Chapter 11  MAGNETISM IN MATTER

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INTRODUCTION

In a dielectric the electric field induces electric dipole moments or aligns permanent electric dipoles. This electric polarization partially cancels the electric field inside the dielectric. Similar behaviour occurs for magnetic materials where atomic magnetic moments align with the external magnetic field. However, in contrast to dielectrics, where the dielectric weakens the electric field, the magnetic field inside ferromagnetic material can be greatly enhanced. Since magnetic dipole moments are responsible for magnetic effects it is useful to review the properties of the magnetic dipole.

MAGNETIC DIPOLE

The magnetic dipole moment $\vec{\mu}$ of a current loop having $N$ turns, of area $A$ and current $I$, was defined as

$$\vec{\mu} = NIA\hat{n}$$

where $\hat{n}$ is the normal to the current loop taken in a right-handed direction. It is useful to discuss the magnetic field due to a magnetic dipole. Computation of the magnetic field from a magnetic dipole is a straightforward application of what you have learned but the calculation is complicated except on the axis of the dipole.

Figure 1 compares the electric field due to an electric dipole with the magnetic field due to a magnetic dipole. This difference has important consequences.

Figure 1  Comparison of the electric field due to an electric dipole with the magnetic field due to a magnetic dipole.

leads to dramatically different behaviour in magnetic materials compared to dielectrics.

The torque on a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ was shown to be $\vec{\tau} = \vec{\mu} \times \vec{B}$, while the torque on an electric dipole $\vec{p}$ in an electric field $\vec{E}$ is $\vec{\tau} = \vec{p} \times \vec{E}$. In both cases, the torque aligns the dipole with the corresponding electromagnetic field. In both cases, the electric and magnetic dipole moments align with the corresponding applied external field. An electric dipole produces an electric field adjacent to the dipole that partially cancels the external electric field that aligns the dipole. In contrast, a magnetic dipole produces a magnetic field adjacent to the dipole that enhances the external magnetic field aligning the dipole. This difference has important consequences.

MICROSCOPIC MAGNETISM IN MATTER

As an example of typical atomic magnetic moments, consider a hydrogen atom. Equating the centripetal acceleration and the Coulomb force implies that the electron revolves around the proton with a velocity of $2.2 \times 10^6 m/s$ producing an average current of 1
milliamp in a circular orbit of radius $0.53 \times 10^{-10}m$. That is, the magnetic moment is $\mu = \pi r^2 I = 0.93 \times 10^{-23}Am^2$. In a model where all the atomic current loops are aligned, then the currents from adjacent atoms cancel in the interior leaving an effective current only at the surface of the material, as shown in figure 2.

That is, the material looks like a macroscopic-sized magnetic dipole due to the effective atomic surface current times the cross-sectional area of the material which can be $10^{16}$ times larger than the area of a single atom. This surface current, called an *amperian current*, is similar to the real current in a solenoid. This surface current produces an axial magnetic field in the material similar to that produced by a solenoid.

It is useful to define the *magnetization* $\mathbf{M}$ which is the magnetic moment per unit volume.

$$\mathbf{M} = N\mathbf{\mu}$$

where there are $N$ magnetic dipoles per unit volume each of magnetic moment $\mu$.

Consider Ampère’s law for magnetostatics

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_{\text{Enclosed current}} \mathbf{j} \cdot d\mathbf{S}$$

The current must include all possible currents, both free and bound currents, that pass through the surface bounded by the closed curve $C$. The only bound currents that contribute to the right-hand integral are those where the closed loop passes through the area prescribed by the circulating atomic current.

As shown in figure 3, consider that there are $N$ atomic dipoles per unit volume that circulate in an orbit of radius $r$ and current $I$. Then for a path length $l$ there will be a net bound current around the line for all magnetic dipoles in the volume $\pi r^2 l \cos \theta$. That is

$$I_{\text{Bound}} = N\pi r^2 I l \cos \theta = N\mathbf{\mu} \cdot \mathbf{l} = \mathbf{M} \cdot \mathbf{l}$$

which is independent of the size and other details of the atomic magnetic moments. Thus for a closed loop, the net bound current passing through the enclosed loop is

$$I_{\text{Bound}} = \oint \mathbf{M} \cdot d\mathbf{l}$$

Inserting this bound current into Ampère’s law for magnetostatics gives

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_{\text{Enclosed current}} \mathbf{j}_{\text{free}} \cdot d\mathbf{S} + \mu_0 \oint \mathbf{M} \cdot d\mathbf{l}$$

where $\mathbf{j}_{\text{free}}$ is the applied current density.

