The Birth of Modern Physics

1900: Culmination of development of Classical Physics

• Newton’s Laws of Motion
• Laws of Thermodynamics
• Maxwell’s Equations

1905: Birth of Modern Physics

• Einstein’s Theory of Special Relativity
• Einstein proposed the photon $E = h\nu$ leading to the development of quantum physics
• Einstein introduced the quantum principle to statistical mechanics
Einstein’s Postulates

1) *The laws of nature are the same in all inertial frames of reference.*

This simple postulate, has remarkable consequences. When combined with Maxwell’s equations describing the nature of light, then it implies that

2) *The velocity of light in vacuum is the same in all inertial frames of reference.*

This second postulate was confirmed by the Michelson-Morley experiment. However, it was not this experimental result that led Einstein to the theory of special relativity. He deduced the special theory of relativity from consideration of Maxwell’s equations of electromagnetism. Although Einstein’s postulates appear reasonable, they lead to some surprising implications.
Einstein’s Theory of Relativity

Einstein's theories of relativity have had an enormous impact on twentieth century physics and philosophy of science. However, it is removed from everyday experience so it is not as apparent as other developments this century. It is important to any well-educated person that they are cognisant of the consequences of this theory of nature.

On the 109th anniversary of the genesis of General Relativity, the theory has held up under extensive experimental scrutiny. Einstein could well smile.
• X rays
• Radioactivity
• Discovery of the electron
• Atomic nucleus
• Origin of the quantum constant
• Photon hypothesis
• Quantization of atomic energies
• Bohr model of the atom
Technological developments during the 1890’s included:

- Photography
- Induction spark coil
- Vacuum pump

These led to chance discoveries that formed the foundation upon which quantum physics originated.
Origins of Quantum Physics

- Black-body radiation
  - Planck constant (Planck)
  - \( E = h \nu \) (Einstein)

- Photoelectric effect (Hertz)

- Radioactivity

- Electron

- Nucleus (Rutherford)

- Bohr Atom

- Atomic spectra

- X-rays

- Atomic number (Moseley)
In 1895, Röntgen discovered that an invisible ray, called the X-ray, was produced at a positive electrode held at high voltage in a high-vacuum tube. These X-rays are undeflected by electric and magnetic fields and the X-rays penetrate through matter.
Crystal X-ray diffraction

X-rays are very short wavelength electromagnetic waves. This was proven in 1912 by Von Laue who proposed using a regular crystalline lattice to produce characteristic interference patterns, analogous to multi-slit interference, as shown in figure 3. Constructive interference occurs at angles where the scattered X-rays from different layers are exactly a multiple of $\lambda$ different in phase; i.e. when $2d \sin \theta = n\lambda$. Bragg exploited this phenomenon in Manchester to study crystallography; a development that eventually led to understanding molecular binding, DNA etc.
In 1896, Becquerel discovered that uranium compounds emit a radiation that exposes photographic plates. The penetration radiation was used to photograph a metal cross. Pierre and Marie Curie showed that thorium also was radioactive. This was the start of the field of nuclear physics. In 1998 I had the pleasure of speaking in Warsaw at the centennial commemoration of the discovery of Po by Marie Curie.
In 1897 J.J. Thomson, in Cambridge, showed that cathode rays were due to a new particle, called the electron, that is negatively charged. Cathode rays were known to be due to a charged beam because the rays were deflected by a magnetic field. Thomson used the Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ to measure the charge to mass ratio, $q/m$, of the particles using a cathode ray tube, figure 5. He found that the $q/m$ ratio was always the same independent of the nature of the cathode. Faraday’s work on electrolysis had shown that the fundamental unit of charge was $1.6 \times 10^{-19}$ C. Assuming that this was the charge on the electron, implied that the mass of the electron was about $1/2000$ that of the hydrogen atom. This small mass was a surprise when first proposed by Thomson. In 1909, Millikan performed the famous Millikan Oil-drop experiment that measured the charge on the electron directly and proved Thomson to be correct.
Ernest Rutherford discovered that radioactivity comprised three different species which he named $\alpha$, $\beta$, and $\gamma$ activity. He showed that $\alpha$ particles are positive, heavy and are helium atoms, $\beta$ particles are negative and like electrons, while $\gamma$ rays are uncharged presumably very short wavelength EM radiation.

