G O S I A: An error in the relativistic transformation

Doug Cline, Tom Rao, and Adam Hayes
Department of Physics and Astronomy, University of Rochester
June 23, 2010

1 Abstract:

A Rochester study of the calculated particle-\(\gamma\) angular correlations has identified a serious error in the current version of GOSIA [Gosia_20081208] when used to calculate projectile excitation. The calculated \(\gamma\)-ray angular distribution for projectile excitation has the symmetry axis in the correct direction but the relativistic transformation is made assuming that the projectile is moving at an angle \(-\theta_{\text{projectile}}\), that is, the wrong sign is used. Both the symmetry axis and relativistic correction are calculated correctly for target excitation. This error presumably has existed for many years but has not been identified previously at Rochester because 4\(\pi\) \(\gamma\)-ray detector arrays like Gammasphere, or annular symmetry have been used for which the error cancels. This coding error has important implications for current exotic-beam Coulomb excitation studies that exploit projectile excitation. A corrected version of the code Gosia_20100616.f has been created that eliminates the coding error and gives the correct results when tested using the Rochester battery of Gosia tests codes. This paper outlines the consequences of this coding error, the changes made to eliminate the error, and the results of the tests of the corrected version.

2 Introduction

At Rochester we have reopened an investigation of the possibility of making viable measurements of magnetic moments of excited states via the deorientation effect for recoil in vacuum. In 1989 we abandoned such studies because of the complexity of the process and the intractability of using it for extracting viable magnetic moment measurements from deorientation effect data. The new investigation exploits our high statistics Coulomb excitation data set for \(^{178}\text{Hf}\) on \(^{208}\text{Pb}\). Gosia simulation studies of typical angular correlations have been performed to investigate how to best cluster the 100 Ge detectors in Gammasphere coupled with the symmetries of the heavy-ion detector CHICO to achieve the highest sensitivity to magnetic moments while simultaneously minimizing the number of \(\gamma\)-ray spectra that require peak fitting. An unexpected consequence of this study is that it identified that for projectile excitation the relativistic transformation to the angular correlation is made in the wrong direction as described below. A July 15 2010 Rochester memo reporting this error resulted in Nigel Warr creating an updated version Gosia_20100616.f that corrects this problem.

3 Symmetries of the \(p - \gamma\) angular correlation

The symmetry of the hyperbolic orbit, plus time reversal symmetry, imply that the statistical tensors will exhibit symmetry properties with regard to both the plane of the orbit and the main axis of the hyperbolic orbit. Coordinate system \(B\), shown in figure 1, has the \(z\) axis along the main axis of the hyperbola and exhibits the following symmetry of the statistical tensor

\[ \rho_{k\kappa} = (-1)^k \rho_{k,-\kappa} \]

This implies that the \(\rho_{k,\kappa}\) are real for \(k + \kappa\) even, and imaginary for \(k + \kappa\) negative. As a consequence the angular distribution of deexcitation \(\gamma\)-rays following excitation by projectile scattering at an angle \(\theta_{\text{projectile}}\) has the symmetry axis along the main axis of the classical hyperbolic scattering orbit.
Figure 1: Coordinate system used to evaluate the Coulomb excitation amplitudes. The origin is chosen in the center of mass of the target nucleus. The $x$-axis is perpendicular to the plane of the orbit, the $z$-axis is along symmetry axis of the incoming projectile orbit pointing towards the projectile, while the $y$-axis is chosen such that the $y$-component of the projectile velocity is positive. The scattering angle $\theta$ of the projectile is shown. Note that Alder and Winther (North Holland 1975) call this coordinate system B.

For inelastic scattering the scattered projectile mass is unchanged, and for heavy-ion Coulomb excitation the excitation energy is negligible compared with the incident beam energy. Therefore, to a good approximation the target recoils along the main axis of the hyperbola in the negative direction. As a consequence, to a good approximation, *in the emitting nucleus frame of reference the symmetry axis of the $p-\gamma$ ray angular correlation is along the target recoil direction for both target or projectile excitation.*

The relativistic transformation of the angular distribution required to correct for photon emission by a moving emitter is along the target recoil direction for target excitation and thus the symmetry of the angular correlation along the main axis of the hyperbolic orbit is retained for target excitation. By contrast, for projectile excitation the relativistic transformation produces forward focussing along the projectile recoil direction which is very different from the symmetry axis. That is, for projectile excitation the relativistic transformation of the $\gamma$-ray angular correlation is along the projectile recoil direction which for similar projectile and target masses can be nearly perpendicular to the target recoil direction destroying the symmetry of the angular correlation about the main axis of the hyperbola. Thus for projectile excitation only the up-down symmetry about the scattering plane remains.