In the absence of magnetic material, the magnetic field, called $\mathbf{B}_{\text{applied}}$, is given by

$$\oint_C \mathbf{B}_{\text{applied}} \cdot d\mathbf{l} = \mu_0 \int_{\text{Enclosed current}} \mathbf{j}_{\text{free}} \cdot d\mathbf{S}$$

It is useful to define a magnetic susceptibility given by the relation

$$\mathbf{M} = \chi_m(\mathbf{B}_{\text{applied}}) \mu_0$$

Inserting this into Ampère’s Law gives that

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \oint_C \mathbf{B}_{\text{applied}} \cdot d\mathbf{l} + \chi_m \mu_0 \oint_C \mathbf{B}_{\text{applied}} \cdot d\mathbf{l}$$

That is, since the line integral $C$ is the same, the actual magnetic field in matter is given by

$$\mathbf{B} = (1 + \chi_m)\mathbf{B}_{\text{applied}}$$

Define the relative permeability $\kappa_m$

$$\kappa_m = 1 + \chi_m$$

Then this gives that the actual magnetic field in matter is related to the applied magnetic field by

$$\mathbf{B} = \kappa_m \mathbf{B}_{\text{applied}}$$
This is analogous to the dielectric constant with the notable difference that the actual magnetic field inside matter is $\kappa_m$ times the field in the absence of matter, whereas, the actual electric field inside matter is reduced by the factor $\kappa_E$, that is;

$$\mathbf{B}_{\text{matter}} = \kappa_m \mathbf{B}_{\text{applied}}$$

$$\mathbf{E}_{\text{matter}} = \frac{\mathbf{E}_{\text{applied}}}{\kappa_E}$$

Ampère’s Law in the presence of matter can be rewritten using the relation between $B$ and $B_{\text{applied}}$:

$$\oint \frac{1}{\kappa_m} \mathbf{B} \cdot d\mathbf{l} = \mu_0 \mathbf{j}_{\text{current}} \cdot \mathbf{S}$$

This relation is very important for solving magnetic circuits as will be illustrated later. Note that this implies a much stronger $B$ field for a given enclosed current in the case of a magnetic material with a large relative permeability.

### MAGNETIC BEHAVIOR OF MATTER

In a non-uniform magnetic field it was demonstrated that magnetism comes in three flavors; diamagnetic materials are weakly repelled from the high field region whereas paramagnetic materials are attracted weakly and ferromagnetic materials attracted very strongly to the high-magnetic field region. That is, diamagnetism has a small and negative $\chi_m$, that is, the induced magnetic moment is small and opposite to the applied field, paramagnetism where $\chi_m$ is small and positive, and ferromagnetism where $\chi_m$ is up to $10^5$, that is, $10^{10}$ time larger than the other two cases. Figure 4 lists examples of the magnetic susceptibility of typical materials. Whereas the relative permeability $\kappa_m$ is very close to unity for diamagnetism and paramagnetism, it can be as large as $10^5$ for a ferromagnetic material.

An understanding of atomic physics is needed to understand magnetic properties of atoms. Quantum physics leads to the conclusion that electrons in the atom prefer to pair off in contra-rotating orbits.

**Diamagnetism** is a consequence of Faraday’s law; when the magnetic flux through the atom is changed, an induced emf slightly changes the orbital angular momentum of both paired electrons in such a direction as to oppose the change in magnetic flux. Thus both of the paired electrons give a net negative magnetic moment that is induced opposing the applied field. This means that the magnetic susceptibility is negative, but the induced moment is very small. Effectively, the two identical magnitude contra-rotating atomic currents cancel when there is no magnetic field, but they become unbalanced due to an imbalance caused by the Faraday effect. The net magnetic moment is proportional to the applied magnetic field. Note that diamagnetism occurs in all atoms but it is swamped by paramagnetism or ferromagnetism when these other mechanisms coexist.

**Paramagnetism** occurs for atoms with one unpaired electron. An isolated electron in an orbit has a magnetic moment. The torque on the magnetic moment of this unpaired electron orbit in a magnetic field tends to align the atoms just like polarization of dielectrics in an electric field. Cohesive forces in matter, or thermal motion, both tend to oppose alignment of the atomic magnetic moments. The net effect is proportional to the applied magnetic field just like the analogous situation in dielectrics.

**Ferromagnetism** occurs only for selected atoms like iron, cobalt, nickel, gadolinium and dysprosium. The magnetic moment of these atoms is large enough to cause strong interactions between neighboring atoms resulting in small localized regions where the magnetic moments of adjacent atoms are completely aligned leading to very large magnetic fields in the ferromagnetic material. These regions of magnetic alignment are called magnetic domains and are about $10^{-2}mm$ in size.