Scientists had noted that electrons penetrated matter suggesting that matter may be as empty as the interstellar void. At Manchester, in 1911, Rutherford had Geiger and Marsden study the scattering of $\alpha$ particles from a thin gold foil. To their amazement, they observed backscattering of alpha particles. As Rutherford remarked, it was incredible as if one had fired a 15” shell at a piece of tissue paper and the shell scattered backwards. From the study of the scattering as a function of scattering angle, Rutherford, Geiger and Marsden deduced the size of the nucleus.
The larger the scattering angle, the closer the $\alpha$ particle approaches the nucleus. Beyond a certain scattering angle the distance of closest approach becomes less than the size of the nucleus, then the electric field deviates from the $\frac{1}{r^2}$ dependence, plus the scattered alpha particles are absorbed in the interior of the scattering nucleus, resulting in the scattering deviating from the simple Rutherford relation as illustrated in figure 4. From their measurements they discovered that the size of a gold nuclei is about $10^{-14}$ m. This is four orders of magnitude smaller than the radius of the atom: they had discovered the atomic nucleus. This led Niels Bohr, who collaborated with Rutherford at Manchester, to propose, in 1913, the Bohr Model of the atom. This model was remarkably successful as discussed later. First it is necessary to discuss several parallel developments.
Origins of Quantum Physics

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  - Planck constant (Planck)
  - $E = h\nu$ (Einstein)
- Photoelectric effect (Hertz)
- Radioactivity (Rutherford)
- Electron
- Nucleus (Rutherford)
- Bohr Atom
- Atomic number (Moseley)
- Atomic spectra
- X-rays
Origin of the Quantum Constant: Black-body radiation

\[ \lambda_{\text{max}} = \frac{2.898}{T} \text{mm.K} \quad \text{(Wien displacement law)} \]
Black-body radiation

Planck struggled to explain the black-body spectrum and eventually found a way out of this problem by assuming that electromagnetic radiation was not emitted with a continuous range of energies, but that EM radiation is emitted in discrete bundles of energy called quanta. As illustrated in figure 8, the black body spectral distribution is reproduced assuming that the energy carried by a single quantum must be an integer multiple of a certain quantity:

\[ E = h\nu = \frac{hc}{\lambda} \]

where \( \nu \) is the frequency of the radiation and \( h \) is Planck's constant, \( h = 6.626 \times 10^{-34} \) J·sec. That is, Planck assumed that energy comes in discrete bundles of energy, called quanta, equal to \( h\nu \). By making this extreme assumption, in an act of desperation, Planck was able to reproduce the experimental black body radiation spectrum.

Planck Formula

\[ I(\lambda, T) = \frac{2\pi hc^2 \lambda^{-5}}{e^{hc/\lambda kT} - 1}. \]
Cosmic 2.74° K Black-body spectrum

Cosmic background spectrum at the north galactic pole

\[ I(\lambda, T) = \frac{2\pi h c^2 \lambda^{-5}}{e^{hc/\lambda k T} - 1} \]
Planck Quantum Hypothesis

\[ E = h\nu = \frac{hc}{\lambda} \]

The assumption that energy was exchanged in bundles hinted that the classical laws of physics were inadequate in the microscopic domain. This radical revision led to the theory of quantum mechanics that left the older generation physicists, including Planck himself, in the dust as the new generation physicists, like Einstein, Bohr, Heisenberg, Born, and Schrödinger, developed a revolutionary new theory.
Origins of Quantum Physics

Black-body radiation
Planck constant
(Planck)

E = hv
(Einstein)

Planck's constant

Photoelectric effect (Hertz)

Radioactivity
Nucleus
(Rutherford)

Atom

Electron

Bohr Atom

Atomic spectra

X-rays

Atomic number
(Moseley)
Photo-electric effect

The 1887 experimental proof of electromagnetic waves by Hertz was the epitome of classical physics, it proved that Maxwell was correct. Ironically, it was Hertz’s measurements that also led to the downfall of classical physics. Hertz observed that a piece of metal was electrified when irradiated with ultraviolet light. J.J. Thomson showed that electrons are ejected by electromagnetic waves by what is now called the photoelectric effect.

A certain amount of energy, provided by the EM wave, is required to liberate one electron from a metal, this is called the work function. Classically the energy of the emitted electron should depend on the intensity of the E field of the EM wave and thus the photoelectric effect should be much more important as the intensity of the light is increased. Experimentally, the number of electrons, not their energy is proportional to the light intensity.
Photon Hypothesis [Einstein 1905]

In 1905, Einstein not only published the theory of special relativity, he also made the important prediction of the existence of the photon. It is remarkable to realize that he developed these two revolutionary theories in one year when he was only 26 years old. Einstein uncovered a subtle error in Planck’s derivation of black body radiation. In order to correct the derivation of Planck’s distribution, Einstein had to strengthen the quantum hypothesis. Planck demanded that light of frequency $\nu$ be packaged in quanta whose energies were multiples of $h\nu$, but Planck never thought that light would have particle-like behaviour. Maxwell had convinced all physicists that light was a wave phenomena contrary to Newton’s idea that light comprised of corpuscles. Interference and diffraction effects are a convincing manifestations of the wave-like properties of light.
Photo-electric effect

