Q^2$ ,  $R_{\text{vector}} = \frac{\nu}{T}$  in great details.
- Nuclear effects on various targets in res, and quasielastic region as a function of  $Q^2$
- Hadronic Final States in electron scattering

#### Things that are learned in neutrino scattering

- Check on Current Algebra sum rules and understanding duality -
- Axial vector contribution to  $F_2$  at low  $Q^2$
- Different nuclear effects in neutrino scatt.
- Account for  $R_{\text{axial}}$  different from  $R_{\text{vector}}$
- Hadronic final states in neutrino scattering

Collaborative approach between High Energy and Nuclear Physics community

**High x and low  $Q^2$  PDFs for e/neutrino, Resonance form factors, nuclear corrections**

1. Electron scattering exp. at JLAB P03-110 - **5 Days of DATA and -> Lots of analysis+ follow-up with investigation of final states**

2. New Near Detector neutrino exp. at Fermilab-NUMI/JHF - --> **Years of data** e.g. MINERvA + JHF

# Radiative Corrections Checks, e.g. SLAC E140

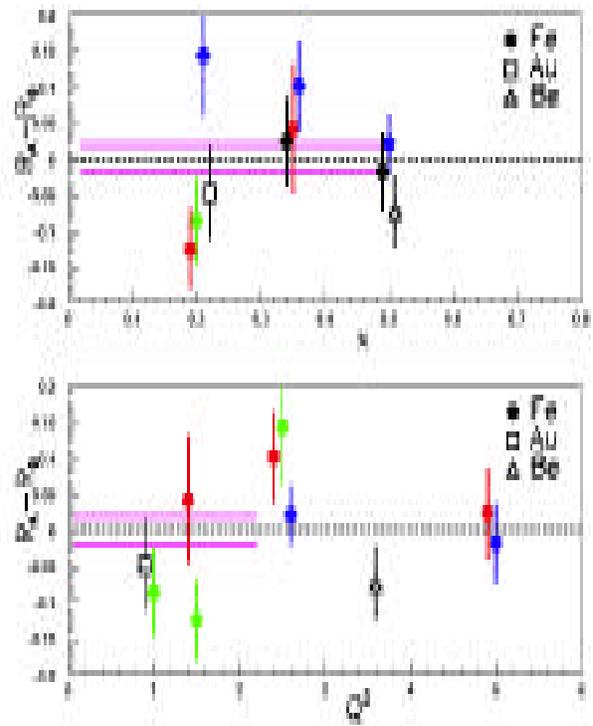
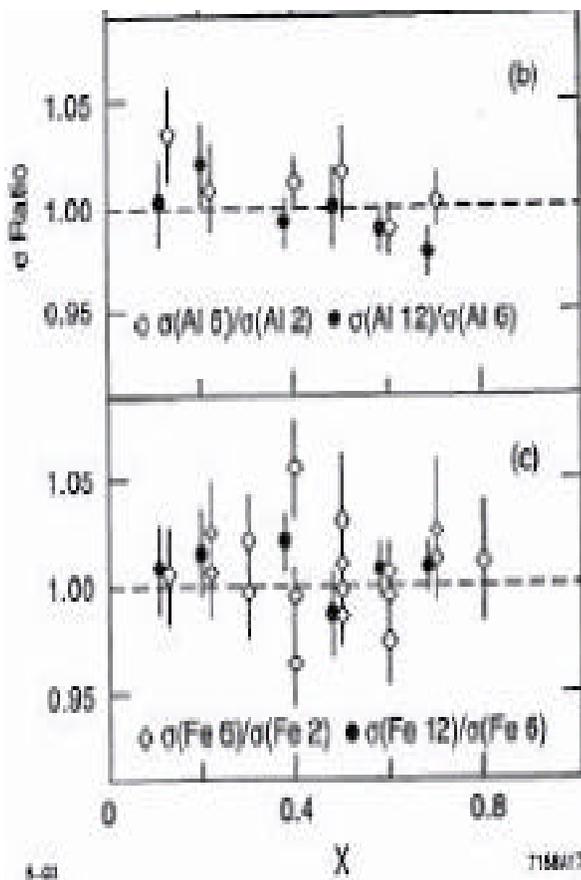
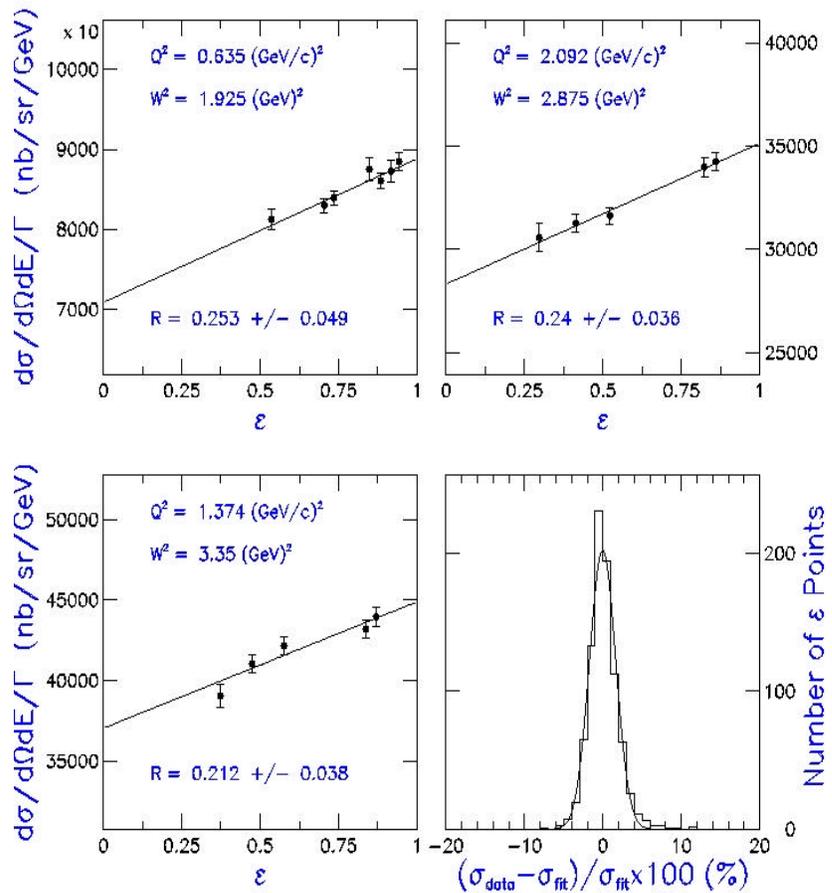


FIG. 1. The SLAC E140 dataset of  $\sigma_4 - \sigma_2$  in deuterium in the Deep Inelastic Region.

## Rosenbluth Separations E94-110 for H Also to be done for D in E02-109

- 180 L/T separations total (most with 4-5  $\epsilon$  points)

- Spread of points about the linear fits is fairly Gaussian with  $\sigma \sim 1.6\%$  - consistent with the estimated pt-pt experimental uncertainty
  - a systematic "tour de force"



# References

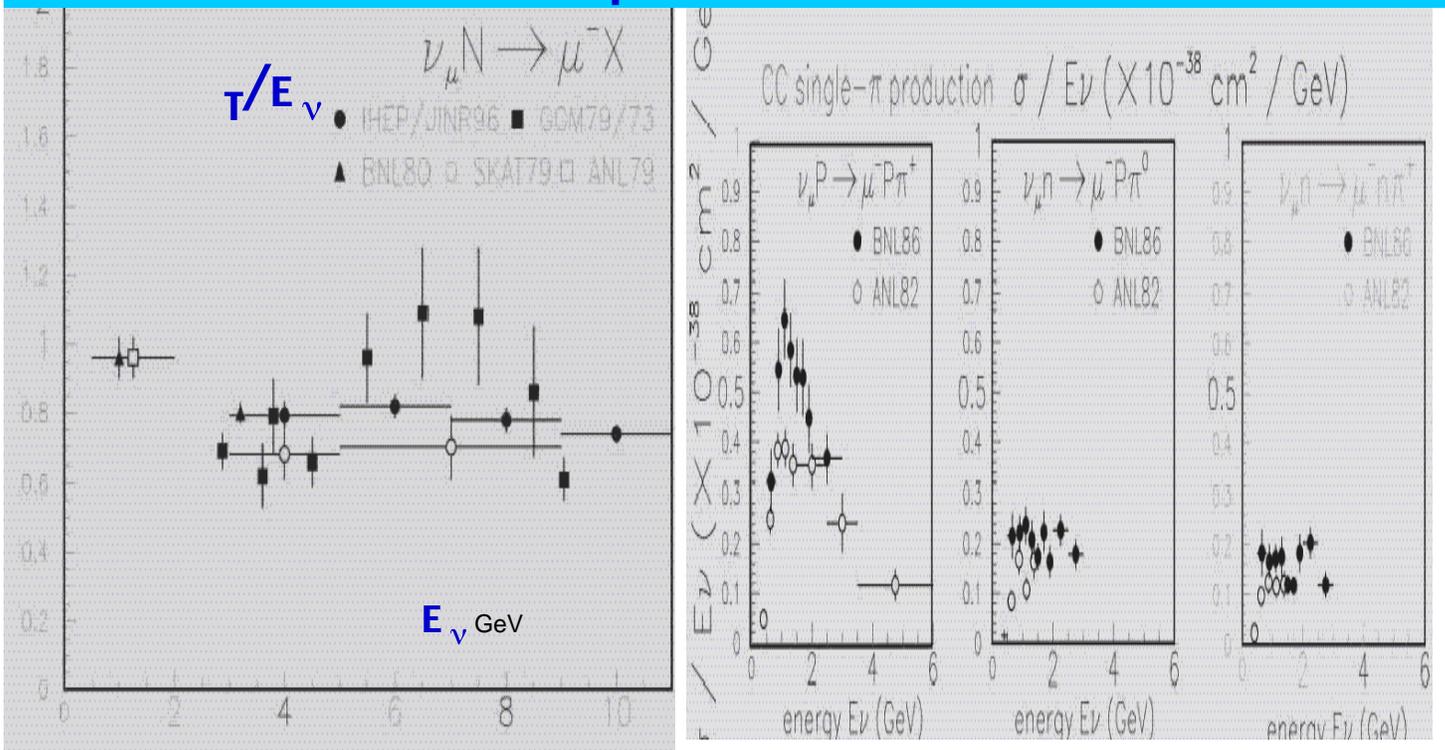
Sum Rules: S. L. Adler, Phys. Rev. 143, 1144 (1966); F. Gilman, Phys. Rev. 167}, 1365 (1968).

Resonance Models: D. Rein and L. M. Sehgal, Annals Phys. 133} 79 (1981) ;D. Rein, Z. Phys. C. 35, 43 (1987); R. P. Feynman, M. Kislinger and F. Ravndal, Phys. Rev. D 3, 2706 (1971).

