Einstein's Universe
The Birth of Modern Physics

1900: Culmination of development of Classical Physics

- Newton’s Laws of Motion
- Laws of Thermodynamics
- Maxwell’s Equations

1905: Birth of Modern Physics

- Einstein’s Theory of Special Relativity
- Einstein proposed the photon $E = h\nu$ leading to the development of quantum physics
- Einstein introduced the quantum principle to statistical mechanics
• Inertial frames

• Special theory of relativity

• Behavior of clocks and rules in motion

• Electromagnetism

• Relativistic energy and momentum

• General relativity
An inertial frame is one in which Newton’s Laws of motion apply. Inertial frames are non-accelerating frames so that pseudo forces are not induced. All reference frames moving at constant velocity relative to an inertial reference also are inertial frames. Newton’s Laws of nature are the same in all inertial frames of reference. As a consequence there is no way of determining absolute motion because no inertial frame is preferred over any other. This is called Galilean-Newtonian invariance.
Let us consider two inertial frames as shown in figure 2 where the primed frame is moving a velocity $v$ with respect to the unprimed frame.

In Newtonian mechanics the transformation between these two inertial frames is given by the Galilean transformation:

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$
The Ether

However, at the end of last century physicists thought they had discovered an absolute frame of reference, the frame of the medium that transmitted light. Maxwell’s laws of electromagnetism predicts that electromagnetic radiation in vacuum travel at $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.998 \times 10^8$ m/s. Maxwell did not address in what frame of reference that this speed applied. In the nineteenth century all wave phenomena were transmitted by some medium, such as waves on a string, water waves, sound waves in air. Physicists thus envisioned that light was transmitted by some unobserved medium which they called the ether. This ether had mystical properties, it existed everywhere, even in outer space, and yet had no other consequences on nature. The ether obviously should be the absolute frame of reference.
Michelson-Morley Experiment 1897

Result was that the relative motion of the Earth within the ether was less than 1/6 the velocity of the Earth.
In 1905, at the age of 26, Einstein published a seminal paper entitled "On the electrodynamics of moving bodies". He considered the relation between space and time in inertial frames of reference that are in relative motion. In this paper he made the following postulate.
Einstein’s Postulates

1) The laws of nature are the same in all inertial frames of reference.

This simple postulate, has remarkable consequences. When combined with Maxwell’s equations describing the nature of light, then it implies that

2) The velocity of light in vacuum is the same in all inertial frames of reference.

This second postulate was confirmed by the Michelson-Morley experiment. However, it was not this experimental result that led Einstein to the theory of special relativity. He deduced the special theory of relativity from consideration of Maxwell’s equations of electromagnetism. Although Einstein’s postulates appear reasonable, they lead to some surprising implications.
Galilean Transformation

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Violates Einstein’s Postulates
Lorentz Transformation

Now if we insist that the Einstein’s postulate that the velocity of light be the same in all frames of reference, and that, in each frame the laws of nature define the same definition of length and time, then we are forced to conclude that the only transformation consistent with these constraints is:

\[
\begin{align*}
  x' &= \gamma (x - vt) \\
  y' &= y \\
  z' &= z \\
  t' &= \gamma \left( t - \frac{vx}{c^2} \right)
\end{align*}
\]

\[
\begin{align*}
  x &= \gamma (x' + vt) \\
  y &= y' \\
  z &= z' \\
  t &= \gamma \left( t' + \frac{vx'}{c^2} \right)
\end{align*}
\]

Where \( \gamma \) is defined as:

\[
\gamma \equiv \frac{1}{\sqrt{1 - \left( \frac{v}{c} \right)^2}}
\]
Lorentz Transformation; Light in vacuum