When an external magnetic field is applied, the magnetic domains that are aligned along the applied B field grow at the expense of the other magnetic domains. Eventually, when all the domains are aligned along the magnetic field, the magnetization of the ferromagnetic material saturates as illustrated in figure 6. In contrast to diamagnetism and paramagnetism, where the magnetization depends linearly on the applied magnetic field, the magnetization for ferromagnetism depends on the applied magnetic field in a very non-linear manner as it approaches saturation. More-
Figure 5 (a) Domain formation in ferromagnetic materials in zero external magnetic field. The arrows indicate the magnetic moments of individual domains. (b) The presence of an external magnetic field influences the domains, making some larger and realigning others.

Figure 6 Saturation of magnetization in a ferromagnetic material.

Figure 7 Plot of the actual magnetic field $B$ in a hard ferromagnetic material versus the applied magnetic field $B_{\text{applied}}$.

Figure 8 Plot of the actual magnetic field $B$ in a soft ferromagnetic material versus the applied magnetic field $B_{\text{applied}}$.

over, the magnetization is $10^8$ to $10^{10}$ times stronger, that is, the magnetic field in the matter is enhanced by the relative permeability that is as large as $10^5$.

Ferromagnetics become paramagnetics at high temperature when the thermal motion destroys the order in the magnetic domains. This is illustrated by heating an iron needle, the ferromagnetism disappears at the Curie temperature.

The discontinuities in the magnetic domains result in discontinuous jumps in the magnetization as the ferromagnetic is magnetized or demagnetized. This is illustrated by the Barkhausen effect where you can hear a hiss when the external magnetic field is rotated due to discontinuous jumps in growth of domains. Energy is lost in this process. This energy loss is characterized by hysteresis in the plot of $\vec{B}$ versus $\vec{B}_{\text{applied}}$ shown in figure 7 for a hard ferromagnetic material and in figure 8 for a soft magnetic material. As will be shown, the area of the hysteresis loop is a measure of the energy lost as the sample is taken around the hysteresis loop. Large area loops as shown in figure 7 are required for a permanent magnet so that a large remanent magnetic field exists when the applied magnetic field is switched off. On the other hand, the cores of transformers require a small area hysteresis loop, figure 8, to minimize...
energy dissipation due to the hysteresis for AC fields.

Since the magnetic susceptibility $\chi_m$ for diamagnetics is about $-10^{-5}$ and is about $+10^{-3}$ for para-
magnetics, then in both cases the relative permeability $\kappa_m = 1.000 \pm 10^{-3}$. For such materials the influence of matter on the magnetic field is negligible. In con-
trast, the relative permeability for a ferromagnetic can be $10^{+3} - 10^{+5}$ and thus the material greatly enhances the applied magnetic field.

**BOUNDARY CONDITIONS**

It was shown that the lines of magnetic field $B$ are continuous. Thus the normal component of $B$ at the boundary must be continuous. That is

$$\vec{B}_\perp = \vec{B}_\perp^f$$

From Ampère’s Law, if there are no free currents flowing at the boundary between materials, then

$$\int_C \vec{B} \cdot d\vec{l} = 0$$

this gives that the parallel component of $\frac{B}{\kappa_m}$ is contin-
uous across the boundary.

These can be compared with the analogous bound-
ary conditions for electrostatics where $\kappa_m \vec{E}_\perp$ and $\vec{E}_\perp$ are continuous. The difference between these two cases can be illustrated by considering a cylinder of matter, shown in figure 9, where the cylindrical axis is aligned with the $B$ field. In the case of the dielec-
tric, a surface charge distribution at the boundary of the end of the cylinder reduces the electric field inside the dielectric producing a discontinuity in the normal component of $E$ at the boundary. By contrast, the magnetic polarization of the cylinder produces an amperian current around the circumference of the cylin-
der that, in the case of a ferromagnetic, greatly enhances the $B$ field within the cylinder relative to the $B$ field outside of the cylinder. As shown in figure 9, for the boundary at the end of the cylinder the $B$ field is continuous. That is, since lines of magnetic field are continuous, the enhanced magnetic field in a ferromagnetic is equally strong outside the boundary where the field leaves the ferromagnetic material. This is of importance for applications of ferromagnetic materials in technology. The parallel component of the $B$ field is unchanged when perpendicular to the mag-
etization axis of the material. At the circumference of the cylinder the parallel component of $B_{applied}$ is unchanged but there is a large change in the actual $B$ inside versus outside the amperian current.