Coherent nuclear effects. R. Belusevic and D. Rein, Phys. Rev. D 46, 3747 (1992)

Modeling: A. Bodek and U. K. Yang, hep-ex/0203009, Nucl.Phys.Proc.Suppl.112:70-76,2002. A. Bodek and U. K. Yang, hep-ex/0301036 A. Bodek, U. K. Yang, hep-ex/0210024 , J. Phys. G. Nucl. Part. Phys. 29, 1 (2003) and references therein.

**Examples of Low Energy Neutrino Data: Total (inelastic and quasielastic) Charged Current cross section: Flux errors are about 10% to 20% , and Single charged and neutral pion production**

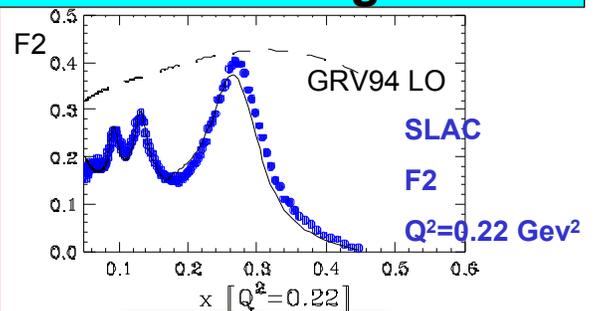


$E_\nu$

Next generation experiments need these cross sections to 2% to get precise neutrino mixing angles

## Need to build up a model for all $Q^2$ for both vector and axial structure of the nucleon, in both electron and neutrino scattering

- Aim to build up a model to describe all  $Q^2$  from high down to very low energies
- [DIS, resonance, photoproduction( $Q^2=0$ ) ]
- Described in terms of quark-parton model, PDFs and also in terms of elastic and resonance form factors
- With PDFs, it is straightforward to convert charged-lepton scattering cross sections into neutrino cross sections. (just matter of different couplings)
- With Form Factors, use isospin relations, CVC:  $I=1/2$  and  $I=3/2$
- **Need:**  $R_{\text{vector}}$ ,  $R_{\text{axial}}$  and axial form factors and structure functions at low  $Q^2$
- **Need:** nuclear effects in both vector and axial structure functions and form factors



### Challenges

- Understanding of high  $x$  PDFs at very low  $Q^2$ ?
- Requires understanding of non-perturbative QCD effects, though SLAC, JLAB data.
- Understanding of Quasielastic + resonance scattering in terms of quark-parton model, form factors (Need to understand duality, QCD, low  $Q^2$  sum rules, transition between DIS and resonance)

Start with: Quasielastic Scattering: C.H. Llewellyn Smith (SLAC).  
SLAC-PUB-0958 Phys.Rept.3:261,1972

$$\frac{d\sigma}{dq^2} \left( \nu n \rightarrow l^- p \right) = \frac{M^2 G^2 \cos^2 \theta_c}{8\pi E_\nu^2} \left[ A(q^2) \mp B(q^2) \frac{(s-u)}{M^2} + \frac{C(q^2)(s-u)^2}{M^4} \right]. \quad (2)$$

In this expression, G is the Fermi coupling constant and  $\theta_c$  is the Cabibbo mixing angle ( $G = 1.16639 \times 10^{-5} \text{GeV}^{-2}$ ). The functions A, B, and C are convenient combinations of the nucleon form factors.

Contraction of the hadronic and leptonic currents yields: **Non zero**

$$A = \frac{(m^2 - q^2)}{4M^2} \left[ \left(4 - \frac{q^2}{M^2}\right) |F_A|^2 - \left(4 + \frac{q^2}{M^2}\right) |F_V^1|^2 - \frac{q^2}{M^2} |\xi F_V^2|^2 \left(1 + \frac{q^2}{4M^2}\right) - \frac{4q^2 \text{Re} F_V^{1*} \xi F_V^2}{M^2} \right] \quad (3)$$

Axial
Vector
Vector
Vector

small
zero
zero
zero

$$- \frac{m^2}{M^2} \left( |F_V^1 + \xi F_V^2|^2 + |F_A + 2F_P|^2 + \left(\frac{q^2}{M^2} - 4\right) \left( |F_S|^2 + |F_T|^2 \right) \right)$$

$$B = -\frac{q^2}{M^2} \text{Re} F_A^* (F_V^1 + \xi F_V^2) - \frac{m^2}{M^2} \text{Re} \left[ \left( F_V^1 \mp \frac{q^2}{4M^2} \xi F_V^2 \right)^* F_S - \left( F_A + \frac{q^2 F_P}{2M^2} \right)^* F_T \right] \quad (4)$$

interference vector axial

$$C = \frac{1}{4} \left( |F_A|^2 + |F_V^1|^2 - \frac{q^2}{M^2} \left| \frac{\xi F_V^2}{2} \right|^2 - \text{zero} \right), \quad (5)$$

where  $m$  is the final state lepton mass. Ignoring second-class currents (those which violate G-parity) allows us to set the scalar and tensor form factors to zero. According to the CVC

Vector form factors  
From electron  
scattering  
Via CVC

$$F_V^1(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1} [G_E^V(q^2) - \frac{q^2}{4M^2} G_M^V(q^2)] \quad (6)$$

$$\xi F_V^2(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1} [G_M^V(q^2) - G_E^V(q^2)]. \quad (7)$$

The electromagnetic form factors are determined from electron scattering experiments:

UPDATE: Replace by  
 $G_E^V = G_E^P - G_E^N$

$$G_E^V(1^2) = \frac{1}{\left(1 - \frac{q^2}{M_V^2}\right)^2} \quad G_M^V(q^2) = \frac{1 + \mu_p - \mu_n}{\left(1 - \frac{q^2}{M_V^2}\right)^2}.$$

UPATE: Replace by  
 $G_M^V = G_M^P - G_M^N$

The situation is slightly more complicated for the hadronic axial current.  $F_A(q^2 = 0) = -1.261 \pm .004$  is known from neutron beta decay. The  $q^2$  dependence has to be inferred or measured. By analogy with the vector case we assume the same dipole form:

$$M_A = 1.032 \pm .036 \text{ GeV [7].}$$

$$F_A(q^2) = \frac{-1.23}{\left(1 - \frac{q^2}{M_A^2}\right)^2}. \quad Q^2 = -q^2 \quad (9)$$

$g_A = -1.267, M_A$  need to  
Be updated from  
Neutrino scatter.

$$F_P(q^2) = \frac{2M^2 F_A(q^2)}{M_\pi^2 - q^2}. \quad \begin{array}{l} \text{Fp important for} \\ \text{Muon neutrinos only at} \\ \text{Very Low Energy} \end{array} \quad (10)$$

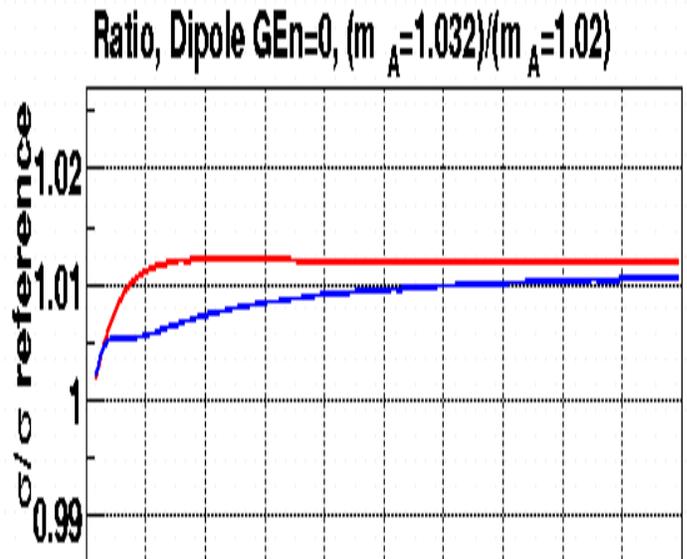
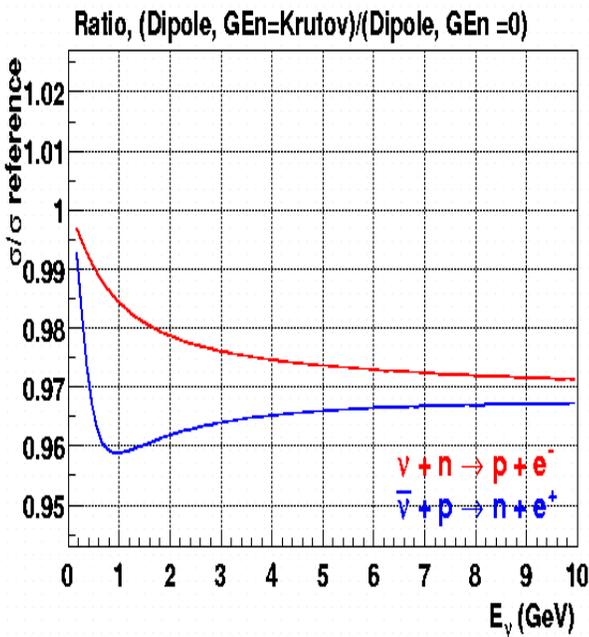
From C.H. Llewellyn Smith (SLAC). SLAC-PUB-0958 Phys.Rept.3:261,1972

For data on nuclei, need nuclear Corrections.