These are called the Lorentz transformations and they satisfy Einstein’s requirement that the laws of physics are the same in both frames of reference. To check this, consider a light wave in the fixed frame travelling in the $x$ direction with velocity $c$, then $x = ct$ or $x/t = c$. Substitute for $x$ in the Lorentz transformations gives:

$$x' = \gamma(c - v)t$$

$$t' = \gamma(c - v)\frac{t}{c}$$

Thus the velocity in the moving frame is:

$$x' = ct'$$
Lorentz Transformation; $\gamma$ factor

The factor $\gamma$, defined above, occurs prominently in the Lorentz transformations. Note that $\gamma$ is always greater than or equal to 1; that is, $\gamma \to 1.0$ as $v \to 0$, and increases to infinity as $\frac{v}{c} \to 1$ as illustrated in figure 3. A useful fact is that for $\frac{v}{c} << 1$;

$$\gamma \to 1 + \frac{1}{2} \left( \frac{v}{c} \right)^2$$

(Limit for $v << c$)
Consider that a clock, fixed at $x'_0$ in a moving frame, measures the time interval between two events in the moving frame, i.e. $\Delta t'_p = t'_1 - t'_2$. The times in the fixed frame is given by:

$$t_1 = \gamma \left( t'_1 + \frac{vx'_0}{c^2} \right)$$
$$t_2 = \gamma \left( t'_2 + \frac{vx'_0}{c^2} \right)$$

Thus the time interval is given by:

$$t_2 - t_1 = \gamma (t'_2 - t'_1)$$

$$\Delta t = \gamma \Delta t'_p$$ (Time dilation.)

The time between events in the frame of reference of the clock, $\Delta t'_p$ is called the proper time which always is the shortest time measured for a given event.
This time dilation can be understood by looking at the simple geometry of the problem and noting that light in proper frame of the clock has the shortest distance to travel as shown in figure 5. That is, in the frame b the component of velocity in the direction of the mirror is $\sqrt{c^2 - v^2}$ using Pythagoras theorem. Thus the transit time towards and back from the mirror must be

$$\Delta t = \frac{2D}{c\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

But in the proper frame

$$\Delta t'_p = \frac{2D}{c}$$

Thus we get that

$$\Delta t = \frac{2D}{c\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma \Delta t'_p$$

which is identical to that given previously.
The Lorentz transformation leads to a contraction of the length of an object in a moving frame. The length of a ruler in its own frame of reference is called the *proper length*. Consider that we place an accurately known rod of proper length $L_p = x'_2 - x'_1$ that is at rest in the moving primed frame. The locations of both ends of this rod are measured at a given time $t_o$ in the stationary frame since the rod is moving, that is, $t_1 = t_2$. The corresponding locations in the moving frame are:

\[
\begin{align*}
  x'_2 &= \gamma (x_2 + vt_2) \\
  x'_1 &= \gamma (x_1 + vt_1)
\end{align*}
\]

Since $t_2 = t_1$, the measured lengths in the two frames are related by:

\[
x'_2 - x'_1 = \gamma (x_2 - x_1)
\]

That is, the lengths are related by:

\[
L = \frac{1}{\gamma} L_p
\]

(Lorentz contraction)
Length Contraction
Simultaneity

Another surprising consequence is that the concept of simultaneity is frame dependent. Again look at the Lorentz transformation. Measure the time difference between two carefully synchronised clocks at two different locations $x_1$ and $x_2$ in the stationary frame.

\[
\begin{align*}
t'_1 &= \gamma \left( t_1 - \frac{vx_1}{c^2} \right) \\
t'_2 &= \gamma \left( t_2 - \frac{vx_2}{c^2} \right)
\end{align*}
\]

Assume that the two clocks were carefully synchronised in the fixed frame. Now at a given time $t'_1 = t'_2$ in the moving frame, then the two clocks will be seen, from the moving frame, to have a time difference

\[t_2 - t_1 = \frac{v}{c^2} (x_2 - x_1)\]

That is:

\[\Delta t = L_p \frac{v}{c^2}\]
Simultaneity
Relativistic snake

Snake proper length = 100 cm, Snake velocity v = 0.36c, \(1/\gamma = 0.80\)

**Fixed frame**

**Snake frame**

Figure 15.5 The snake paradox, as seen in the student’s frame \(S\). The cleavers fall simultaneously at time \(t = 0\).

