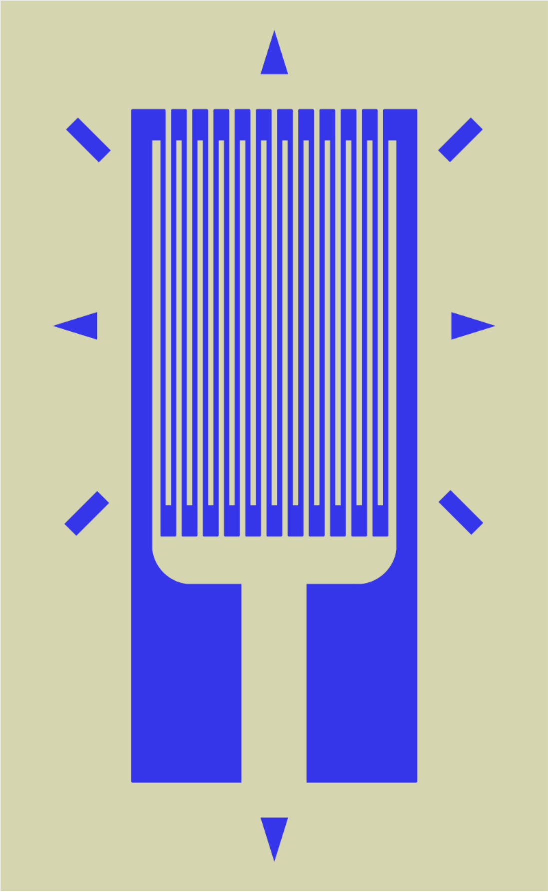
Today in Physics 122:

- Resistor networks
- Simple applications: the Simmons strain gauge.
- Midterm logistics



Resistors in circuits

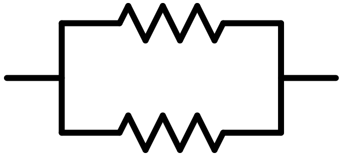
- Like capacitors, resistors are used ubiquitously in electrical circuits. The appear in circuit diagrams as

where the lines outside the zigzag are understood to be perfect conductors.

- Again like capacitors, there are two ways two resistors can be connected:

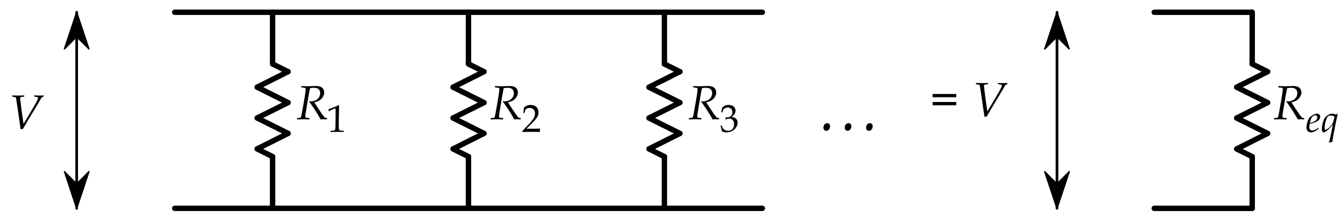
series,

and **parallel**.



Resistors in circuits (continued)

By definition, resistors connected in parallel have the same voltage:

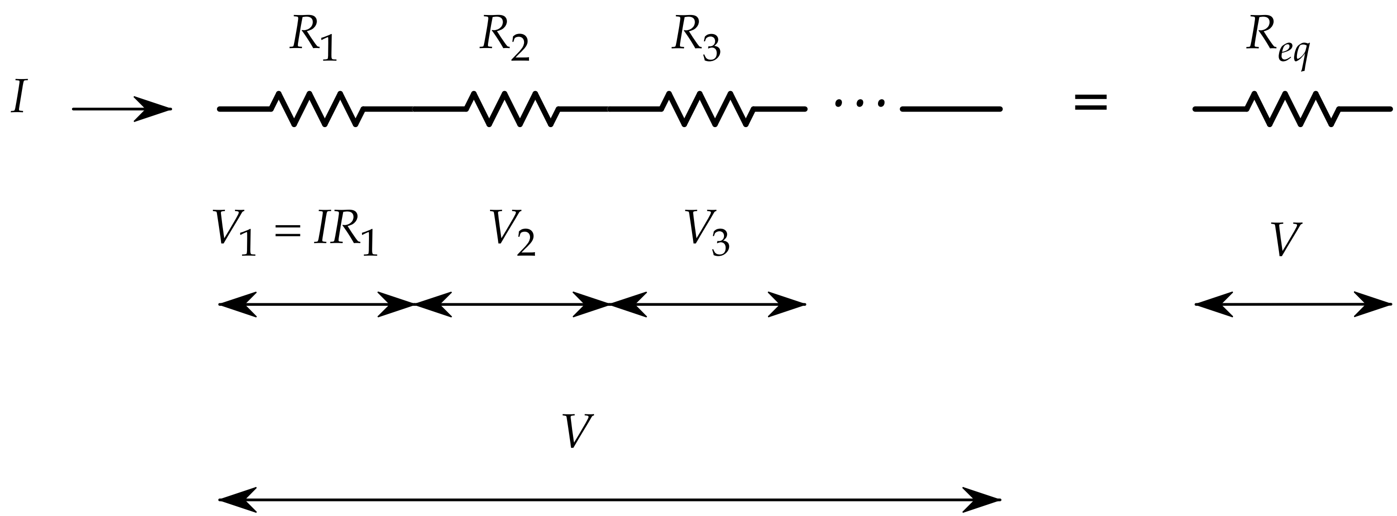


- The current drawn by resistor n is V/R_n , and the total current drawn by the parallel combination is

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \ldots$$
$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \right) V \equiv \frac{1}{R_{eq}} V$$

Resistors in circuits (continued)

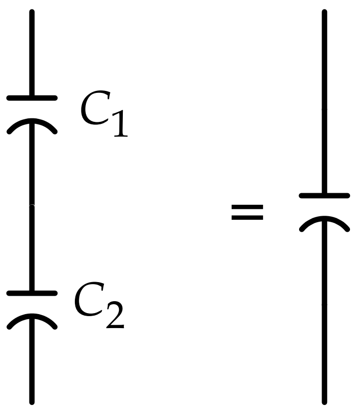
Resistors in series have different voltages, but all the same current I, since they can't accumulate charge:



so $$V = IR_1 + IR_2 + IR_3 + \ldots = I\left(R_1 + R_2 + R_3 + \ldots\right) \equiv IR_{eq}$$

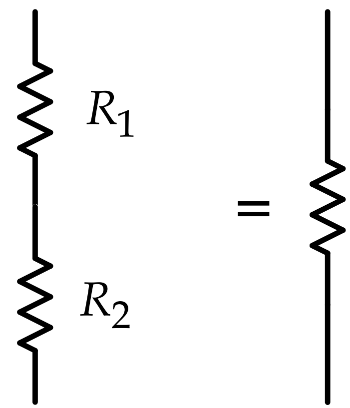
Combinations of *R*s

Thus the equivalent capacitances and resistances of series and parallel pairs exhibit a nice complementarity:

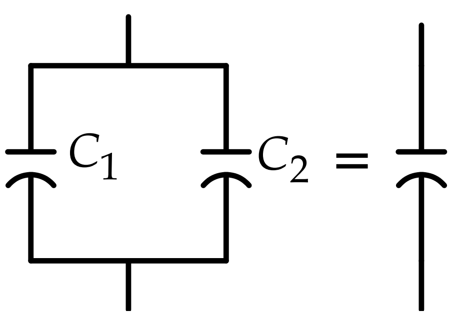


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

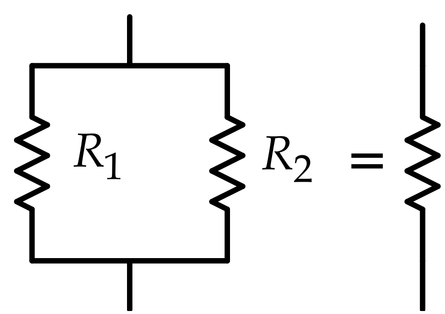
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$