4 Gosia simulations of the $p-\gamma$ correlations using Gosia_20081208

The following tests identify the coding problem in Gosia_20081208 and earlier versions.

4.1 Test case 1: 890 MeV $^{178}$Hf beam on $^{208}$Pb

The symmetry axis of the emitting nucleus should point along the direction of the recoiling target nucleus, that is the major axis of the hyperbolic orbit. The symmetry axis is best identified by studying the angular distribution for the weakly populated $2^+$ $\gamma$-band state decaying the to $0^+$ ground state. The forward focussing due to the relativistic transformation from the moving emitter frame to laboratory frame is best illustrated by studying the angular distribution for decay of an excited $0^+$ state to the ground-band $2^+$ state since the angular distribution in the emitter frame is isotropic. Two such transitions are used in all of the following tests. The test case 1 has normal single-valued kinematics characteristic of scattering when the mass of the projectile is less than the target nucleus.
Figure 2: Target Coulomb excitation of $^{208}$Pb by a $890\text{ MeV}$ $^{178}$Hf beam scattered at $\theta_{\text{proj}} = 60^\circ$. Left: shows a $2^+$ state at 1.175$\text{ MeV}$ decaying to the $0^+$ ground state. Right shows a 1.0$\text{ MeV}$ $0^+$ state decaying to the ground-band $2^+$ state. The calculated yield using Gosia_20081208 is plotted versus $\theta_\gamma$ which is measured with respect to the incident beam direction.

4.1.1 Target excitation:

Gosia simulations have shown that the symmetry axis of the $p-\gamma$ angular correlations is along the major axis of the hyperbolic orbit, which also is the target recoil direction, both for target and projectile Coulomb excitation. Figure 2 illustrates an example for Coulomb excitation of a $^{208}$Pb target by a $^{178}$Hf beam scattered at a laboratory angle of $\theta_{\text{proj}} = 60^\circ$. It was assumed that $^{208}$Pb has a rotational ground state band, plus an excited $2^+$ state at an excitation energy of 1.175$\text{ MeV}$ and a $0^+$ state at 1.0$\text{ MeV}$ that decays to the ground-band $2^+$ state, that is, for simulation purposes the level scheme of this pseudo lead nucleus is the same as that for $^{178}$Hf. The excited $^{208}$Pb target nucleus recoils at an angle of $\theta_{\text{target}} = -36^\circ$ which is anti-aligned with the $\theta_{\text{orbit}} = 144^\circ$ of the major axis of the hyperbolic scattering orbit. Figure 2 left shows that for target excitation the two equally strong peaks for $\gamma$-ray emission by the recoiling $^{208}$Pb target nucleus are at $\theta_\gamma = +4^\circ$ and $-76^\circ$, that is, at angles that are $\pm 40^\circ$ of the target recoil direction of $\theta_{\text{target}} = -36^\circ$. The other two peaks have equal but weaker strength and lie at $\theta_\gamma = 94^\circ$ and $194^\circ$, that is, angles that are $\pm 50^\circ$ to the $144^\circ$ angle of the major axis of the hyperbola. As predicted the relativistic transformation is aligned with the target recoil direction resulting in a 15.5% increase of the two forward peaks and 15.5% decrease for the two rearward peaks. Figure 2 right shows the angular distribution for the decay of the $0^+$ state, which is isotropic in the emitter frame, and thus only the $\pm 15.5\%$ relativistic forward focussing is manifest.