Figure 15.6 The snake paradox, as measured in the snake’s frame \(S'\). The cleavers move to the left with speed \(V\), and the right one falls 2.5 ns before the left one. Even though the cleavers are only 80 cm apart, this lets them land 125 cm apart.
Length Contraction

Trolley car stationary

Trolley moving to the left at $v = 0.866c$
A problem that troubled physicists for many years is called the twin paradox. If you have two identical twins, Jack and Jill and send Jill off in a spaceship at a speed of $\gamma = 4$ for 20 years, as measured by Jack’s clock, and then have her return taking another 20 years, according to Jack. Thus, Jack has aged 40 years by the time his twin sister returns home. However, Jill’s clock measures $20/4 = 5$ years for each half of the trip so that she thinks she travelled for 10 years total time according to her clock. Thus she has aged only 10 years on the trip, that is, now she is 30 years younger than her twin brother.
Twin Paradox

Note that, according to Jill, the distance she travelled out and back was 1/4 the distance according to Jack, so she perceives no inconsistency in her clock, and the speed of the spaceship. This was called a paradox because some people claimed that Jill will perceive that the earth and Jack moved away at the same relative speed in the opposite direction and thus according to Jill, Jack should be 30 years younger, not her. Some people claimed that this problem is symmetric and therefore both twins must still be the same age since there is no way of telling who was moving away from whom. This argument is incorrect because Jill was able to sense that she accelerated to $\gamma = 4$ which destroys the symmetry argument. In fact it is true that Jill will be younger. Two atomic clocks were flown around the world and this effect was observed. The effect is observed with accelerated beams of unstable nuclei such as the muon. Thus the Twin paradox is not a paradox.
Electromagnetism

The unity of electromagnetism is a prediction of the Theory of Relativity

Figure 8  Charge $q$ moving with velocity $v$ parallel to a long straight current $I$ in the stationary, unprimed, frame. The primed frame is moving with velocity $v$ with the charge $q$.

$$\vec{F} = q \left( \vec{E} + \vec{v} \times \vec{B} \right)$$

Lorentz force implies there must be an E field in the rest frame of the charge
Electromagnetism

In the rest frame of the charge $q$, the theory of relativity gives that the velocities of the positive and negative line charges densities, are

$$u'_+ = \frac{u_o - v}{1 - \frac{u_o v}{c^2}}$$

$$u'_- = \frac{u_o + v}{1 - \frac{u_o v}{c^2}}$$

respectively. Since $u'_- > u'_+$ then $\gamma(u'_-) > \gamma(u'_+)$. Thus the negative line charge density $\lambda'_- = \gamma(u'_-)^2 \lambda_p$ is larger than the positive charge density $\lambda'_+ = \gamma(u'_+) \lambda_p$ due to Lorentz contraction. That is, there is a net non-zero negative line charge density $\lambda'_+ + \lambda'_-$ in the rest frame of the charge $q$ since the negative line charge is compressed into a shorter distance than the positive line charge. This leads to an electric force attracting the positive charge $q$ towards the negatively charged line current. Using Coulomb’s law to calculate the electric force in the rest frame of the charge $q$, and transforming back to the stationary frame gives the same answer for the force acting on the charge as obtained using the magnetic force.
Electromagnetism

Remember that typical currents in a wire imply drift velocities of the conduction charges of only $10^{-5}\text{m/s}$, that is, $(\frac{v}{c})^2 \sim 10^{-25}$. However, the line charge density typically is $1.6 \times 10^4 \text{C/m}$ implying enormous electric forces on a nearby charge. In the stationary frame of reference, one has exact cancellation of enormous equal and opposite electric forces. The magnetic force results from a $10^{-25}$ imbalance in the positive and negative charge densities caused by the Lorentz transformation to a moving frame.
Relativistic Momentum

It can be shown from the theory of special relativity that the momentum $p$ is given by:

$$\vec{p} = \gamma m_o \vec{v}$$

where $m_o$ is the rest mass of the object, that is, the normal inertial mass measured when the object is stationary. The object behaves as if it has an effective mass $m = \gamma m_o$. In the non-relativistic limit $\gamma \to 1.00$, and $m = m_o$. However, as $v/c \to 1$, $m \to \infty$, that is the effective mass $m$ becomes infinite. Since $\vec{F} = \frac{d\vec{p}}{dt} = m \vec{a}$ the acceleration goes to zero as $v \to c$. In other words it is impossible to accelerate beyond the speed of light even if enormous forces are applied to accelerate an object.
Relativistic Energy

The above force equation gives that the kinetic energy is:

\[ \text{Kinetic energy} = \gamma m_o c^2 - m_o c^2 \]

Note that this equals the Newtonian value \( \frac{1}{2} m_o v^2 \) when \( v \ll c \).