In order to reproduce Planck’s result, Einstein had to treat black-body radiation as if it consisted of a gas of particles, called photons, each photon having energy \( E = h\nu \). This was a revolutionary concept that returned to Newton’s corpuscle theory of light, in spite of the fact that all the research since had shown that light is a wave. Einstein realized that there were direct tests of his photon hypothesis, one of which is the photo-electric effect. According to Einstein, each photon has an energy \( E = h\nu \) and according to this approach, in contrast to the classical case where the energy of the electron depends on the intensity of the light, Einstein predicted that the ejected electron will have a kinetic energy

\[
KE = h\nu - W
\]

where \( W \) is the work function which is the energy needed to remove an electron from a solid. Einstein’s theory was vindicated in 1915 by Millikan who showed that, as predicted, the energy of the ejected photoelectron depended on the frequency and not intensity of the light.
The problem was how could one simultaneously explain wave-like properties if light is composed of corpuscles. As Thomson stated, the two models were like a struggle between a shark and a tiger, each is supreme in its own element, but helpless in that of the other. Einstein still believed in wave-like properties, but he also believed that light could manifest either wave-like or corpuscle-like behaviour. The measurement of the photo-electric effect forced physicists to take Einstein’s photon hypothesis seriously. Note that Einstein was awarded the Nobel prize for his theory of the photoelectric effect, not the theory of relativity.
In 1923 Compton discovered another example of the photon-like behaviour of light. Scattering of X-rays from matter occurred as if individual photons, with momentum $p = \frac{h \nu}{c}$ are scattered off individual electrons in two-body collisions that satisfied the mechanical laws of energy and momentum.
Photon Hypothesis

The production of X-rays is a manifestation of the photon. An electron is accelerated and then when it hits the anode, its energy is converted to a photon. Another example, seen frequently in nuclear physics, is pair production. A \( \gamma \) ray photon is completely converted into a positron plus negative electron, and the energy and kinematics obey the simple mechanical rules expected if the \( \gamma \) ray is a particle-like photon.
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- Electron
- Bohr Atom
- Atomic number (Moseley)
- Atomic spectra
- X-rays
Quantization of Atomic Energies

Atoms emit light in discrete quanta. The study of atomic spectra was a well-studied field. Physicists had used prisms or diffraction gratings to discover that the reciprocal wavelengths of emission line spectra obey a simple relation:

$$\frac{1}{\lambda} = RZ^2 \left[ \frac{1}{m^2} - \frac{1}{n^2} \right]$$

where $R$ is a constant, called the Rydberg constant, and $m$ and $n$ are integers. For a given series of emission lines, $m$ is a constant.

The origin of these regularities was not understood. Classically one would expect that all possible orbits would be allowed for an electron circling a nucleus in the atom just as the planets can orbit the sun with an arbitrary radius.
Origins of Quantum Physics

Black-body radiation
Planck constant (Planck)

E = \hbar \nu
(Einstein)

Photoelectric effect (Hertz)

Radioactivity
Nucleus (Rutherford)

Electron

Atomic spectra
X-rays

Bohr Atom

Atomic number (Moseley)
Bohr Model of the Atom

The first thoughts of quantum behavior, by Einstein and Planck, concerned the nature of light, not the structure of the atom. However, in 1913, following Rutherford’s 1911 discovery of the atomic nucleus, Niels Bohr proposed a model of the atom that incorporates the Rutherford atom and the concept of the photon. His model was remarkably successful.
Classical Model of Atom

A classical planetary orbit model with electrons in circular orbits around a nucleus, using Coulomb’s Law, predicts that the potential energy, $U$, for the electron outside of a nucleus of charge $Ze$ is given by:

$$U = -\frac{1}{4\pi \varepsilon_o} \frac{Ze^2}{r} \quad \text{(Coulomb potential energy)}$$

The total electron energy $E$ is the kinetic plus potential energy, that is:

$$E = \frac{1}{2} mv^2 - \frac{1}{4\pi \varepsilon_o} \frac{Ze^2}{r} \quad \text{(Total electron energy)}$$