Can fix the  $Q^2$  dependence either way (by changing  $m_A$  or using correct vector form factors). However the overall *cross sections will be 14% too high if one chooses wrong*

**Gen (right)/Gen=0 (wrong) gives 6% lower cross section**

**Wrong  $m_A=1.1$  (used by K2K) Over  $m_A=1.02$  (Ratio) gives 8% higher cross Section (1% for each 0.01 change in  $m_A$ )**



**MRSR2 or CTEQ4M predictions using NLO QCD + TM + higher twist describe the data reasonably well -**

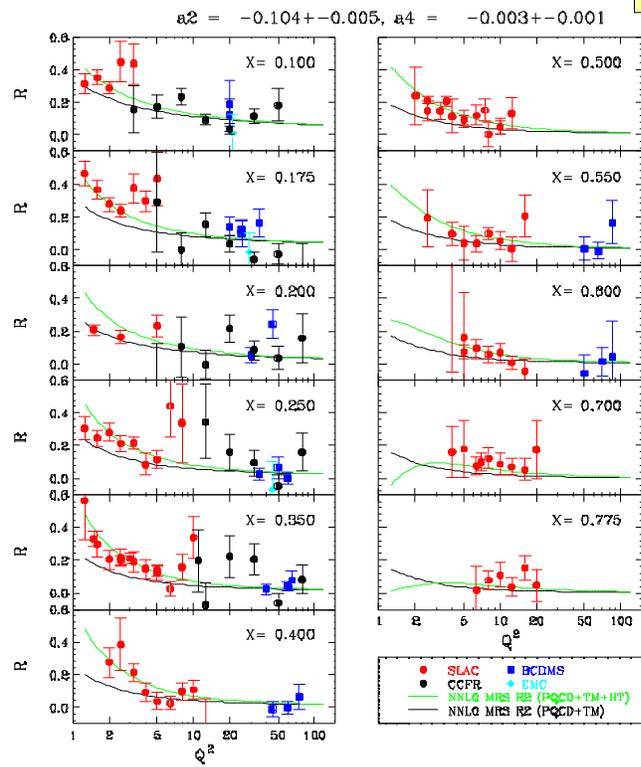
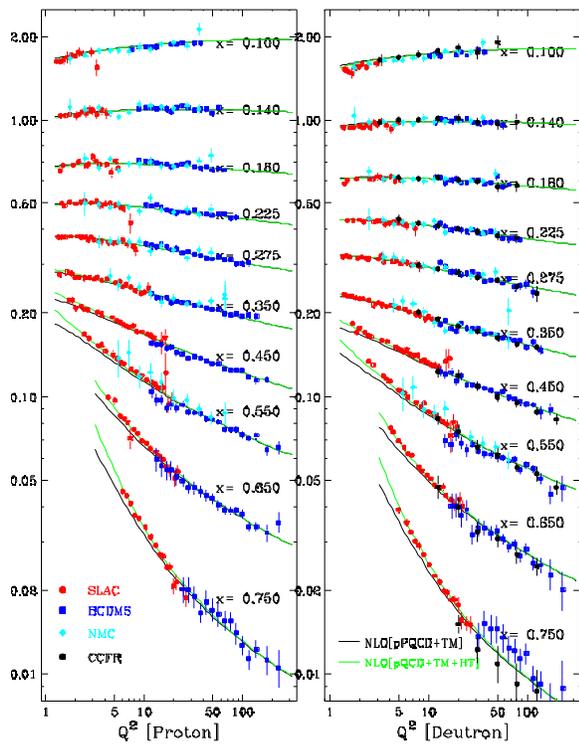
Bodek/Yang *Phys. Rev. Lett* 82, 2467 (1999) ; *Phys. Rev. Lett.* 84, 3456 (2000)  
 Higher Twist  $a_2 \cdot C_2(x)/Q^2 + a_4 \cdot C_4(x)/Q^4$   $a_2=0.104$

F2

$$a_2 = -0.104 \pm 0.005$$

$$a_4 = -0.003 \pm 0.001$$

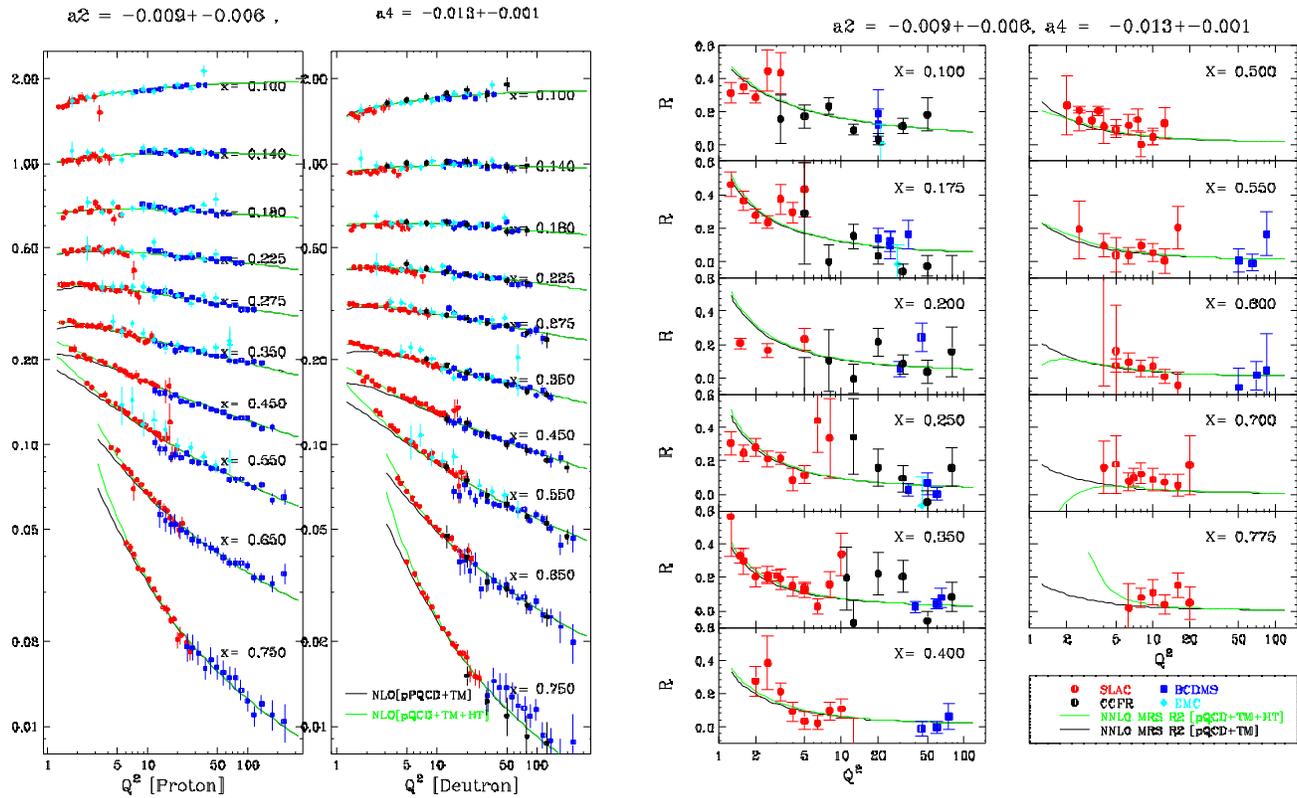
R



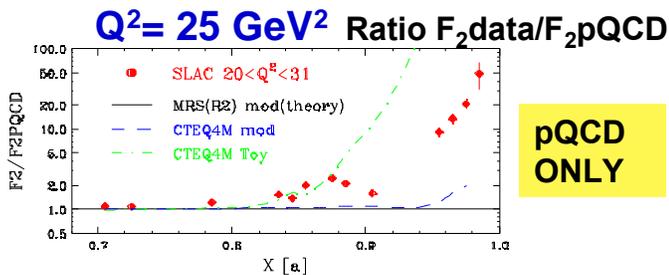
# F2, R comparison with NNLO QCD (Bodek/Yang)

Eur. Phys. J. C13, 241 (2000)

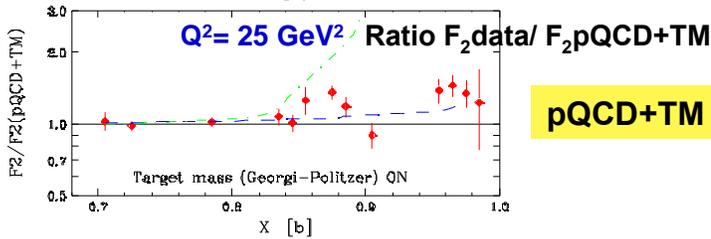
Size of the higher twist with NNLO analysis is really small:  $a_2 = -0.009(\text{NNLO})$  vs  $-0.10(\text{NLO})$



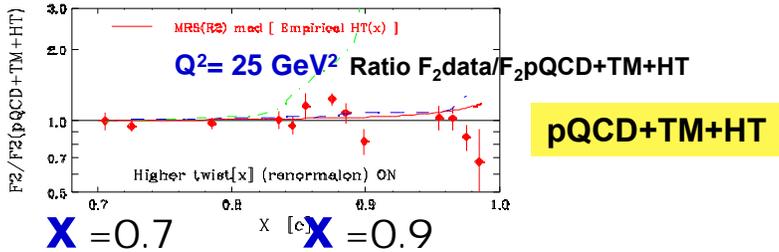
# Very high x F2 proton data (DIS + resonance) (not included in the original fits $Q^2=1.5$ to $25 \text{ GeV}^2$ )



**pQCD ONLY**

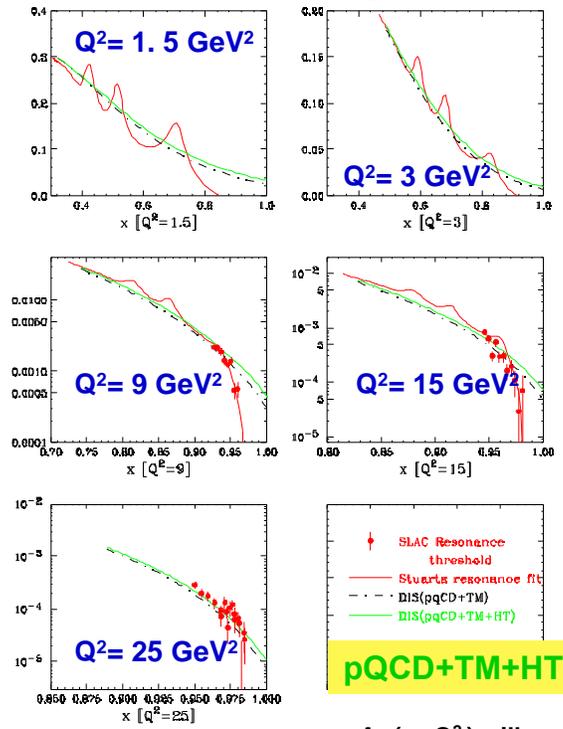


**pQCD+TM**



**pQCD+TM+HT**

## F2 resonance Data versus $F_2\text{pQCD+TM+HT}$

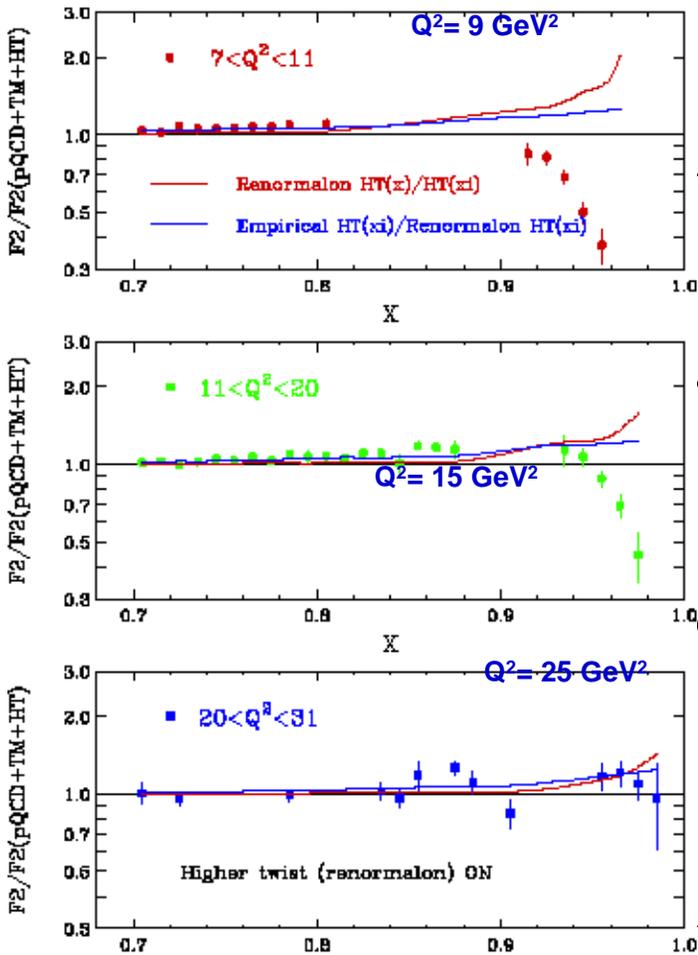


**pQCD+TM+HT**

(Bodek/Yang) NLO pQCD + TM + higher twist describes very high x DIS  $F_2$  and resonance  $F_2$  data well. (duality works)  $Q^2=1.5$  to  $25 \text{ GeV}^2$