$$R_{eq} = R_1 + R_2$$



$$C_{eq} = C_1 + C_2$$



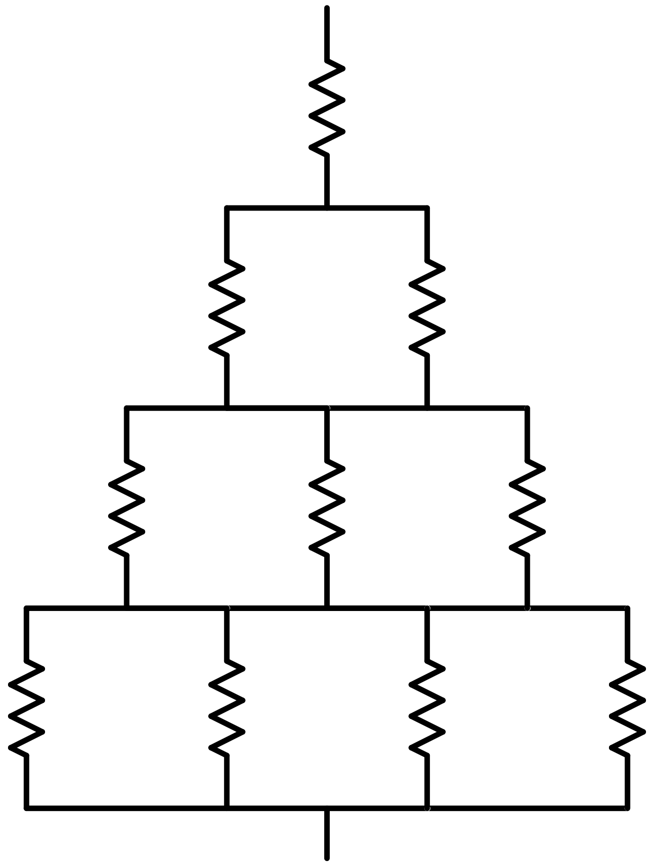
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Combinations of *R*s (continued)

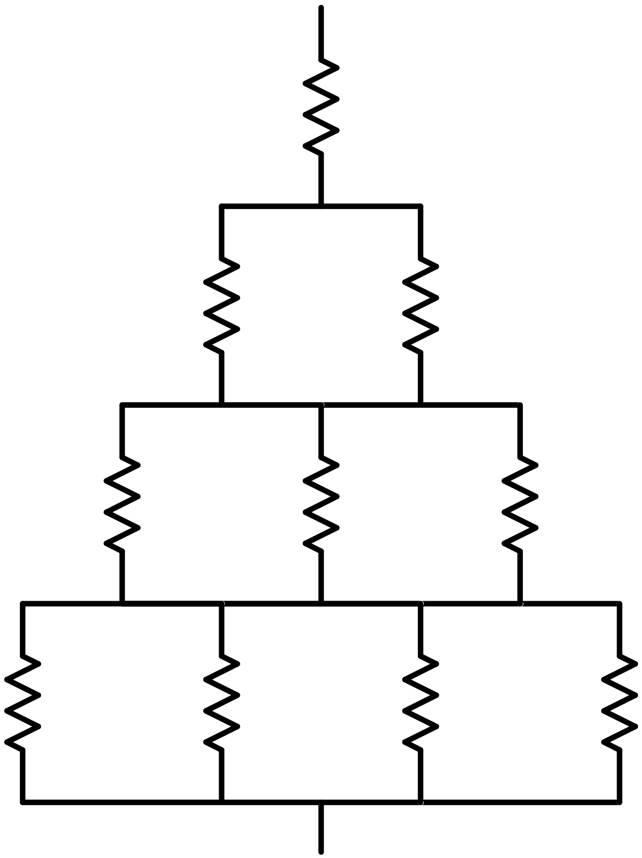
So we can resolve networks of resistors that are composed purely of series and parallel combinations, in much the same way that we can for capacitors.