To illustrate the impact of a ferromagnetic material on the design of magnets and transformers, consider for simplicity the $N$ turn toroidal solenoid shown in figure 10. In the absence of matter inside the toroidal
core, the $B$ field at a radius $r$ inside the windings of the toroid, when carrying a current $I$, is given by Ampère’s Law expressed as:

$$
\oint_{C} \frac{1}{\kappa_m} \vec{B} \cdot d\vec{a} = \mu_0 \int_{\text{Enclosed \ current}} \vec{j}_{\text{free}} \cdot d\vec{S}
$$

That is, since $\kappa_m = 1$, then:

$$B = \mu_0 \frac{NI}{2\pi r}$$

Note that the maximum $B$ field is obtained by making $NI$ large and the circumference $l = 2\pi r$ small.

Filling the inside of the toroid with a ferromagnetic material with a large relative permeability $\kappa_m$ the magnetic field inside the toroid is given by:

$$B = \mu_0 \kappa_m \frac{NI}{2\pi r}$$

This is enhanced by the relative permeability which can be 5500 for iron.

The ferromagnetic core of the toroid maintains a very large $B$ field inside the toroid even if the $N$ turn winding is not uniform. The ferromagnetic material acts like the path of least resistance for the magnetic field. Thus the windings can be concentrated in a region of the core with little influence on the internal magnetic field. Consequently, a transformer can be built by winding a primary coil and separate secondary coil both wrapped on a ferromagnetic core made of minimum hysteresis iron, as shown in figure 11. The core usually is laminated to reduce eddy current losses due to the alternating magnetic field.

**ELECTROMAGNET**

An electromagnet comprises a ferromagnetic core with a gap. Electromagnets are used extensively in science and technology. Consider the electromagnet formed by cutting a gap in the ferromagnetic core as shown in figure 12. Assuming that the applied magnetic field follows the path of the iron, then from Ampère’s Law:

$$
\oint_{C} \frac{1}{\kappa_m} \vec{B} \cdot d\vec{a} = \mu_0 \int_{\text{Enclosed \ current}} \vec{j}_{\text{free}} \cdot d\vec{S}
$$

This gives that

$$\oint_{C} \frac{1}{\kappa_m} \vec{B} \cdot d\vec{a} = \frac{1}{\kappa_m} B_{\text{iron}} L_{\text{iron}} + B_{\text{gap}} L_{\text{gap}} = \mu_0 NI$$

Now we know from the boundary condition that the normal component of $B$ is continuous, that is, it is unchanged inside and outside the iron, then,

$$B_{\text{iron}} = B_{\text{gap}}$$

Therefore

$$B_{\text{gap}} = \frac{\mu_0 NI}{L_{\text{gap}} + \frac{L_{\text{iron}}}{\kappa_m}}$$

Since the iron core has relative permeability of 5500, then the contribution to the line integral for the magnetic path length in the iron is negligible. Thus:

$$B_{\text{gap}} \approx \frac{\mu_0 NI}{L_{\text{gap}}}$$

Thus the iron has essentially focussed all of the contribution of the current-turns into providing the magnetic field in the gap. This is the reason for the use of iron yokes for magnets.

Iron saturates at an internal magnetic field of $B_{\text{sat}} = \mu_0 M = 2.16T$ (21.6$k$gauss), thus typical conventional electromagnets are useful up to about 20$kG$. For stronger magnetic fields it is necessary to rely on very large ampere-turn values, iron is used only for the return yoke to stop the magnetic field from permeating large distances. A practical magnet usually has a larger cross sectional area for the iron yoke, than the area of the air gap, to spread out the magnetic flux which minimizes saturation effects in the iron. Figure 13 shows a typical electromagnet where $I = 1amp$, $N = 10,000$, $L_{\text{iron}} = 2$
Figure 13  Example of a practical electromagnet

\[ A_{\text{air}} = 50\text{cm}^2, \quad A_{\text{iron}} = 100\text{cm}^2, \] then \[ B_{\text{air}} = 1.3T = 13\text{gauss}. \] If the iron core was removed then the magnetic field in the air gap would drop to about 800\text{gauss}.

Solenoids made using wires that are superconducting are used to provide very strong and large volume magnetic fields. A large cyclotron using conventional iron core and normal copper windings can consume a megawatt of power. An equivalent magnet made using superconductors consumes much less energy; once the magnetic field is established the only power requirements are for the refrigerator, the current continues flowing indefinitely with zero resistance.

SUMMARY

Ferromagnetic material greatly enhances magnetic fields, whereas paramagnetic and diamagnetic materials change magnetic fields by a negligible amount. The important feature of ferromagnetic materials is that they enhance the applied magnetic field as opposed to dielectrics where the dielectric polarization partially cancels the applied electric field. The enhancement factor is given by the relative permeability \( \kappa_m \) which can be as large as \( 10^5 \).

**Reading assignment:** Read Giancoli, Chapter 28.7-28.10.