4.1.2 Projectile excitation:

Figure 3 shows the case of the identical kinematics to above except that the levels used above now are assigned to be in the $^{178}$Hf nucleus, that is, the case of projectile excitation of the $^{178}$Hf. Figure 3 left shows that for Coulomb excitation of the $2^+$ state the symmetry axis of the $^{178}$Hf projectile excitation again is along the target recoil direction of $\theta_{\text{target}} = -36^\circ$, that is, it is aligned with the major axis of the hyperbolic orbit. However, the four peaks of the $\gamma$-ray angular correlation have different intensities due to the fact that the axis of the relativistic transformation is not aligned with the target recoil direction. Figure 3 right shows that the $\pm 12.1\%$ angular distribution for decay of the $0^+$ state is a maximum at $\theta = -60^\circ$ and a minimum at $120^\circ$. These angles have the opposite sign but same magnitude as the correct value of $\theta_{\text{proj}} = 60^\circ$. That is, the correction is displace by $120^\circ$ from the correct direction resulting in about a 20% error in the calculated intensity.
4.2 Test case 2: 890 Mev $^{178}$Hf on $^{60}$Ni

This example involves Coulomb excitation of $^{178}$Hf by $^{60}$Ni for which the kinematics is two-valued, while the inverse reaction at the same center of mass angle has kinematics that is single-valued. The conclusions for this case are the same as mentioned above, namely that the relativistic forward focussing is made assuming that the projectile is moving at an angle $-\theta_{\text{projectile}}$, that is, the wrong sign is used.

5 Updated Gosia code: Gosia_20100616

In response to the July 15 2010 version of this memorandum describing the error, Nigel Warr identified a coding problem in the function RECOIL, right at the beginning of the subroutine, where it has: `CALL ROTATE(Alab,Attl,-Theta,7,2)`. He states that he guesses that this should be $+\Theta$ for projectile excitation and $-\Theta$ for target excitation. The same function is called again at the end with the opposite sign to rotate it back. So this has to be handled symmetrically. As Theta is here a formal parameter to the subroutine RECOIL, it can simply pass the appropriate sign to it. This is probably easier, because there is no information in RECOIL as to whether it is a projectile or target excitation. The coding was changed to define a variable \( trec2 \) in ANGULA and have:

\[
\text{trec2} = \text{SIGN}(\text{TREC}, \text{DBLE}(\text{IZI}(\text{IEXP})))
\]

\[
\text{IF ( ITTE(IEXP).NE.1 ) CALL RECOIL(alab,attl,bt,trec2)} ! Relativistic correction
\]

instead of:

\[
\text{IF ( ITTE(IEXP).NE.1 ) CALL RECOIL(alab,attl,bt,Trec)} ! Relativistic correction
\]

where Trec is the existing "theta of the recoiling nucleus" variable and

\[
\text{IZI(IEXP)}\) is the Z_n for each experiment that you give in EXPT, i.e. it is negative for projectile excitation, so trec2 will be the same as Trec for target excitation and trec2 = -Trec for projectile excitation.

A corrected version of the Gosia code, called Gosia_20100616.f, has been created that eliminates the problem. This version has been tested at Rochester as described below.
Figure 4: Target Coulomb excitation of $^{208}$Pb by a $^{890}$Hf beam scattered at $\theta_{proj} = 60^\circ$. Left: shows a $2^+$ state at 1.175MeV decaying to the $0^+$ ground state. Right: shows a 1.0MeV $0^+$ state decaying to the ground-band $2^+$ state. The calculated yield using Gosia_20100616 is plotted versus $\theta_\gamma$ which is measured with respect to the incident beam direction.

6 Gosia simulations of the $p-\gamma$ correlations using Gosia _20100616

6.1 Test case 1: $^{890}$MeV $^{178}$Hf beam on a $^{208}$Pb target: $\theta_{proj} = 60^\circ$

This is a case used previously where the projectile is lighter than the target nucleus and thus has only one kinematic solution. As done above, the level scheme and matrix elements are assigned either to the target for target excitation and projectile for projectile excitation.

6.1.1 Target excitation:

Figure 4 left shows that the minimum along the symmetry axis is at $\theta_{target} = -36^\circ = 324^\circ$ which is the correct direction, that is, along the target recoil direction as was the case with the prior version of Gosia. Figure 4 right shows that the relativistic forward focussing also is along this target recoil direction. These results are the same as obtained with the older version of Gosia and illustrated in figure 2.