Example. *Replace each capacitor in the pyramid we used [Wednesday](#) with a resistor of value* R. *Find the equivalent resistance of the pyramid.*



Combinations of *R*s (continued)

- First resolve the parallel combinations:



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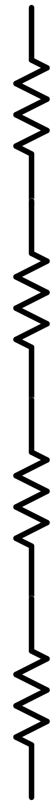


R

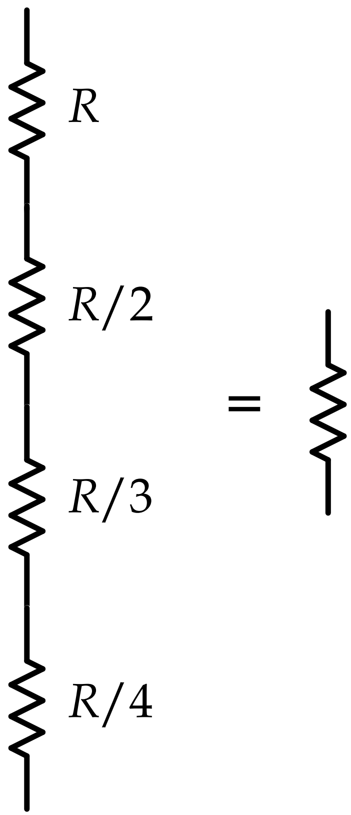
$$\frac{1}{R_1} = \frac{1}{R} + \frac{1}{R} \Rightarrow R_1 = \frac{R}{2}$$

$$\frac{1}{R_2} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \Rightarrow R_2 = \frac{R}{3}$$

$$\frac{1}{R_3} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \Rightarrow R_3 = \frac{R}{4}$$

Combinations of *R*s (continued)

- Then the series combinations:



$$R_{eq} = R + \frac{R}{2} + \frac{R}{3} + \frac{R}{4} = \boxed{\frac{25}{12}R}$$

Application: the Simmons strain gauge

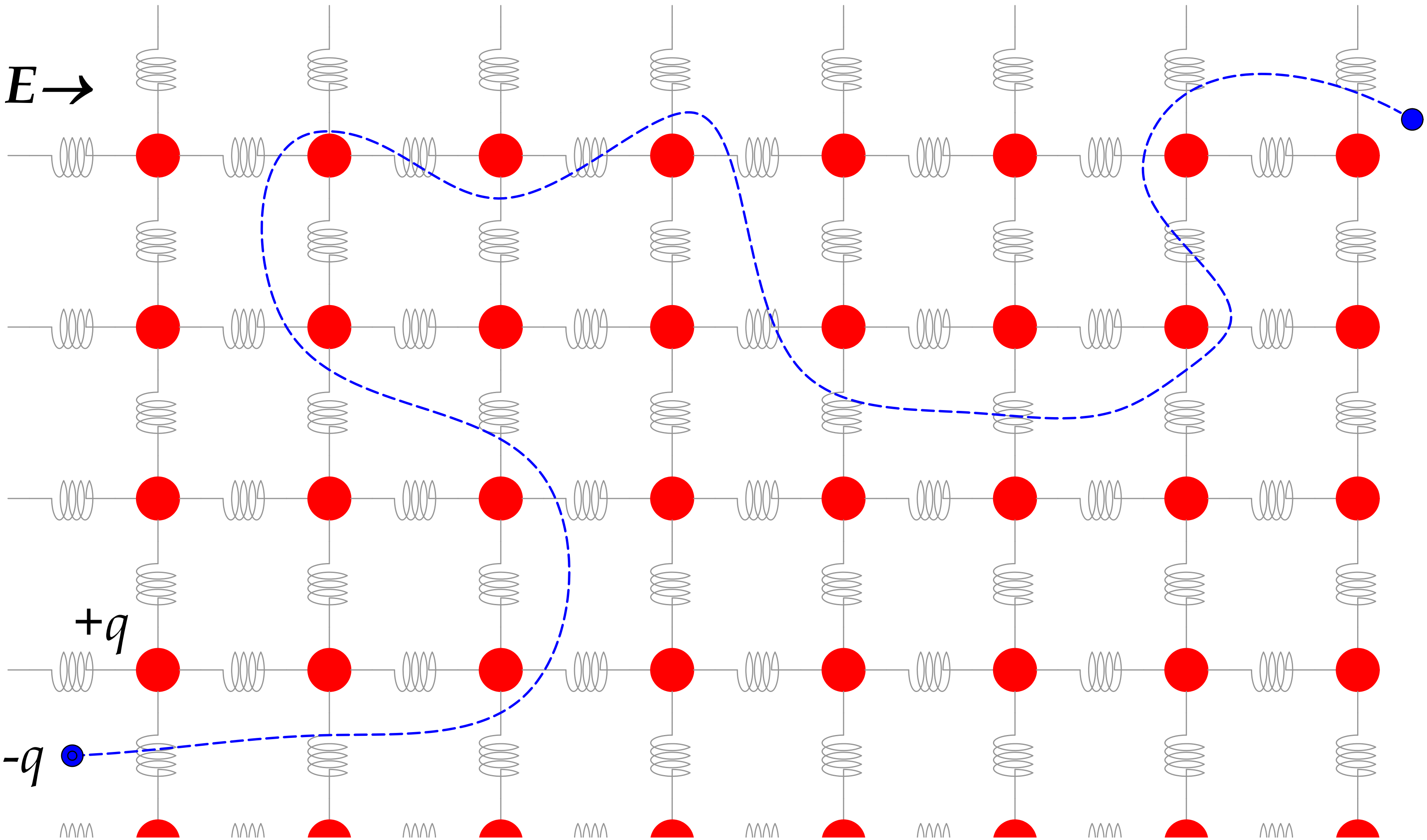
Resistance is a property of the material that the resistor is made out of, and thus changes if the material properties are altered, notably by anything that changes the spacing of the immobile ions that the mobile charges can scatter from. Notable changes are