$A_w(w, Q^2)$  will account for interactions with spectator quarks

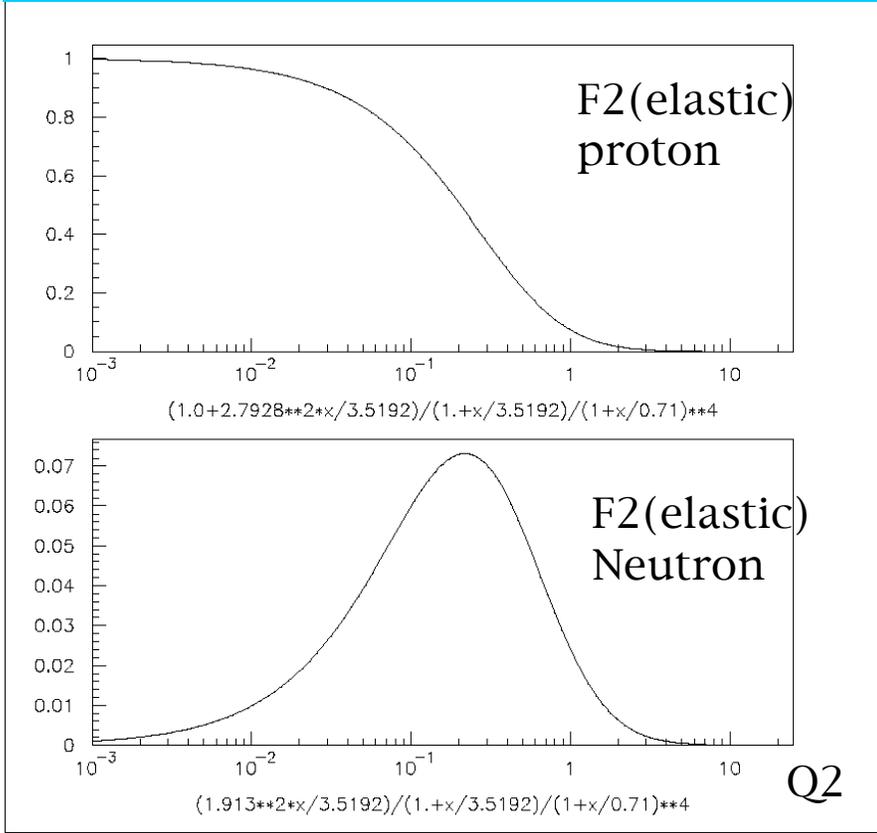
# Look at $Q^2 = 8, 15, 25 \text{ GeV}^2$ very high $x$ data-backup slide\*



Ratio  
 $F_2\text{data}/F_2\text{pQCD}+\text{TM}+\text{HT}$

- Pion production threshold  $A_w(w, Q^2)$
- Now Look at lower  $Q^2$  (8,15 vs 25) DIS and resonance data for the ratio of  $F_2$  data/( NLO pQCD +TM +HT)
- High  $x$  ratio of  $F_2$  data to NLO pQCD +TM +HT parameters extracted from lower  $x$  data. These high  $x$  data were not included in the fit.
  - The Very high  $x(=0.9)$  region: It is described by NLO pQCD (if target mass and higher twist effects are included) to better than 10%

Note that in electron inelastic scattering from Neutrons the quark charges contribute But at  $Q^2=0$ , the elastic form factor is zero)  
\*Backup-slide



Arie Bodek, Univ. of Rochester

Momentum sum rule breaks down and all QCD sum rules break down below  $Q^2=1$ .

However, the Adler sum rule, which comes from Current Algebra (which includes the elastic part) is exact and is equal to the NUMBER of  $U_v - D_v = 1$ .  $\rightarrow (F_2(x)/x)$ .

It is valid all the way to  $Q^2=0$ .

# Example Modeling of Continuum Region

## Modeling in Leading Order from $Q^2=0$ to very high $Q^2$

A. Bodek and U. K. Yang, hep-ex/0203009, Nucl.Phys.Proc.Suppl. 112:70-76,2002. - GRV98 and  $\xi_w$

A. Bodek and U. K. Yang, hep-ex/0301036 - GRV98 and  $\xi_w$

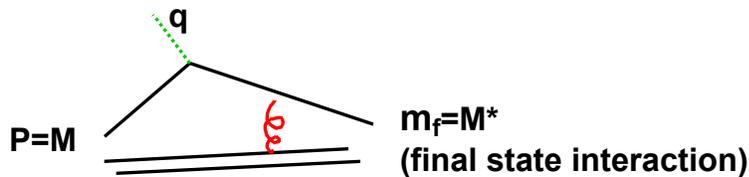
A. Bodek, U. K. Yang, hep-ex/0210024, J. Phys. G. Nucl. Part. Phys. 29, 1 (2003) - GRV94 and Xw

## **Based on QCD NLO and NNLO studies for $Q^2 > 1 \text{ GeV}^2$**

- o **Studies in NLO +TM +HT - Yang and Bodek: Phys. Rev. Lett 82, 2467 (1999) ; Phys. Rev. Lett. 84, 3456 (2000)**
- o **Studies in NNLO +TM +HT - Yang and Bodek: Eur. Phys. J. C13, 241 (2000)**

## Pseudo NLO approach

- Original approach (NNLO QCD+TM) was to explain the non-perturbative QCD effects at low  $Q^2$ , but now we reverse the approach: **Use LO PDFs and “effective target mass and final state masses” to account for initial target mass, final target mass, and missing higher orders**



Resonance, higher twist, and TM

$$= \frac{Q^2 + m_f^2 + O(m_f^2 - m_i^2)}{M_V (1 + (1 + Q^2/V^2))^{1/2}} \rightarrow X_{bj} = Q^2 / 2 M_V$$

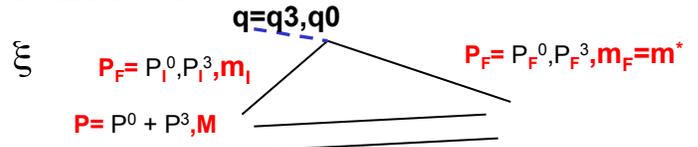
$$\xi_w = [Q^2 + B] / [M_V (1 + (1 + Q^2/V^2))^{1/2} + A]$$

**A** : initial binding/target mass effect  
plus higher order terms

**B**: final state mass  $m_f^2$ ,  $\Delta m^2$  and photo-  
production limit ( $Q^2 = 0$ )

**Initial quark mass  $m_i$  and final mass  $m_F = m^*$  bound in a proton of mass  $M$  -- Summary: INCLUDE quark initial Pt) Get  $\xi$  scaling (not  $x=Q^2/2Mv$ ) for a general parton Model**

$\xi$  Is the correct variable which is Invariant in any frame :  $q_3$  and  $P$  in opposite directions.



	$P_i, P^0$	$q_3, q_0$
	quark	photon
$\xi = \frac{P_i^0 + P_i^3}{P^0 + P^3}$		

$$(q + P_i)^2 = P_F^2 \quad q^2 + 2P_i \cdot q + P_i^2 = m_F^2$$

**Special cases:**

- (1) Bjorken  $x$ ,  $x_{BJ} = Q^2/2Mv$ ,  $\xi \rightarrow x$   
For  $m_F^2 = m_i^2 = 0$  and High  $v^2$ ,
- (2) Numerator  $m_F^2$ : Slow Rescaling  $\xi$  as in charm production
- (3) Denominator: Target mass term  
 $\xi$  = Nachtmann Variable  
 $\xi$  = Light Cone Variable  
 $\xi$  = Georgi Politzer Target Mass var. (all the same  $\xi$ )

$$\xi_w = \frac{Q^2 + m_F^2 + A}{\{Mv[1 + \sqrt{1 + Q^2/v^2}] + B\}} \quad \text{for } m_i^2, Pt = 0$$

**Most General Case: (Derivation in Appendix)**

$$\xi_w = [Q'^2 + B] / [Mv(1 + (1 + Q^2/v^2)^{1/2}) + A] \quad (\text{with } A=0, B=0)$$

where  $2Q'^2 = [Q^2 + m_F^2 - m_i^2] + \{ (Q^2 + m_F^2 - m_i^2)^2 + 4Q^2(m_i^2 + P^2t) \}^{1/2}$

**Bodek-Yang: Add B and A to account for effects of additional  $\Delta m^2$**

**from NLO and NNLO (up to infinite order) QCD effects.** For case  $\xi_w$  with  $P^2t = 0$  see R. Barbieri et al Phys. Lett. 64B, 1717 (1976) and Nucl. Phys. B117, 50 (1976)

## Modified GRV98 PDFs

## Fit with $w$ and $K_{val}$ and $K_{sea}$

Only 5 parameters for all DIS data at all  $Q^2$ :  $A$ ,  $B$ ,  $C_{sea}$ ,  $C_{2V}$  and  $C_{1V}$

### 1. GRV98 LO ( $Q^2_{min}=0.80 \text{ GeV}^2$ )

- describe  $F_2$  data at high  $Q^2$

### 2. Replace the $X$ with a new scaling, $X = [Q^2] / [2M]$

$$W = [Q^2 + B] / [M_v (1 + (1 + Q^2/v^2)^{1/2}) + A]$$

### 3. Multiply all PDFs by a $K$ factor of for photo prod. limit and higher twist

$$[ ( ) = 4 / Q^2 * F_2( w, Q^2 )]$$

### 4. Freeze the evolution at $Q^2 = Q^2_{min}$

$$- F_2(x, Q^2 < 0.80) = K F_2( w, Q^2=0.80)$$

### ➤ Different $K$ factors for valence and sea

$$K_{sea} = Q^2 / [Q^2 + C_{sea}]$$

$$K_{val} = [1 - G_D^2(Q^2)] * [Q^2 + C_{2V}] / [Q^2 + C_{1V}]$$

$$\text{where } G_D^2(Q^2) = 1 / [1 + Q^2 / 0.71]^4$$

(elastic nucleon dipole form factor)