6.1.2 Projectile excitation:

Figure 5 left shows that the symmetry axis is in the correct direction $\theta_{target} = -36^\circ = 324^\circ$. However figure 5 left shows that the intensity of the second peak at around 100$^\circ$ is much stronger than calculated by the incorrect version of Gosia shown in figure 3 left. That is, the correct relativistic transformation produces a significantly different result. Figure 4 right shows that the relativistic forward focussing maximum now occurs in the correct direction $\theta_{proj} = 60^\circ$ which is very different from what is shown in figure 3 right which was obtained using the faulty version of Gosia.

Thus for normal kinematics the $p-\gamma$ angular correlations for both target and projectile excitation are correctly predicted using the corrected version of Gosia.

6.2 Test case 2: $^{890}$MeV $^{178}$Hf beam on a $^{60}$Ni target: $\theta_{proj} = 9.758^\circ$

This is a case of inverse kinematics where the projectile mass is greater than the target nucleus mass and thus the kinematic solutions are two-valued. The maximum scattering angle occurs at $\theta_{proj} = 19.664^\circ$. 

5
Figure 5: Projectile Coulomb excitation of a $890\,MeV\,^{178}\text{Hf}$ beam scattered at $\theta_{\text{proj}} = 60^\circ$ by a $^{208}\text{Pb}$ target. Left: shows a $2^+\text{ state at } 1.175\,MeV$ decaying to the $0^+\text{ ground state. Right shows a } 1.0\,MeV\,0^+\text{ state decaying to the ground-band } 2^+\text{ state. The calculated yield using Gosia_20100616 is plotted versus } \theta$, which is measured with respect to the incident beam direction.

6.2.1 Backward-scattering solution: Target excitation:

For this solution the excited target recoils at an angle of $\theta_{\text{target}} = -10^\circ = 350^\circ$. Figure 6 left shows that for target excitation the minimum along the symmetry axis is in the correct direction, that is, $\theta_{\text{target}} = -10^\circ = 350^\circ$. Figure 6 right shows that the forward focussing due to the relativistic correction is in the target recoil direction of $\theta_{\text{tar}} = -10^\circ$ which is correct. Note the very large relativistic forward focussing for this case where using a $5\,MeV/A$ beam results in a target nucleus recoil energy of about $10\,MeV/A$ giving a recoil velocity of $v/c = 15.3\%$ and a large forward to backward focussing ratio of 1.85.

6.2.2 Backward-scattering solution: projectile excitation:

For this solution the unexcited target recoils at an angle of $\theta_{\text{target}} = -10^\circ = 350^\circ$. Figure 7 left shows that for projectile excitation the symmetry axis is in the correct direction, that is, $\theta_{\text{target}} = -10^\circ = 350^\circ$. Figure 7 right shows that the forward focussing due to the relativistic correction now is in the projectile direction of $\theta_{\text{proj}} = 9.758^\circ$ which is correct.

6.2.3 Forward-scattering solution: Target excitation:

For $\theta_{\text{proj}} = 9.758^\circ$ and the forward scattering solution the target recoils at an angle of $\theta_{\text{target}} = -69.9^\circ = 290.1^\circ$. For target excitation the symmetry axis of the $p - \gamma$ angular distribution occurs at $\theta_{\text{target}} = -69.9^\circ = 290.1^\circ$ as shown in figure 8 left. The maximum of the relativistic forward focussing also occurs at $\theta_{\text{target}} = -69.9^\circ = 290.1^\circ$ as shown in figure 8 right.

6.2.4 Forward-scattering solution: Projectile excitation:

For projectile excitation at $\theta_{\text{proj}} = 9.758^\circ$ the forward scattering solution gives the target recoils at an angle of $\theta_{\text{target}} = -69.9^\circ = 290.1^\circ$. For projectile excitation the symmetry axis of the $p - \gamma$ angular distribution occurs at $\theta_{\text{target}} = -69.9^\circ = 290.1^\circ$ as shown in figure 9 left. However, the maximum of the relativistic forward focussing occurs at $\theta_{\text{proj}} = 9.758^\circ$ as shown in figure 9 right.