- changing temperature, which alters all of the spacings in 3D as well as altering the amount by which individual ions are vibrating around, and
- **strain**: stretching or compressing one set of spacings more than the others. Compare the paths of the current carrier in the two different states of strain, in the following two pages.

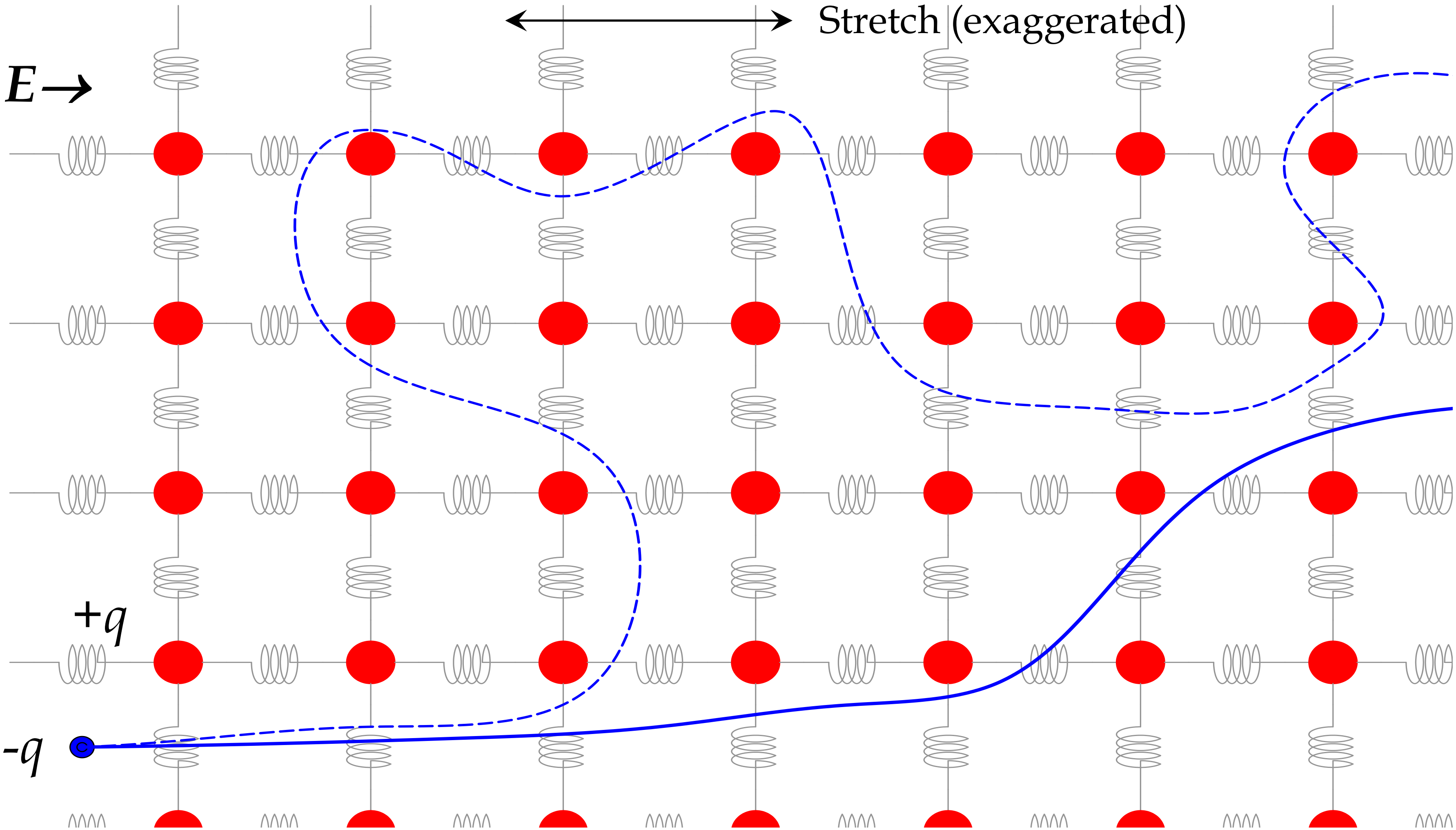
Small strains or temperature changes produce small changes in resistance. Nevertheless, even tiny changes can be detected, with cleverly designed, simple circuits.