Balancing the centripetal force and Coulomb force requires that:

$$\frac{mv^2}{r} = \frac{1}{4\pi \varepsilon_o} \frac{Ze^2}{r^2}$$

This implies that the kinetic energy is half the potential energy. Substituting into the equation for the total energy gives that for circular orbits:

$$E = -\frac{1}{8\pi \varepsilon_o} \frac{Ze^2}{r}$$

Note that some books write this in terms of the binding energy $W = -E$. 
Bohr noticed that, using the known radius of the hydrogen atom, this gave the correct binding energy for the electron. Unfortunately, classical physics allows all possible radii and corresponding binding energies, not the quantized values required to explain atomic spectra. Secondly, Maxwell’s equations predict that a circulating electron will radiate electromagnetic waves dissipating energy. The great contribution of Bohr was to make two hypotheses contradicting the laws of classical physics in order to correct for these problems.
Bohr Hypotheses

a) Stationary orbits

The electrons are in stationary orbits that do not radiate if not disturbed from the outside.

b) Quantization of orbits

The angular momentum $l$ around a nucleus in a stationary state is equal to an integer multiple of $\frac{h}{2\pi}$ independent of the charge of the nucleus. That is:

$$l = n\frac{h}{2\pi}$$

(Quantized angular momentum.)

where $n$ is a positive integer and is called the quantum number.
Quantization of Orbits

Substituting $mv = \frac{l}{r}$ into the centripetal-Coulomb force balance equation above gives a relation for the orbit radius $r$

$$r = \frac{4\pi \varepsilon_0 l^2}{Ze^2m}$$

which can be substituted into the electron energy equation to give:

$$E = -\frac{m}{2} \left( \frac{1}{4\pi \varepsilon_0} \frac{Ze^2}{l} \right)^2$$

Using Bohr's quantization hypothesis $E$ gives that the binding energy of orbit $n$:

$$E_n = -\frac{m}{2} \left( \frac{1}{2\varepsilon_0} \frac{Ze^2}{n\hbar} \right)^2 = -\frac{Z^2E_0}{n^2}$$

(Quantized energy levels)

where $n$ is the integer quantum number characterizing the orbit and $E_0 = 13.6eV$. As shown in figure 13, the energy levels for binding of the electron, in the Coulomb potential of the atomic nucleus, are quantized as characterized by the quantum number $n$.

The radius of the $n$ quantum orbit is given by:

$$r = \frac{\varepsilon_0 \hbar^2 n^2}{Ze^2m\pi} = n^2 \frac{a_o}{Z}$$

(Quantized orbit radii)

where $a_o = 0.0529nm$. Note that the radius of the electron orbit is proportional to the square of the integer quantum number $n$. 
Quantized EM transition energies

\[ h\nu = E_1 - E_2 = Z^2 E_0 \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \]

(Quantized EM transition energies.)
Atomic Number

Moseley 1914

This first workable model of the atom and the explanation of the hydrogen spectrum was Bohr’s masterstroke. In 1914 Henry Moseley, a Manchester graduate student, interpreted his X-ray data using the Bohr model to show that every element from aluminum to gold is characterized by an integer charge $Z$ as illustrated in figure 15. He showed that, in general, $Z = A/2$ except for a couple of cases where the atomic mass happens to not follow the atomic number, $Z$, sequence. He also predicted that three elements were missing. Moseley was unable to measure the long-wavelength X-ray spectra from elements lighter than Al. Unfortunately, the brilliant Moseley was killed in 1915 in action at the Gallipoli campaign at the age of 28.
Assumptions of Bohr Model

The Bohr model is based on the following assumptions.

1) Electromagnetic radiation is quantized with \( E = h\nu \).

2) Electromagnetic radiation exhibits behavior characteristic of the emission of photons with energy \( E = h\nu \) and momentum \( p = \frac{h\nu}{c} \). That is, it exhibits both wave-like and particle-like behavior.

3) Electrons are in stationary orbits that do not radiate in contradiction to classical physics.

4) The orbits are quantized such that the electron angular momentum is an integer multiple of \( \frac{\hbar}{2\pi} \).

5) Atomic electromagnetic radiation is emitted with photon energy equal to the difference in binding energy between two atomic levels involved. \( hf = E_1 - E_2 \)

The first two assumptions are due to Planck and Einstein while the last three were made by Niels Bohr.
Philosophical problems of the Bohr Model

The problems with the Bohr model were the philosophical problems of violating the tenets of classical physics in explaining hydrogen-like atoms, the theory was prescriptive, not deductive, and the model failed to reproduce the properties of non hydrogen-like atoms. There was a hiatus until the early 1920’s when de Broglie made the key suggestion of wave-particle duality that put quantum theory on a logical and firm foundation. Quantum theory is the most important scientific development of the twentieth century. This will be the topic of the next lecture.