The rest energy \( E_o \) is defined as

\[ E_o = m_o c^2 \]  

(Rest energy)

Note that this has the units of energy. Define a total relativistic energy \( E \):

\[ E = KE + m_o c^2 = \gamma m_o c^2 \]

\[ E = mc^2 \]

This is the famous Einstein relativistic energy that relates the equivalence of mass and energy. The total relativistic energy \( E \) is a conserved quantity in nature. It is an extension of the conservation of energy.
In nuclear and particle physics there are many examples where mass is converted to energy and vice versa, such as the creation and annihilation of the positron-electron pair, or energy released by nuclear reactions such as fission.

A dramatic example of Einstein’s equation is a nuclear reactor. One gram of material, the mass of a paper clip, provides $E = 9 \times 10^{13}$ joules. This is the daily output of a 1 GWatt nuclear power station or the explosive power of the Nagasaki or Hiroshima bombs.
General Relativity

In 1916 Einstein developed a more general theory called the Theory of General Relativity. This theory allows accelerating, non-inertial, frames. It introduces the *Principle of equivalence* which states that a homogeneous gravitational field is completely equivalent to a uniform accelerated reference frame. For example, in an elevator accelerating upwards your weight will increase by ma where a is the acceleration of the elevator. There is no way of differentiating between this accelerating system and a similar gravitational acceleration. The mathematics involved in this theory is difficult and there are few experimental consequences. However the following are results that confirm Einstein’s General theory of relativity.
Einstein, predicted that light beams also would be bent by gravitational attraction based on the principle of equivalence as illustrated in figure 9. This was observed during a solar eclipse in 1919. Recently astronomers have seen focusing of light due to bending of light by massive stellar objects.
In 1979 a twin quasar was discovered which was the first known example of the gravitational lens effect. The twin consisted of identical quasar objects only 6 arc seconds apart. The spectra and red shifts were identical, that is they are at the same distance from the Earth. The twins are so identical that they can only reasonably be accounted for as two images of the same object, exactly as would be expected if a massive galaxy lay on the line of sight causing the lens effect.

Nature 1979
Seeing a Gravitational Illusion

The radio map, right, is the first ever made of a complete Einstein ring, a phenomenon predicted by Einstein’s general theory of relativity. Using six radio telescopes in England and the Hubble Space Telescope, astronomers discovered and captured this infrared image of two galaxies, one lying at a great distance directly behind the other. The closer galaxy acting as a gravitational lens is the bright spot in the center of this infrared image. The distant galaxy behind it is not directly visible, but its light, curving around the nearer galaxy, has been spread into a ring.

According to Einstein’s theory, a massive object, like a galaxy, warps surrounding space and causes light to curve around it, much as a glass lens bends a beam of light. When a telescope on or near the Earth is perfectly aligned with a galaxy acting as a gravitational lens and a more distant object like a quasar, the light from the farther object curves around the lensing galaxy and appears as a bright, unbroken ring.
Figure 14  Einstein’s Cross comprises four images of a distant quasar that has been imaged by a closer galaxy acting as a gravitational lens. The image was taken by the ESA Faint Object Camera on the NASA Hubble telescope.
Just as a moving clock exhibits time dilation, also, an accelerated clock runs slower than an unaccelerated clock. This has been observed in what is called the gravitational red shift. This is only possible because of extremely high-sensitivity techniques developed by nuclear physicists.
A weight placed on a flat and frictionless rubber sheet causes a deformation of the sheet—a change in its two-dimensional geometry. A smaller weight rolling along the sheet describes a curved trajectory. In a similar way, the Sun affects the geometry of three-dimensional space, thereby causing planets and comets to describe curved trajectories.

Predict **Black Holes**, star-like objects from which light cannot escape.
Is Relativity a April 1 hoax?
Is Relativity a April 1 hoax?

No, it has been proven in many experiments.
So who is crazy, you or me?
Einstein’s Theory of Relativity

Einstein’s theories of relativity have had an enormous impact on twentieth century physics and philosophy of science. However, it is removed from everyday experience so it is not as apparent as other developments this century. It is important to any well-educated person that they are cognisant of the consequences of this theory of nature.

On the 109th anniversary of the genesis of Einstein’s Theory of Relativity, the theory has held up under extensive experimental scrutiny. Einstein could well smile.