P113 University of Rochester NAME Soln key - SM
S. Manly Fall 2003

Final Exam (December 18, 2003)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

<u>Problem 1 (6 pts, show your work):</u>

While solving a physics problem you perform a series of algebraic manipulations that lead to a mathematical expression for a distance. If F=force, a=acceleration, v=velocity, m=mass, t=time, and μ_s=coefficient of static friction, N=a normal force, use dimensional analysis to show which of the following expressions could be correct. Indicate those expressions that could represent a distance by circling them (there may be more than one). You may circle more than one answer.

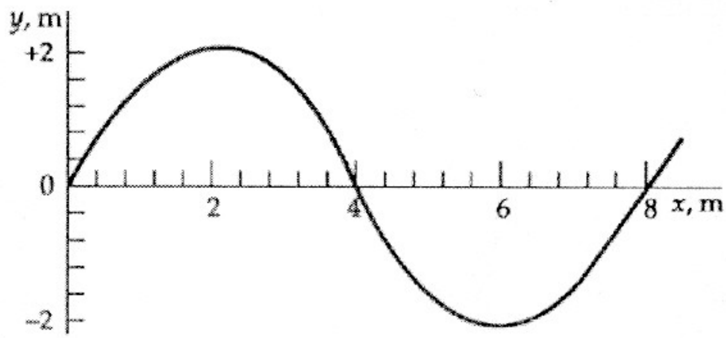
a) $\frac{vt\mu_s N}{F}$ b) $\frac{v^2}{2a\mu_s}$ c) $\frac{mva}{tN}$ d) $\frac{ma^2t^2}{N\mu_s}$ e) $\frac{at^2\mu_s}{N}$

$\frac{m}{s}\, s = m$ $\frac{\frac{m^2}{s^2}}{m/s^2}$ $\frac{kg\,\frac{m}{s}\,\frac{m}{s^2}}{s\ kg\,m/s^2}$ $\frac{kg\,\frac{m}{s^2}\,\frac{m}{s^2}\,s^2}{kg\,m/s^2}$ $\frac{\frac{m}{s^2}\,s^2}{kg\,m/s^2}$

<u>Problem 2 (8 pts, show your work):</u>

The graph below shows a wave traveling to the right with a velocity of 4 m/s. The equation that best represents the wave is

a) y(x,t)=2sin(πx/4 - πt) meters
b) y(x,t)=2sin(16πx - 8πt) meters
~~c)~~ y(x,t)=2sin(πx/4 + πt) meters
~~d)~~ y(x,t)=4sin(πx/4 - πt) meters
~~e)~~ y(x,t)=4sin(16πx - 8πt) meters



$y(x,t) = A\sin(kx - \omega t)$

$k = \frac{2\pi}{\lambda}$ $\omega = \frac{2\pi}{T} = 2\pi f$

Amplitude = 2 ~~d~~ ~~e~~

Moves to right ~~c~~

$\lambda = 8 \Rightarrow k = \frac{2\pi}{8} = \frac{\pi}{4} \Rightarrow$ a

$V = 4 \Rightarrow T = 2$ $\omega = \frac{2\pi}{2} = \pi \Rightarrow$ a

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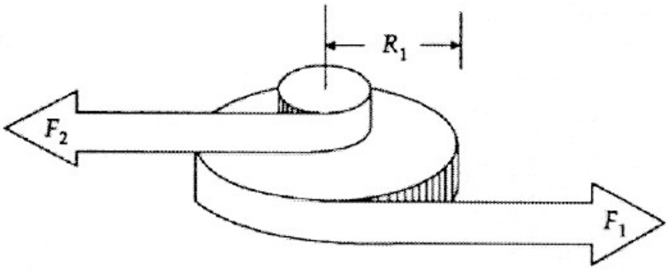
1)	/6
2)	/8
3)	/7
4)	/5
5)	/10
6)	/8
7)	/7
8)	/8
9)	/6
10)	/7
11)	/8
12)	/10
13)	/10
tot	/100

Problem 3 (6 pts, show your work):

