Chapter 8  MAGNETISM

- Permanent magnetism
- Electromagnetism
- The Lorentz force
- Units
- Motion of point charges in uniform magnetic fields
- Magnetic forces on electric currents
- Force on a current loop
- Summary

PERMANENT MAGNETISM

The fact that the stone magnetite attracts iron has been known for about 2000 years, while the alignment of magnets in the earth’s magnetic field is the principle of the compass which has been used for navigation since the twelfth century. Magnetism should be a familiar to all of you, be it for navigation with a compass or attaching notes to the refrigerator door.

Magnets behave like electric dipoles. One end of a magnet is called the north pole because it seeks the earth’s geographic north pole. The other end of the magnet is called a south pole since it seeks the earth’s south pole. The same polarity magnetic poles repel while opposite magnetic poles attract, analogous to the attraction and repulsion of electric charges. Thus the north pole of a magnet is attracted to the south pole of another magnet. The earth has a magnetic field that corresponds to a huge magnet with the magnetic south pole of this magnet lying under northern Canada, about 1100 miles from the earth’s geographic north pole, while the north pole of this magnet is near the earth’s geographic south pole. Thus, the north-seeking pole of a compass needle points towards the earth’s geographic north pole because of the attraction to a magnetic south pole produced by the earth. As shown in figure 1, one has to be careful to differentiate between a north-seeking pole and the earth’s actual magnetic pole lying under northern Canada.

Your concept of a magnetic field probably comes from the alignment of a compass needle. We envision that the magnetic field points towards the earth’s north pole which implies that it points from the north pole of a bar magnet towards the south pole of the magnet. This is analogous to the electric field which points from the positive to negative charge of an electric dipole. Historically the magnetic field is designated by the symbol $\mathbf{B}$. This magnetic field is a vector field analogous to the electric field for an electric dipole. The demonstration illustrates the magnetic lines of force of a bar magnet by the alignment of iron filings and a compass needle along the magnetic field. The north-seeking pole of the compass points in the direction of the $\mathbf{B}$ field.

The magnetic field distribution of a bar magnet appears similar to that of an electric dipole as shown in figure 2. However, there is one crucially important difference; in electrostatics one can isolate a single pole, that is either the positive or negative charge. This is not possible for a magnetic dipole; if you split a magnetic dipole, you always finish up with two smaller magnetic dipoles. There has been much speculation about the possible existence of isolated magnetic poles, called magnetic monopoles, but none have been ob-
Magnetism appeared completely unrelated to electricity until 1819 when Hans Christian Oersted discovered that an electric current induces a magnetic field. As shown in figure 3, and the demonstration, the magnetic field induced by an electric current circles clockwise around a current-carrying conductor.

Following Oersted’s discovery, André Ampère showed that electric currents produce the same magnetic behavior as observed with lodestone. In particular, a circular current loop produces the same magnetic field at large distance as does a magnetic dipole as demonstrated and shown in figure 4. Ampère speculated that magnetism is produced by electric currents in matter. Now we know that this is the basis of the modern theory of magnetism; electrons circulating in the atom produce microscopic current loops that result in the characteristic field of a magnetic dipole.

The fact that the origin of the magnetic field is due to electric current loops, explains why magnetic monopoles do not exist. Note that the magnetic field of the current loop leads to continuous lines of magnetic field that loop through the current loop and return outside the loop. There are no sources of sinks of magnetic field in contrast to electrostatics where the electric field originates at a positive charge and ends at a negative charge.

LORENTZ FORCE

The magnetic field is manifest by the forces on a magnetic dipole placed in the magnetic field. The most direct demonstration of the existence of a magnetic field is the force acting on a moving charge in this magnetic field. As demonstrated, a current of electrons, \( q = -1.602 \times 10^{-19} \text{C} \), moving with velocity \( v \) in a cathode ray tube, is deflected by the magnetic field produced by a bar magnet. As shown in figure 5, a charge \( q \) moving with velocity \( \mathbf{v} \) in a magnetic field \( \mathbf{B} \) is deflected perpendicular to \( \mathbf{B} \) and \( \mathbf{v} \) according to the relation:

\[
\mathbf{F}_{\text{mag}} = q \mathbf{v} \times \mathbf{B}
\]

This important result is the magnetic force law. Note that it is important to make sure that you define the cross product in the correct direction, that is, as given by the right-hand rule or clockwise as illustrated in figure 6.