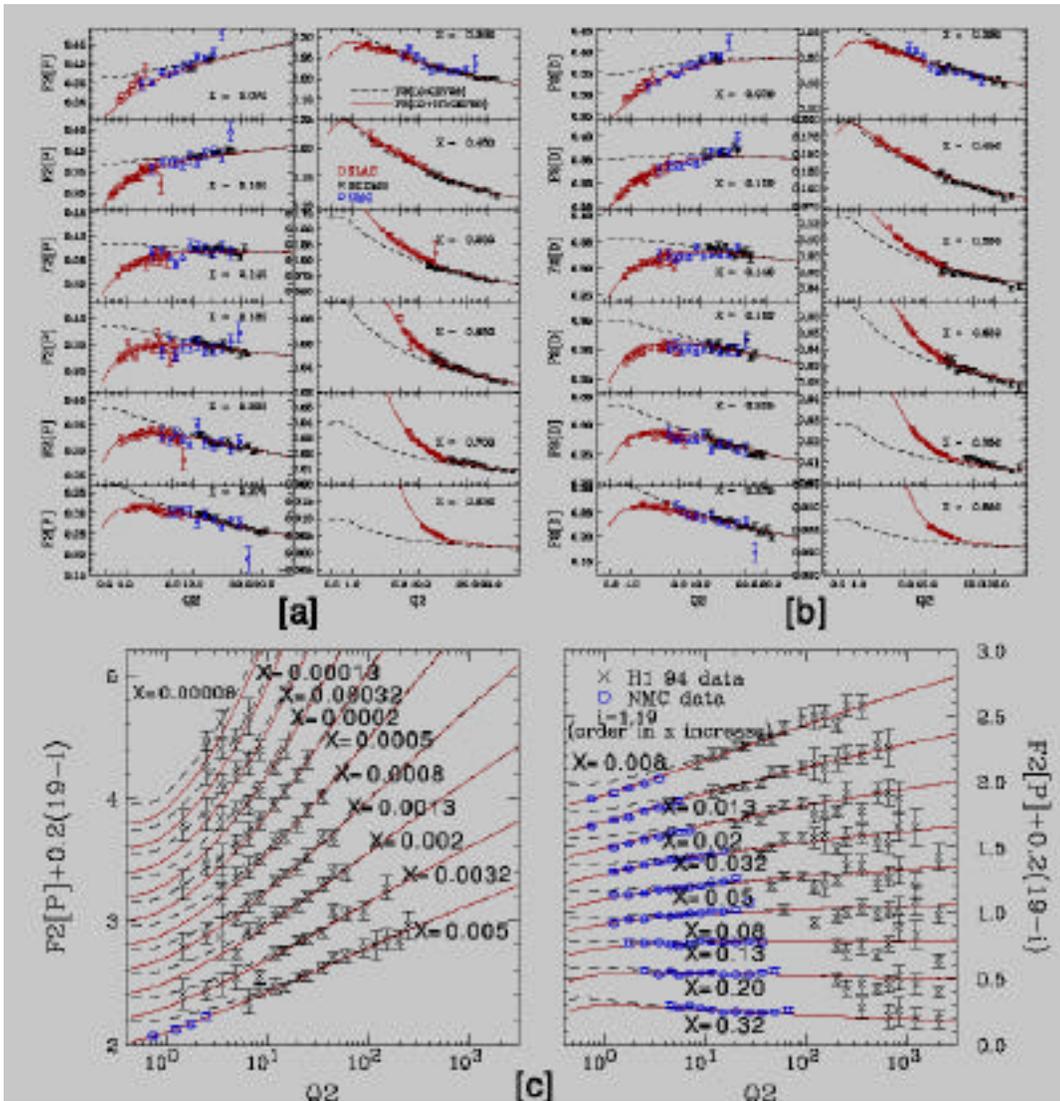
(Form Motivated by Adler Sum Rule)

### ☐ Do a fit to SLAC/NMC/BCDMS $F_2$ P, D + low $x$ HERA/NMC $F_2$ data. Very good fits are obtained

$$A=0.418, B=0.222, C_{sea} = 0.381$$

$$C_{1V} = 0.604, C_{2V} = 0.485$$

$$\chi^2/DOF = 1268 / 1200$$



$\chi^2 = 1268 / 1200$  DOF

Dashed=GRV98 LO QCD  
 $F_2 = F_{2QCD}(x, Q^2)$

Solid=modified  
 GRV98 LO QCD

$F_2 = K(Q^2) * F_{2QCD}(\xi w, Q^2)$

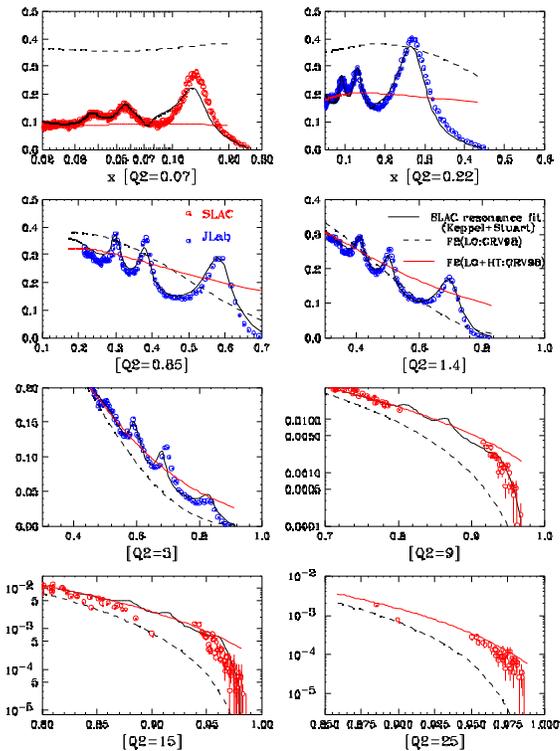
SLAC, NMC, BCDMS (H,D)

HERA 94

Data ep

Fit with  $w$   
 modified  
 GRV98 PDFs

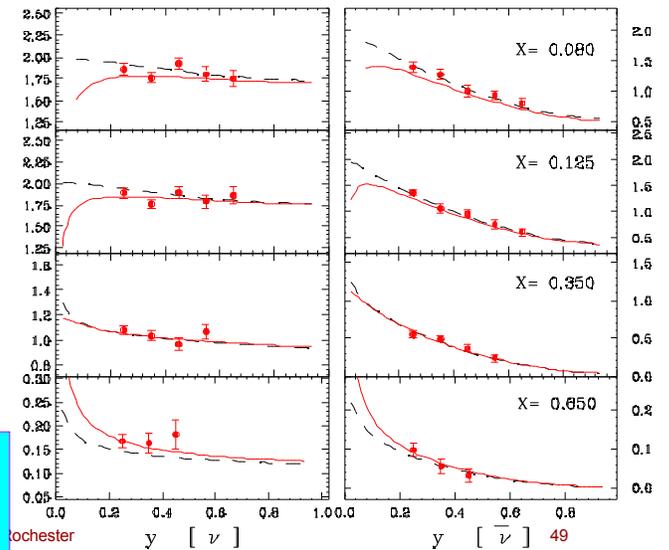
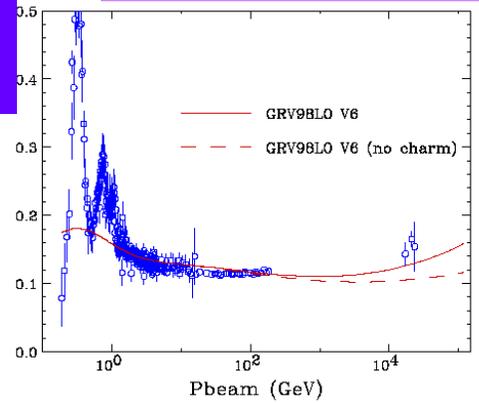
## F<sub>2</sub>(P) resonance



## Neutrino Xsection on iron at 55GeV (CCFR)

Fit with  $w$   
Predictions  
modified GRV98  
PDFs

## Photo-production (P)



rochester

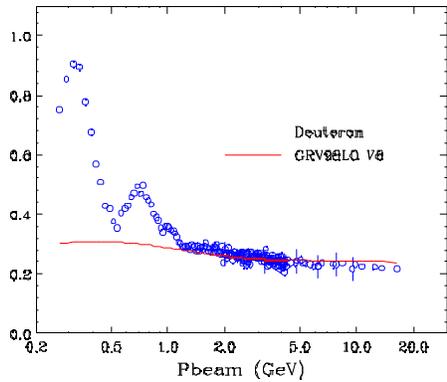
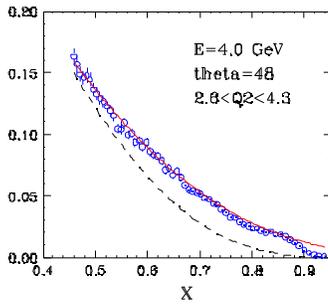
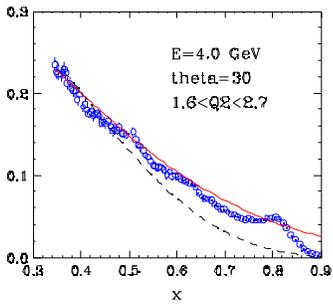
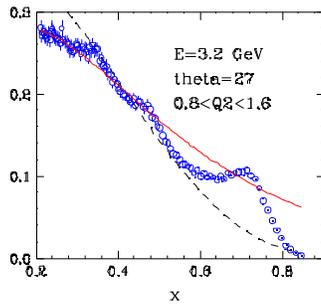
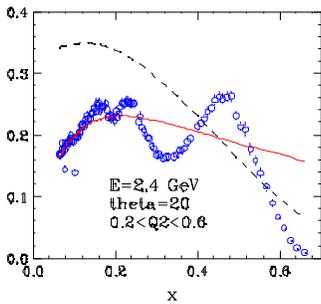
y [  $\nu$  ]

y [  $\nu$  ] 49

Fit with  $w$  Predictions  
modified GRV98 PDFs

$F_2(d)$  resonance

Photo-production (d)