Thus test cases 1 and 2 show that the corrected version Gosia_20100616 predicts the symmetry axis and the relativistic forward focussing in the correct directions for normal kinematics and for both solutions with inverse kinematics.
Figure 6: Backward scattering solution for target Coulomb excitation of $^{60}$Ni by a $890\,MeV$ $^{178}$Hf beam scattered at $\theta_{\text{proj}} = 9.758^\circ$. Left: shows a $2^+$ state at $1.175\,MeV$ decaying to the $0^+$ ground state. Right shows a $1.0\,MeV$ $0^+$ state decaying to the ground-band $2^+$ state. The calculated yield using Gosia_20100616 is plotted versus $\theta_\gamma$ which is measured with respect to the incident beam direction.

Figure 7: Backward scattering solution for projectile Coulomb excitation of a $890\,MeV$ $^{178}$Hf beam scattered by $^{60}$Ni at $\theta_{\text{proj}} = 9.758^\circ$. Left: shows a $2^+$ state at $1.175\,MeV$ decaying to the $0^+$ ground state. Right shows a $1.0\,MeV$ $0^+$ state decaying to the ground-band $2^+$ state. The calculated yield using Gosia_20100616 is plotted versus $\theta_\gamma$ which is measured with respect to the incident beam direction.
Figure 8: Forward scattering solution for target Coulomb excitation of $^{60}$Ni by a $^{178}$Hf beam scattered at $\theta_{\text{proj}} = 9.578^\circ$. Left: shows a 2$^+$ state at 1.175 MeV decaying to the 0$^+$ ground state. Right shows a 1.0 MeV 0$^+$ state decaying to the ground-band 2$^+$ state. The calculated yield using Gosia_20100616 is plotted versus $\theta_\gamma$, which is measured with respect to the incident beam direction.

Figure 9: Forward scattering solution for projectile Coulomb excitation of $^{89}$Ni by a $^{178}$Hf beam scattered at $\theta_{\text{proj}} = 9.578^\circ$ by a $^{60}$Ni target. Left: shows a 2$^+$ state at 1.175 MeV decaying to the 0$^+$ ground state. Right shows a 1.0 MeV 0$^+$ state decaying to the ground-band 2$^+$ state. The calculated yield using Gosia_20100616 is plotted versus $\theta_\gamma$, which is measured with respect to the incident beam direction.
6.3 Test Case 3:

Adam Hayes has tested the corrected version of Gosia using six test cases which include integrations and fits to simulated data. All tests so far give only rounding error differences.

7 Conclusions

7.0.1 Gosia_20081208 and earlier versions:

1) The \( p - \gamma \) angular distribution and relativistic transformation both are calculated correctly by GOSIA for target excitation.
2) The \( p - \gamma \) angular distribution is calculated correctly by GOSIA for projectile excitation but the relativistic transformation is made using the wrong sign for \( \theta_{proj} \).
3) This is not an error in the input to Gosia because the angular distribution is calculated correctly whereas simultaneously the relativistic transformation is incorrect for projectile excitation.
4) This error can have a large effect on current projectile Coulomb excitation studies using exotic beams that do not use 4\( \pi \) \( \gamma \)-ray detector arrays or involve axial symmetry.
5) Fortunately this coding error is of no consequence for cases where the \( \gamma \)-ray detector has spherical symmetry, i.e subtends 4\( \pi \) solid angle, or the particle detector has axial symmetry. As a consequence this coding error has not impacted prior Coulomb excitation work performed by the Rochester group.
6) Probably this error was introduced in early versions of Gosia but was not recognized because our earlier projectile excitation studies have used 4\( \pi \) \( \gamma \)-ray detector arrays or axial symmetry for which the coding error cancels.

7.0.2 Corrected version: Gosia_20100616:

1) The \( p - \gamma \) angular distribution and relativistic transformation both are calculated correctly by GOSIA_20100616 for both target and projectile Coulomb excitation for both normal and inverse kinematics. This new version of Gosia also gives the correct results when subjected to the battery of Gosia test cases developed at Rochester by Adam Hayes.
2) The coding change should be applied to Gosia2.
3) The community should be informed immediately of the coding error and encouraged to update to the newest versions of Gosia. This is important because of the current emphasis on projectile Coulomb excitation using exotic beams.