The combination of the magnetic force and the electric force on a moving charge is called the Lorentz Force:

\[
\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})
\]

The Lorentz force actually defines the electric and magnetic fields. It states that the total electromagnetic force on a moving charge separates into two terms, a velocity-independent term, called the electric force,
and a velocity-dependent term, which is the magnetic force. Calculation of electromagnetic forces requires knowledge of both the $\mathbf{E}$ and $\mathbf{B}$ fields to use in the Lorentz force. The electric field was defined as the electric force per unit charge. Similarly, the magnetic field is related to the magnetic force per unit charge, but this relation is more complicated in that it is necessary to define a magnetic field where the magnetic force is mutual perpendicular to both the velocity and magnetic field vectors.

It is important to note that the electric and magnetic field must be specified in the same spatial frame of reference, because Einstein’s Theory of Relativity shows that the electric and magnetic fields change under transformation to moving frames of reference. As an example, consider a frame of reference where $\mathbf{E}$ is zero and $\mathbf{B}$ is nonzero. A moving charge will feel a magnetic force $q\mathbf{v} \times \mathbf{B}$. Transforming to a frame of reference travelling in which the moving charge is stationary, then the velocity is zero so the magnetic force must be zero. However, a real force still exists, and is described as an electric force. That is, an electric field must have been created by the transformation of the magnetic force into this second frame of reference. Einstein’s theory of relativity shows this relation and that electric and magnetic fields are manifestations of the same phenomena.

**Is the magnetic force conservative?**

The Lorentz force gives that the magnetic force is $\mathbf{F}_{mag} = q\mathbf{v} \times \mathbf{B}$

The work done moving at velocity $\mathbf{v}$ during time $dt$ against the magnetic force $\mathbf{F}_{mag}$ is given by:

$$\Delta W = \mathbf{F}_{mag} \cdot \mathbf{v} dt = q\mathbf{v} \times \mathbf{B} \cdot \mathbf{v} dt = 0$$

That is, the work done during the time $dt$ is zero because the force always is perpendicular to the velocity $\mathbf{v}$.

Consider the kinetic energy of the moving point charge.

$$KE = \frac{1}{2}mv^2$$

thus

$$\frac{d(KE)}{dt} = \frac{1}{2}m\frac{d(v^2)}{dt} = m(\mathbf{v} \cdot \frac{d\mathbf{v}}{dt})$$

But

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt}$$

therefore,

$$\frac{d(KE)}{dt} = \mathbf{v} \cdot \mathbf{F} = 0$$

Thus although the magnetic force can change the trajectory of the particle, it does not change the magnitude of the kinetic energy, it only changes the direction.

**UNITS**

The SI unit of magnetic field $\mathbf{B}$ is the *Tesla*, when $F$ is given in Newtons, $q$ in Coulombs, and $v$ in m/s. A commonly used unit of magnetic field is the Gauss where 1tesla = $10^4$gauss. The earth’s magnetic field is about 0.6gauss, $(6 \times 10^{-2}T)$, pointing north and about 70° downward to the earth’s surface at Rochester. A large horse-shoe permanent magnet produces fields of about 0.1T, while a large electromagnet produces of field of about 1T. Magnetic fields as large as $10^4T$ are produced by the atomic electrons at the surface of the nucleus.

**MOVING POINT CHARGES IN A UNIFORM MAGNETIC FIELD**

Magnetic forces on moving charges have many important applications ranging from electronics used in medicine to nuclear and astrophysics. The simplest example is motion of a charged particle in a uniform magnetic field.

A charge moving with velocity $\mathbf{v}$ in a uniform magnetic field $\mathbf{B}$ experiences a force perpendicular to $\mathbf{v}$ and $\mathbf{B}$. For the special case where $\mathbf{v}$ and $\mathbf{B}$ are perpendicular, the charge will follow a circular trajectory where the centripetal acceleration is provided by the magnetic force:

$$F_{mag} = qvB = \frac{mv^2}{R}$$

Thus the radius of curvature of the motion $R$ is:

$$R = \frac{mv}{qB}$$

The period of the circular motion $T$ is given by:

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$$

This is called the *cyclotron period*. The corresponding *cyclotron frequency* is:
Cyclotron

The cyclotron accelerator, invented by Lawrence and Livingston in 1932, is based on the fact that the cyclotron frequency is independent of the velocity of the charged ion. That is, it depends on the charge to mass ratio and the strength of the magnetic field. Higher velocity particles traverse a larger radius trajectory such that the cyclotron frequency remains constant. The cyclotron uses a large uniform magnetic field inside of which there are two D-shaped electrical cavities separated by a small gap. A sinusoidal electric field is applied between the two D electrodes at the cyclotron frequency. At each half cycle, charges with the correct phase are accelerated in the gap between the electrodes and then they coast around 180° during which electric field reverses direction. The magnet makes it possible to repeatedly use the one oscillating electric field to repeatedly accelerate the charged particle. The kinetic energy of the particles leaving the cyclotron is given by the maximum radius of the orbit;

\[ R = \frac{mv}{qB} \]