S. Adler, Phys. Rev. 143, 1144 (1966) Sum rule for W1

$$\alpha = W_1$$

$$\beta = W_2$$

(B) Sum Rule for  $\beta^{(\pm)}$  **W<sub>2</sub>**

The sum rule on  $\beta^{(\pm)}$  of Eq. (14) is obtained by adding together two separately derived sum rules on the axial-vector and the vector parts of  $\beta^{(\pm)}$ ,  $\beta_A^{(\pm)}$ , and  $\beta_V^{(\pm)}$ :

$$1 = g_A(q^2)^2 + \int_{M_N+M_\pi}^{\infty} \frac{W}{M_N} dW \times [\beta_A^{(-)}(q^2, W) - \beta_A^{(+)}(q^2, W)], \quad (53a)$$

$$1 = F_1^V(q^2)^2 + q^2 F_2^V(q^2)^2 + \int_{M_N+M_\pi}^{\infty} \frac{W}{M_N} dW \times [\beta_V^{(-)}(q^2, W) - \beta_V^{(+)}(q^2, W)]. \quad (53b)$$

In terms of the structure functions defined in Eq. (41),

$$\beta_A^{(\pm)}(q^2, W) = [q^2 A_1^{(\pm)}(q^2, W) + (q^2)^2 A_2^{(\pm)}(q^2, W) + q^2 D_A^{(\pm)}(q^2, W) + 4M_N^2/(W^2 - M_N^2 + q^2)], \quad (54)$$

$$\beta_V^{(\pm)}(q^2, W) = q^2 [V_1^{(\pm)}(q^2, W) + q^2 V_2^{(\pm)}(q^2, W)] \times 4M_N^2/(W^2 - M_N^2 + q^2).$$

[The structure functions  $A_2^{(\pm)}(q^2, W)$  and  $D_V^{(\pm)}(q^2, W)$  vanish identically in the strangeness-conserving case, because of conservation of the vector current.] Since the derivations of Eqs. (53a) and (53b) are identical, we will treat explicitly only the axial-vector case, Eq. (53a).

**$\beta^- = W_2$  (Anti-neutrino -Proton)**  
 **$\beta^+ = W_2$  (Neutrino-Proton)**

(C) Sum Rule for  $\alpha^{(\pm)}$  **W<sub>1</sub>**

The sum rule on  $\alpha^{(\pm)}$  of Eq. (15) is obtained by adding together the two identities

**axial**  $C_1^{\pm} = \left(1 + \frac{q^2}{4M_N^2}\right) G_A(q^2) + \int_{M_N+M_\pi}^{\infty} \frac{W}{M_N} dW [\alpha_A^{(-)}(q^2, W) - \alpha_A^{(+)}(q^2, W)], \quad (55a)$

**vector**  $C_2^{\pm} = \left(\frac{q^2}{4M_N^2}\right) G_V(q^2) + \int_{M_N+M_\pi}^{\infty} \frac{W}{M_N} dW [\alpha_V^{(-)}(q^2, W) - \alpha_V^{(+)}(q^2, W)]. \quad (55b)$

Here  $\alpha_A^{(\pm)}$  and  $\alpha_V^{(\pm)}$  are, respectively, the axial-vector and the vector parts of  $\alpha^{(\pm)}$ ,

$$\alpha_A^{(\pm)} = A_1^{(\pm)}(q^2, W), \quad \alpha_V^{(\pm)} = V_1^{(\pm)}(q^2, W). \quad (56)$$

$$g_V(q^2) = F_1^V(q^2) + 2M_N F_2^V(q^2),$$

$$C_1^{\pm} = \int d\mu [A_1^{(-)} - A_1^{(+)}] = \int \frac{W}{M_N} dW [A_1^{(-)}(q^2, W) - A_1^{(+)}(q^2, W)], \quad (62)$$

**W<sub>1</sub> Sum has not been investigated**  
 **$\alpha^- = W_1$  (Anti-neutrino -Proton)**  
 **$\alpha^+ = W_1$  (Neutrino-Proton)**

Additional Adler Sum Rules  
Have not been investigated

STEPHEN L. ADLER

$\alpha^- = W_1$  (Anti-neutrino -Proton)  
 $\alpha^+ = W_1$  (Neutrino-Proton)

Backup slide  $C_I^1 + C_I^2 = (1 + q^2/4M_N^2)g_A(q^2)^2 + (q^2/4M_N^2)g_V(q^2)^2 + \int_{M_N+M_\pi}^{\infty} \frac{W}{M_N} dW [\alpha^{(-)}(q^2, W) - \alpha^{(+)}(q^2, W)];$  (15)

(iii) the local commutation relation of Eq. (4b) implies

$$\frac{g_V(q^2)g_A(q^2)}{M_N} = \int_{M_N+M_\pi}^{\infty} \frac{W}{M_N} dW [\gamma^{(-)}(q^2, W) - \gamma^{(+)}(q^2, W)].$$
 (16)

$\gamma^- = W_3$  (Anti-neutrino -Proton)  
 $\gamma^+ = W_3$  (Neutrino-Proton)

We write

Strangeness-Changing Case

$W_3$  Sum rules, and Strangeness Changing Sum  $W_1$  and  $W_2$  rules have not been investigated.

$$d^2\sigma \left( \begin{pmatrix} \nu \\ \bar{\nu} \end{pmatrix} + (p, n) \rightarrow \begin{pmatrix} l \\ \bar{l} \end{pmatrix} + \beta \begin{pmatrix} S=1 \\ S=-1 \end{pmatrix} \right) / d\Omega_l dE_l = \frac{G^2 \sin^2\theta_C E_l}{(2\pi)^2 E_\nu} \times [q^2 \alpha_{(p,n)}^{(\pm)}(q^2, W) + 2E_\nu E_l \cos^2(\frac{1}{2}\phi) \beta_{(p,n)}^{(\pm)}(q^2, W) \mp (E_\nu + E_l) q^2 \gamma_{(p,n)}^{(\pm)}(q^2, W)].$$
 (17)

Then,

(i) the local commutation relations of Eq. (1a) and Eq. (1c) imply

$$(4,2) = \int \frac{W}{M_N} dW [\beta_{(p,n)}^{(-)}(q^2, W) - \beta_{(p,n)}^{(+)}(q^2, W)];$$
 (18)

$\beta^- = W_2$  (Anti-neutrino -Proton)  
 $\beta^+ = W_2$  (Neutrino-Proton)

(ii) the local commutation relations of Eq. (4a) and Eq. (4c) imply

$$[\sqrt{3}(C_Y^1 + C_Y^2) + \frac{1}{2}(C_I^1 + C_I^2), \sqrt{3}(C_Y^1 + C_Y^2) - \frac{1}{2}(C_I^1 + C_I^2)] = \int \frac{W}{M_N} dW [\alpha_{(p,n)}^{(-)}(q^2, W) - \alpha_{(p,n)}^{(+)}(q^2, W)];$$
 (19)

(iii) the local commutation relation of Eq. (4b) implies

$$(0,0) = \int \frac{W}{M_N} dW [\gamma_{(p,n)}^{(-)}(q^2, W) - \gamma_{(p,n)}^{(+)}(q^2, W)].$$
 (20)

The integrals of Eqs. (18)–(20) have discrete contributions at  $W = M_\Lambda$  and/or  $M_\Sigma$  and a continuum extending from  $W = M_\Lambda + M_\pi$  or from  $W = M_\Sigma + M_\pi$  to  $W = \infty$ . We have not explicitly separated off the discrete contributions to the integrals, as was done in Eqs. (14)–(16) for the strangeness-conserving case. It would, of course, be straightforward to do this.

F. Gilman, Phys. Rev. 167, 1365 (1968)

Adler like Sum rules for electron scattering.

$$\alpha = W_1/M_N,$$

$$\beta = W_2/M_N.$$

The vector current part of the original sum rule of Adler for neutrino scattering can be written

$$\int_0^\infty dq_0 [\beta^{(-)}(q_0, q^2) - \beta^{(+)}(q_0, q^2)] = 1. \quad (18)$$

The functions  $\beta^{(\pm)}(q_0, q^2)$  are defined just as in Eq. (7) except that in place of the electromagnetic currents  $J_\mu(0)$  and  $J_\mu(0)$  we have put the isospin raising or

lowering  $F$ -spin currents  $\mathfrak{F}_{(1\pm i2)\mu}(0)$  [recall that  $\mathfrak{F}_{3\mu}(0)$  is just the isovector part of the electromagnetic current].

If we explicitly separate out the nucleon Born term in Eq. (18), we have

$$[F_1^V(q^2)]^2 + q^2 \left( \frac{\mu^V}{2M_N} \right)^2 [F_2^V(q^2)]^2 + \int_{M_\pi + (q^2 + M_\pi^2)/2M_N}^\infty dq_0 [\beta^{(-)}(q_0, q^2) - \beta^{(+)}(q_0, q^2)] = 1, \quad (19)$$

where the superscript  $V$  denotes the fact that we are dealing with the isovector part of the current; the isovector anomalous magnetic moment  $\mu^V = \mu_p' - \mu_n' = 3.70$ . As  $q^2 \rightarrow 0$ , we see from Eq. (10) or (17) that only the first term,  $[F_1^V(q^2)]^2$ , on the left-hand side of Eq. (19) survives, and as  $q^2 \rightarrow 0$  it goes to 1, in agreement with the left-hand side.

In the derivation<sup>3</sup> of Eq. (18) only two assumptions enter: (1) the commutation relation Eq. (3a) of the  $F$ -spin densities, and (2) an unsubtracted dispersion relation for the forward Compton scattering amplitudes (which are the coefficients of  $p_\mu p_\nu$  and  $q_\mu q_\nu$  in the expansion of  $T_{\mu\nu}$ ) corresponding to  $\beta(q_0, q^2)$ . It is of course the second assumption which is most open to question. However, we note the following:

(a) The fact that as  $q^2 \rightarrow 0$  the left- and right-hand sides of Eq. (19) as it now stands automatically become equal rules out a  $q^2$ -independent subtraction. This just means we have done nothing grossly wrong, e.g., introduced a kinematic singularity in  $q^2$  in one of our amplitudes.

\*Backup slide

What about the fact that Adler sum rule is for  $Uv-Dv$  as measured in vector and axial scattering, on light quarks, what about Strangeness Changing -

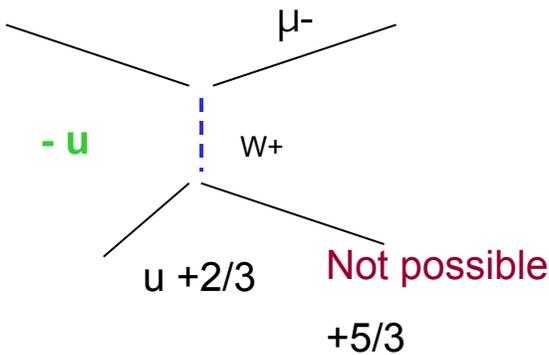
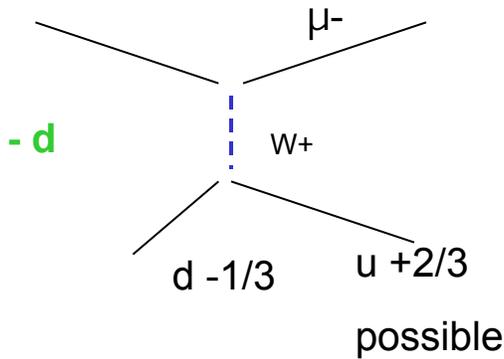
One could get the factors for  $Dv$  and  $Uv$  separately by using the Adler sum rules for the STRANGENESS CHANGING ( $DS=+-1$  proportional to  $\sin^2$  of the Cabbibo angle ) (where he gets 4, 2) if one knew the Lambda and Sigma form factors ( $F1v, F2v, Fa$ ) as follows. Each gives vector and axial parts: Here  $\cos^2 \theta_c$  and  $\sin^2 \theta_c$  are for the Cabbibo Angle.