The Simmons strain gauge is a good example of how simple a very useful electrical device can be.

The Simmons strain gauge (continued)

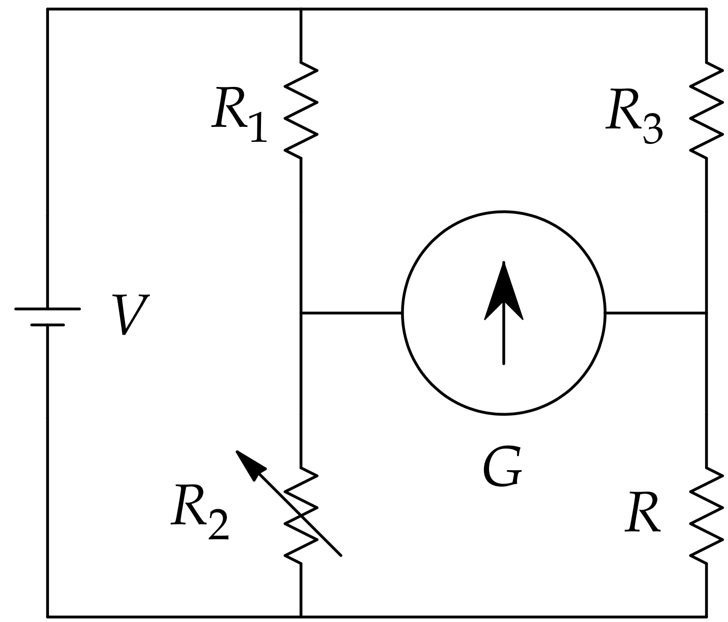


The Simmons strain gauge (continued)



The Simmons strain gauge (continued)

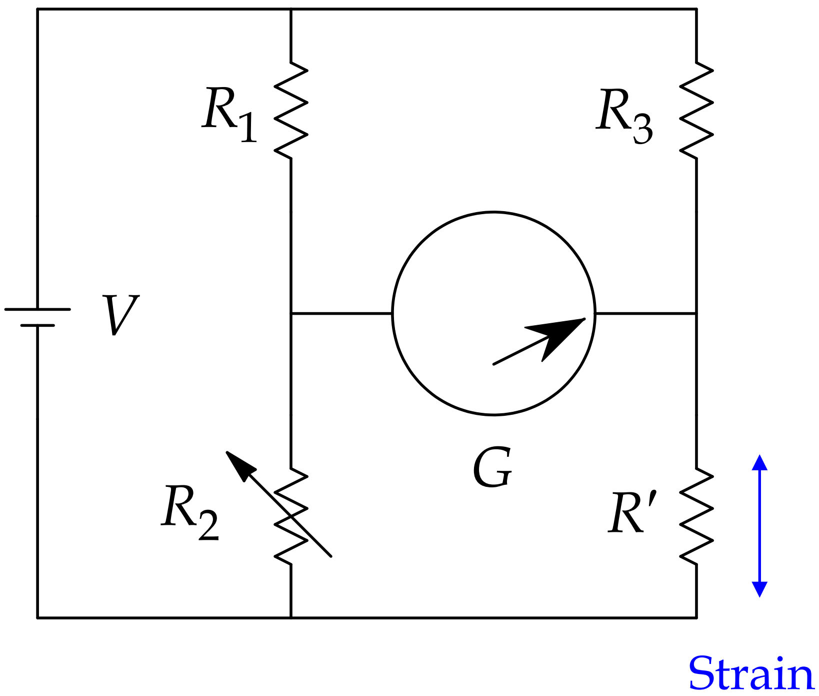
Consider, for instance, a quartet of resistors and a **galvanometer**: a very sensitive current sensor consisting of a coil and a permanent magnet, arranged as at right.