Thus the kinetic energy is;

\[ K = \frac{1}{2}mv^2 = \frac{1}{2}\left( \frac{q^2B^2}{m}\right)R^2 \]

Rochester had two of the largest cyclotrons in the world when they were constructed in 1934 and 1947. The 1934 cyclotron had a 27" diameter magnetic field, and was located in the basement of Bausch and Lomb until 1965 after which it moved to India where it still operates. The 1947 cyclotron had a 130" diameter and produced 250 MeV of energy per proton. It made some of the earliest measurements on the pion. It was scrapped in 1965. Cyclotrons still are used extensively for nuclear physics and for nuclear medicine. Newly developed ion sources can produce charge states as high as \( q = 40e \) for heavy ions making it possible to accelerate the very heaviest ions above the Coulomb barrier.

Magnetic confinement

A charged particle follows a helical path if the velocity is not perpendicular to the magnetic field as shown in figure 9.

One can resolve the velocity into two components, a component \( v_\perp \) perpendicular to \( B \), and a component \( v_\parallel \) lying parallel to \( B \).

\[ \vec{F} = q(\vec{v}_\perp \times \vec{B}) + q(\vec{v}_\parallel \times \vec{B}) \]

In this case, the particle advances along \( B \) with a uniform velocity \( v_\parallel \) since \( \vec{v}_\parallel \times \vec{B} = 0 \), while simultaneously executing a helical path with cyclotron radius of motion \( R \) given by:

\[ R = \frac{mv_\perp}{qB} \]

Cosmic rays are charged ions executing circular motion in the \( 10^{-10}T \) magnetic field of the galaxy. The
Van Allen belts are charged particles emitted by the Sun and trapped in helical orbits by the Earth’s magnetic field. These charged particles oscillate back and forth about 3000km above the Earth. Near the poles they are reflected by magnetic mirrors which is a consequence of the non-uniform magnetic field near the poles. These belts produce Aurora Borealis which are seen near the earth’s magnetic poles. This phenomenon occurs when charged atoms emit light following excitation on entering the atmosphere.

**Discovery of the electron:**

J.J. Thomson discovered the electron at Cambridge in 1897 by using the Lorentz force to measure the charge to mass ratio. This was the first fundamental particle to be identified. His apparatus shown in figure 11 accelerated electrons of charge $e$ through a voltage $V$ and then passed them perpendicular to crossed magnetic and electric fields which were adjusted such that the Lorentz force is exactly cancelled. This determines the velocity of the electron. That is;

$$q(E + v \times B) = 0$$

Therefore

$$v = \frac{E}{B}$$

The velocity is known since the kinetic energy is given by

$$\frac{1}{2}mv^2 = eV$$

where the charge $q = e$. Thus the mass to charge ratio is given by

$$\frac{e}{m} = \frac{E^2}{2VB^2}$$

J.J. Thomson discovered that the charge to mass ratio of the electron was a single valued quantity characteristic of a fundamental particle. Several years later Millikan at Cal Tech measured the charge on the electron $e$, allowing determination of both the mass and charge of the electron.

**Hall Effect**

An especially interesting example is the Hall effect which is one of the only methods of determining the sign of current carriers. Assume that there are $n$ charge carriers per unit volume, each with charge $q$. Thus the current density is

$$\mathbf{j} = nq\mathbf{v}$$

Consider a conductor with cross section $d$ by $a$ with the $\mathbf{B}$ field perpendicular, as shown in figure 12.

The current flowing in the Hall probe $I$ is given by

$$I = \mathbf{j} \cdot ad = nq\mathbf{v} \cdot ad$$

The magnetic force can be written as:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} = \frac{I \times \mathbf{B}}{nad}$$

This magnetic force concentrates charge along the edge of the conductor in such a way that it is balanced by an equal and opposite electric force. That is:

$$q\mathbf{E} = -q\mathbf{v} \times \mathbf{B} = -\frac{I \times \mathbf{B}}{nad}$$
Thus the net force on the in

But we know that electric current density

charges, that is:

\[ \mathbf{F}_{\text{net}} = q \mathbf{v} \times \mathbf{B} \, d\tau \]

The Hall coefficient equals \( \frac{1}{nq} \) which is unusual in that it depends on the sign of the charge carriers in the conductor and charge carrier density. This can be determined by measuring the voltage difference across the width of the Hall probe for a known current \( I \) and magnetic field \( B \). The Hall Effect probe is used as an inexpensive measure of the magnetic field.