1.  $F2nub-p (S=0)/\cos^2 \theta_c = u + dbar$  (has Neutron final state udd **quasielastic**)
  2.  $F2nu-p (S=0)/(\cos^2 \theta_c = d + ubar$  (only inelastic final states **continuum only**)
  3.  $F2nub-p (S+-1)/\sin^2 \theta_c = u + sbar$  (has Lambda and Sigma0 uds **quasielastic**)
  4.  $F2nu-p (S+-1)/\sin^2 \theta_c = s + ubar$  (making uud + sbar **continuum only**)
  5.  $F2nub-n (S+-1) = d + sbar$  (has  $- =dds$  **quasielastic**)
  6.  $F2nu-n (S+-1) = s + ubar$  (making udd + sbar **continuum only**)
- A. strangeness conserving is Equations 1 minus 2 =  $Uv-Dv = 1V+1A = 2$  (and at  $Q^2=0$  has Neutron quasielastic final state) (one for vector and one for axial)
- B. strangeness changing on neutrons is Equation 5 minus 6 =  $Dv = 1V+1A = 2$  (and at  $Q^2=0$  has  $-$  quasielastic)
- C. strangeness changing on protons is Equation 3 minus 4 =  $Uv = 2V+2A = 4$  (and at  $Q^2=0$  has both  $\rho_0$  and  $\omega_0$  quasielastic).

Note according to Physics reports article of Llwellyn Smith -  $I=1/2$  rule has cross section for  $\rho_0$  at half the value of  $\omega_0$ ).

# NEUTRINOS

Only scatter on d quarks

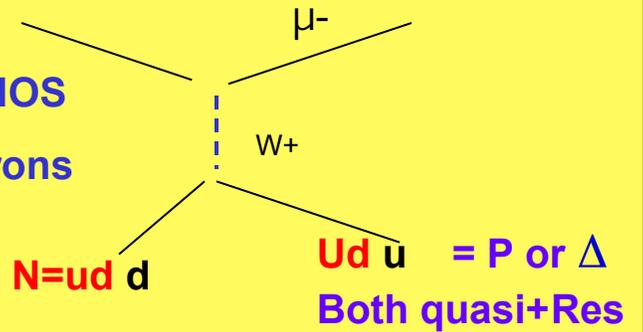


Local duality breaks down at  $x=1$  at all Q2  
 (In neutrino scattering)

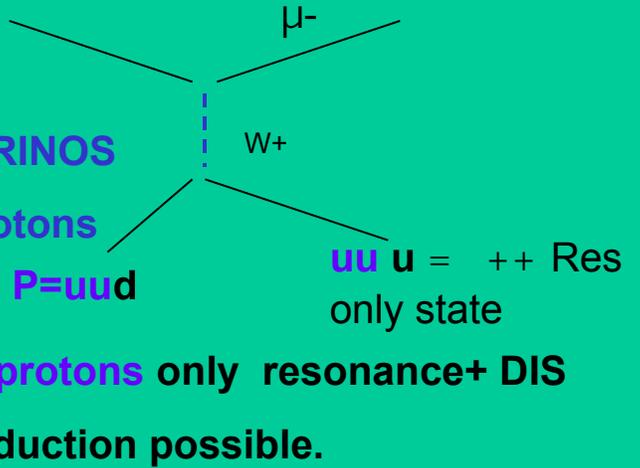
On neutrons both quasielastic

And resonance+DIS production possible.

NEUTRINOS  
 On Neutrons

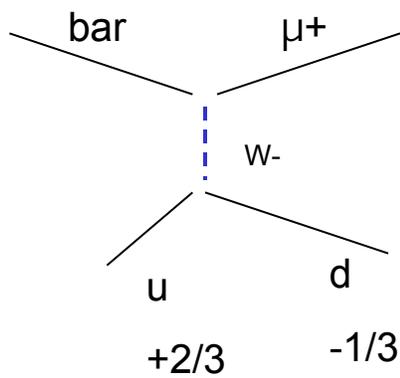


NEUTRINOS  
 On Protons



## ANTI-NEUTRINOS

Only scatter on u quarks

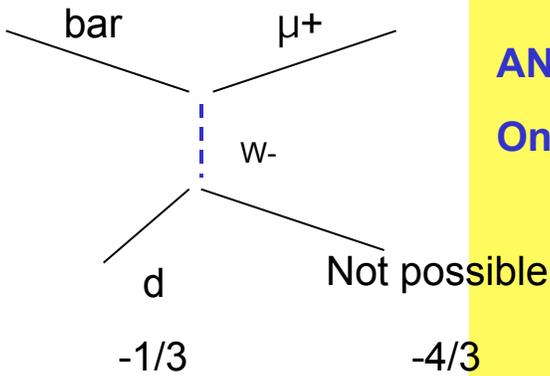
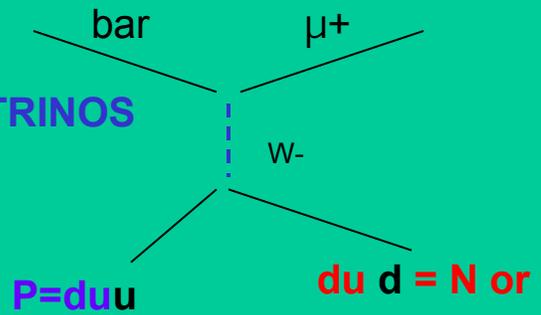


On Protons both quasielastic

And resonance+DIS production possible.

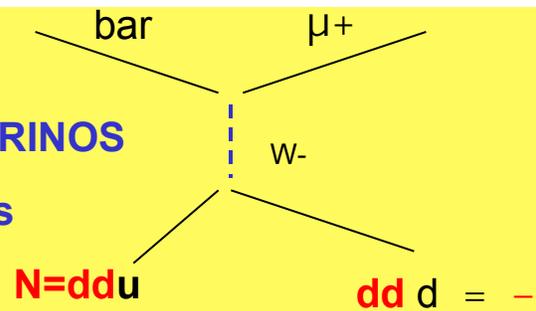
## ANTI+NEUTRINOS

On Protons



## ANTI+NEUTRINOS

On Neutrons



On Neutrons only

resonance+ DIS

production possible.

# Tests of Local Duality at high x, High Q<sup>2</sup> Neutrino Charged Current Scattering Case **\*backup slide**

- INELASTIC High Q<sup>2</sup>,  $x \rightarrow 1$ .  
QCD at High Q<sup>2</sup>: Note  $d$  refers to  $d$  quark in the proton, which is the same as  $u$  in the neutron.  $d/u=0.2$ ;  $x=1$ .
- $F_2(-P) = 2x \cdot d$
- $F_2(-N) = 2x \cdot u$
- $F_2(\bar{-P}) = 2x \cdot u$
- $F_2(\bar{-N}) = 2x \cdot d$
- Elastic/quasielastic + resonance at high Q<sup>2</sup> dominated by magnetic form factors which have a dipole form factor times the magnetic moment
- $F_2(-P) \rightarrow A=0$  (no quasil) + B(Resonance  $c=+2$ )
- $F_2(-N) \rightarrow A G_m$  (quasil) + B(Resonance  $c=+1$ )
- $F_2(\bar{-P}) \rightarrow A G_m$  (quasil) + B(Resonance  $c=0$ )
- $F_2(\bar{-N}) \rightarrow A=0$  (no quasil) + B(Resonance  $c=-1$ )

## In the DIS LIMIT

- $F_2(-P) / F_2(-N) = d/u = 0.2$
- $F_2(-P) / F_2(\bar{-P}) = d/u = 0.2$
- $F_2(-P) / F_2(\bar{-N}) = 1$
- $F_2(-N) / F_2(\bar{-P}) = 1$

## Quasi ELASTIC TERM ONLY

- $F_2(-P) / F_2(-N) = 0$
- $F_2(-P) / F_2(\bar{-P}) = 0$
- $F_2(-P) / F_2(\bar{-N}) = 0/0$
- $F_2(-N) / F_2(\bar{-P}) = 1$

FAILS TEST MUST TRY TO COMBINE Quasielastic and first resonance)

Study of the reaction  $\nu_{\mu}d \rightarrow \mu^{-}pp_{\nu}$

K. L. Miller,<sup>\*</sup> S. J. Barish,<sup>†</sup> A. Engle  
Carnegie-Mellon University, PA

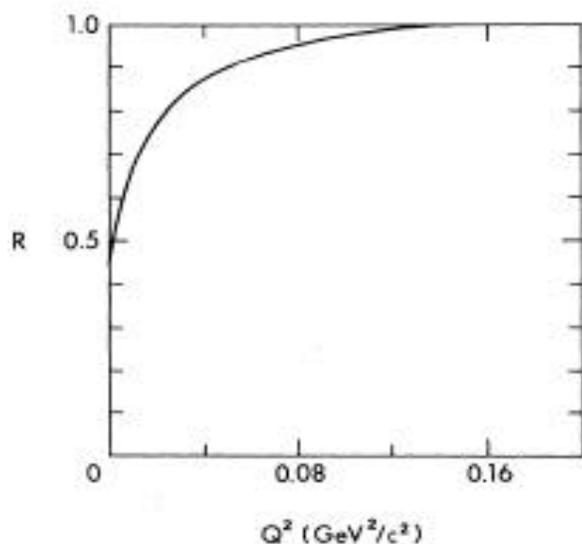


FIG. 3. Deuterium correction factor  $R(Q^2)$ .