- If $R_1/R_2 = R_3/R$, then the voltages across the resistors are in the ratio $I_1R_1/I_1R_2 = I_3R_3/I_3R$, and the galvanometer has the same voltage at each end: no current flows through it.
- This can be set up in the circuit, by adjusting the value of R_2.
- Next week we will discuss this circuit: it's a **Wheatstone Bridge**.

The Simmons strain gauge (continued)

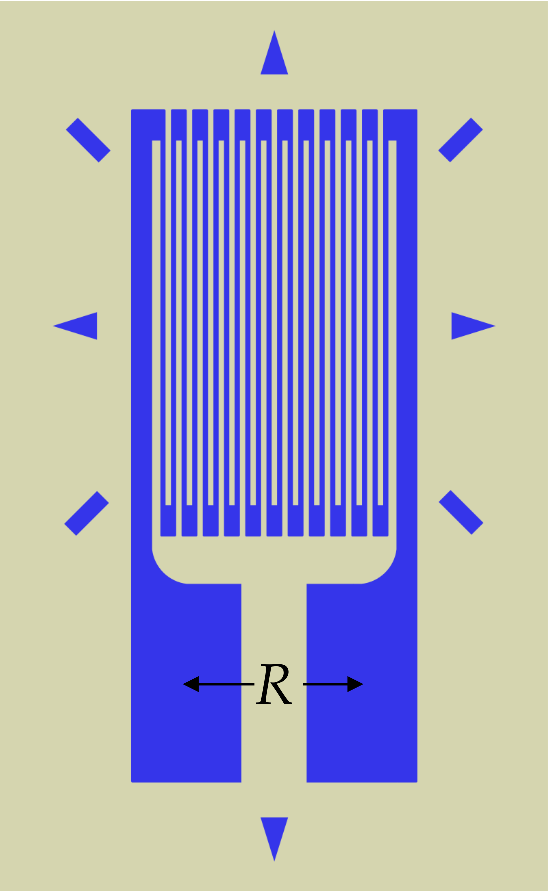
- Then if somebody comes along and alters resistor R – like, say, by straining it – then the delicate balance of voltages is altered, and current flows in the galvanometer.
- In modern versions of galvanometers, this can be used to throw a simple transistor or diode switch, causing the resistance change to trigger something else.
  - Like turning a traffic light from red to green.
  - Like by the strain on R from your car coming to rest on top of it.



The Simmons strain gauge (continued)

This can lead to very inexpensive sensors based on strain, which turn out to be extremely sensitive and robust.

- The pattern at right, for example, is a thin metallic layer created on plastic film, which in turn can be superglued to something to which strain might be applied.
- As designed, it is more sensitive to stretch or compression in the vertical direction (on the page) than the horizontal.
- This whole scheme is called the **Simmons strain gauge**, and was invented in the 1930s.



Midterm exam coming!

This concludes the material to be covered on next Monday's midterm exam. Friday begins new topics.

- ❑ … which will take place right here in our normal class time on 7 October.
- ❑ A practice exam, with solutions, will appear later today on the course website. You will be notified by email.
- ❑ Next lecture will be review and examples.
  - Review = review of the Recipes, and when to use which.
  - The examples will be on request. If you have an example you'd like to see worked out, send it to me by email. I will select from among the suggestions.