**MAGNETIC FORCES ON ELECTRIC CURRENTS**

Electric current is due to motion of individual charges in an element of conductor. Consider \( n \) charges per unit volume each with charge \( q \) and velocity \( \mathbf{v} \). For an infinitesimal volume element \( d\tau \), there will be \( n d\tau \) charges. Each charge is acted upon by a force \( \mathbf{F}_{\text{mag}} = q \mathbf{v} \times \mathbf{B} \). The net force is the sum of the forces on all \( n d\tau \) charges, that is:

\[ \mathbf{F}_{\text{net}} = nq \mathbf{v} \times \mathbf{B} d\tau \]

But we know that electric current density \( \mathbf{j} = nq \mathbf{v} \). Thus the net force on the infinitesimal volume element is:

\[ \mathbf{F}_{\text{net}} = \mathbf{j} \times \mathbf{B} d\tau \]

Note that this is independent of the type, velocity or charge of the charge carriers, only the net current density \( \mathbf{j} \).

Consider a conductor of cross sectional area \( A \) and length \( dl \) carrying current density \( \mathbf{j} \). The net force on this element of conductor is:

\[ \Delta \mathbf{F}_{\text{net}} = \mathbf{j} \times \mathbf{B} A dl \]

The total current flowing in the conductor is \( \mathbf{I} = \mathbf{j} A \) moving in the direction of \( dl \). Thus the vector product can be rewritten as the force per unit element \( dl \) of conductor is:

\[ \Delta \mathbf{F}_{\text{net}} = \mathbf{I} \times \mathbf{B} dl \]

where the direction of the current is defined by the vector \( \mathbf{I} \). The force on a closed current circuit in a magnetic field thus can be written as

\[ \mathbf{F}_{\text{net}} = \oint \mathbf{I} \times \mathbf{B} dl \]

Some textbooks use the element \( d\mathbf{A} \) to define the direction of the current and thus write the force per unit length as:

\[ \mathbf{F}_{\text{net}} = I \oint d\mathbf{A} \times \mathbf{B} \]

The demonstration illustrates that a long current-carrying conductor is deflected perpendicular to the current \( \mathbf{I} \) and \( \mathbf{B} \) as predicted.

**FORCE ON A CURRENT LOOP**

Consider a stiff rectangular loop of wire carrying current \( I \) in a magnetic field \( B \) as illustrated in figure 14. The force on each side of the rectangle can be calculated using the above relation. There is no net translational force in a uniform \( B \) field since the forces \( F_3 \) and \( F_4 \) are equal and opposite as are also \( F_1 \) and \( F_2 \). However, \( F_1 \) and \( F_2 \) lead to a torque \( \mathbf{\tau} \) trying to rotate the loop until the normal points along the \( B \) field. The torque about point \( P \) is just the force \( F_2 = IaB \) times the lever arm \( b \sin \theta \). Thus the torque is:

\[ \mathbf{\tau} = Iab \sin \theta = IAB \sin \theta \]

where \( A \) is the area of the current loop. If the current loop has \( N \) turns then the torque is \( N \) times larger than for a single loop. The torque can be written conveniently in terms of the magnetic dipole moment defined as:

\[ \mathbf{\mu} = NIA\mathbf{n} \]

where the unit vector \( \mathbf{n} \) is defined as perpendicular to the current loop with the current flowing clockwise.
The concept of the magnetic field $\vec{B}$ has been discussed. The important development is the discovery that a current produces a magnetic field circling clockwise about the electric current.

The Lorentz force describing the forces on a charge in electromagnetic fields is:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

It was shown that the magnetic field is a conservative field.

The motion of charged particles in a magnetic field and applications to the cyclotron, and magnetic confinement were discussed.

The magnetic force on an element $dl$ carrying an electric current $I$ in a $B$ field was shown to be:

$$\Delta F_{net} = \vec{I} \times \vec{B}dl$$

A current loop is defined to have a magnetic dipole moment given by:

$$\vec{\mu} = NI\vec{A}\vec{n}$$

The torque on a magnetic dipole of moment $\vec{\mu}$ in a uniform magnetic field $B$ is:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Next lecture will discuss sources of magnetic field, the Biot Savart law, derivation of the important Ampère Law, and calculation of magnetic fields.

**Reading assignment:** Giancoli, Chapters 27