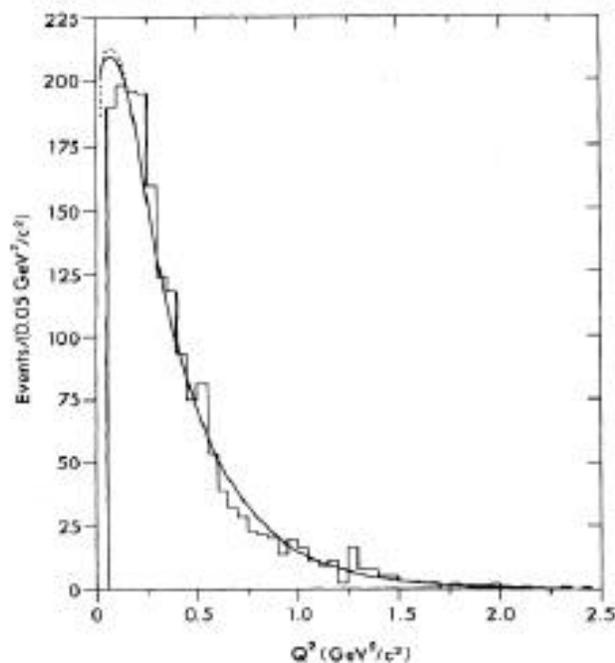
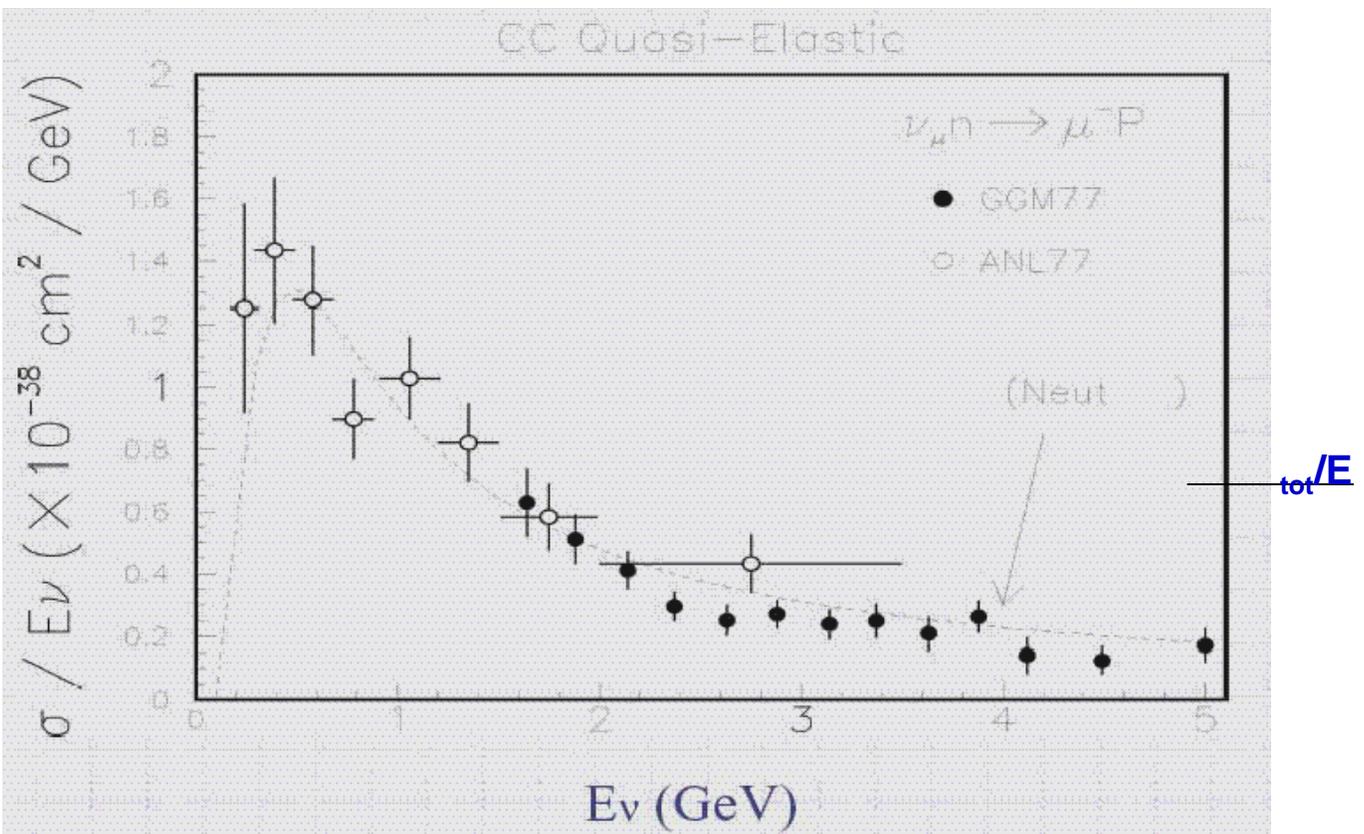


FIG. 4. Weighted  $Q^2$  distribution. The solid curve is from a maximum-likelihood fit to the dipole model ( $M_A = 1.00 \text{ GeV}/c^2$ ). The dotted curve is from a fit to the AVMD model ( $M_A = 1.11 \text{ GeV}/c^2$ ).

Are

**Examples of Current Low Energy Neutrino Data:  
Quasi-elastic cross section -  
Flux errors are about 10% to 20% now**



Next generation experiments need these cross sections to 1% to get precise neutrino mixing angles

# Backup Slides on Importance for Neutrino Experiments

# Importance of Precision Measurements of $P(\nu_\mu \rightarrow \nu_e)$ Oscillation Probability with $\nu_\mu$ and $\bar{\nu}_\mu$ Superbeams

- Conventional “superbeams” of both signs (e.g. NUMI) will be our only windows into this suppressed transition
  - Analogous to  $|V_{ub}|$  in quark sector (CP phase **could be origin of matter-antimatter asymmetry in the universe**)
  - (The next steps:  $\mu$  sources or “beams” are too far away)

Studying  $P(\nu_\mu \rightarrow \nu_e)$  in neutrinos and anti-neutrinos gives us magnitude and phase information on  $|U_{e3}|$

[http://www-numi.fnal.gov/fnal\\_minos/new\\_initiatives/loi.html](http://www-numi.fnal.gov/fnal_minos/new_initiatives/loi.html) A.Para-NUMI off-axis

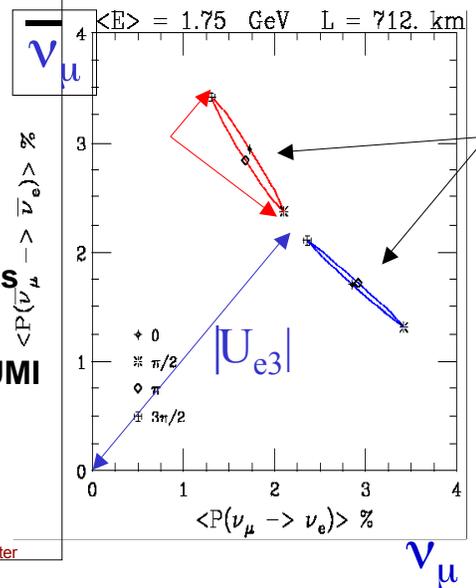
<http://www-jhf.kek.jp/NP02> K. Nishikawa JHF off-axis

<http://www.pas.rochester.edu/~ksmcf/eoi.pdf>

K. McFarland (Rochester) - off-axis near detector NUMI

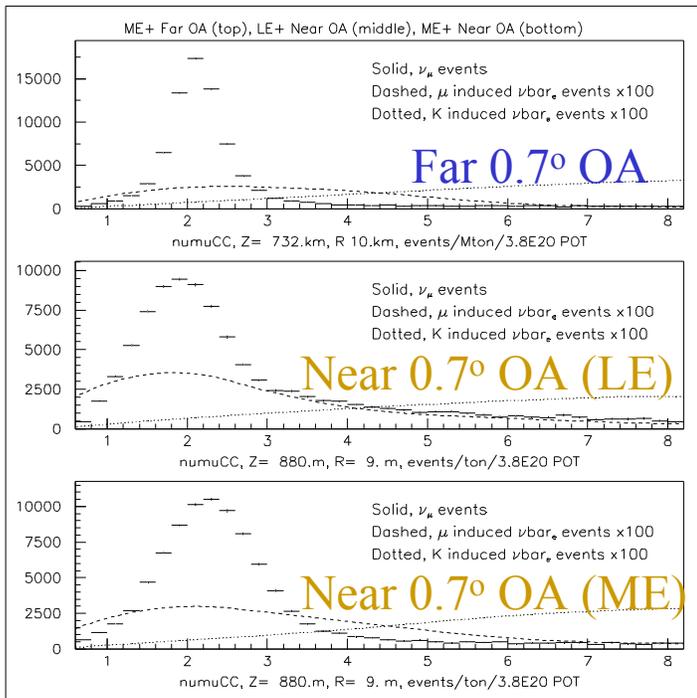
[http://home.fnal.gov/~morfin/midis/midis\\_eoi.pdf](http://home.fnal.gov/~morfin/midis/midis_eoi.pdf).

J. Morfin (FNAL-) Low E neutrino reactions in an on-axis near detector at MINOS/NUMI



Matter effects  
Sign of  $m_{23}$

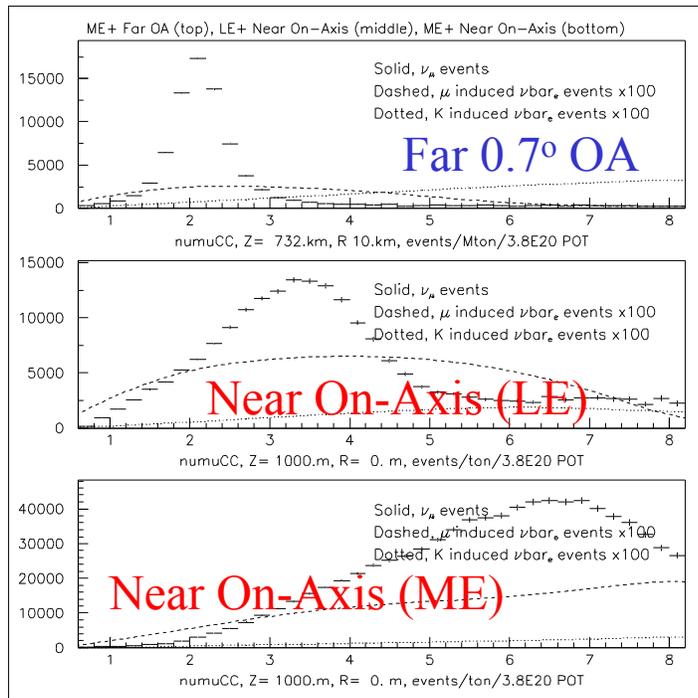
# Event Spectra in NUMI Near Off-Axis, Near On-Axis and Far Detectors (The miracle of the off-axis beam is a nearly mono-energetic neutrino beam making future precision neutrino oscillations experiments possible for the first time)



1 2 3 4 5 6 GeV

Neutrino Energy

Arie Bodek, Univ. of Rochester



1 2 3 4 5 6 GeV

Neutrino Energy