A wheel of radius R_1 has an axle of radius $R_2=R_1/4$. If a force F_1 is applied tangent to the wheel, a force F_2, applied tangent to the axle as shown that will keep the wheel from turning, is equal to

a) $F_1/4$
b) F_1
c) $4F_1$
d) $16F_1$
e) $F_1/16$



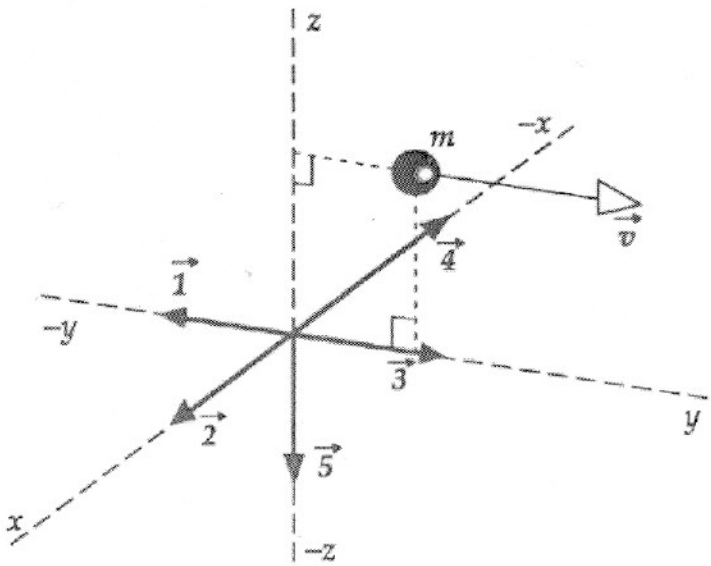
$R_2 F_2 = R_1 F_1$

$\frac{R_1}{4} F_2 = R_1 F_1 \Rightarrow F_2 = 4F_1$

Problem 4 (5 pts, no partial credit):

A particle of mass m is moving with a velocity v in the yz plane as shown in the figure. The vector that most nearly represents the angular momentum about the x axis is

a) 1
b) 2
c) 3
d) 4
e) 5



Problem 5 (10 pts): GRADing for this is A bit subjective

Suppose, as you go home for the holidays, in the midst of a big family holiday feast your grandmother announces that she is about to go on tour as the lead guitarist for the Bare Naked Old Ladies. Everyone is thrilled! After dinner, Grandma, knowing you are now a physics god(ess), asks you to explain the basic physical principles behind the operation of the guitar. Briefly explain what you would tell her. Use sketches and equations as you feel appropriate (Grandma can handle it).

ANSWer CAN Vary

Well GRANDMA, I AM SO glad you asked! I Love Nothing more than talking about physics!!

The guitar, as with MOST stringed instruments, consists of a series of strings with varying mass/length, tension and length with fixed endpoints. (The fixed endpoints are known as boundary conditions.) When one plucks one of the strings you displace a segment of the string transversely. that transverse displacement travels as a transverse wave down the string. When the wave encounters a fixed endpoint it is reflected with a 180° phase change, which, upon superposition, interferes destructively with the incoming wave and satisfies the boundary condition of zero displacement at the fixed ends. The waves going each Direction interfere. For traveling waves that satisfy certain conditions the waves travel back and forth interfering and creating a resonance condition called a STANDING wave. Waves NOT satisfying these conditions will interfere chaotically and die out quickly. For fixed ends, STANDING waves are formed if

$$L = \frac{\lambda}{2} \quad \text{or} \quad L = \frac{2\lambda}{2} \quad \text{or} \quad L = \frac{n\lambda}{2} \quad \text{for } n = 1, 2, 3, \ldots,$$

where L is the Length of the string. Since $v = \lambda f$, this means a given string will propagate STANDING waves of frequencies that satisfy

$$f_n = \frac{nv}{2L} \quad \text{or} \quad f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$ where T = Tension and μ = Mass/Length

Strings vibrating at these particular frequencies create longitudinal pressure waves in air (called sound) of those particular frequencies. This is what you hear GRANDMA! AND you can vary those frequencies by varying the Tension, μ, or length of the string!

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Problem 6 (8 pts):

It is a little known fact that the true bond between Jessica Simpson and Nick Lachey is that they like to do physics experiments in the basement when they are not doing important chores for MTV. Imagine that. Anyway, the other day Jessica and Nick hid from the cameras and reality show producers and went down to the basement to play with balls.

They attached one ball of mass M=0.2 kg to a massless string creating a simple pendulum. The string was length L=1.2 meters and fixed at point C in the sketch below. The pendulum was held in position A where the string was vertical as shown in the sketch below. Another identical ball (with no attached string) was held at the same height. Then the two balls (each at position A) were released and fell to position B where Jessica and Nick measured their respective speeds.

How did the speed of the two balls compare when they reached the height indicated by position B?

By E conservation $mgL = \frac{1}{2}mv^2$ in both cases

Therefore the speeds are the same.

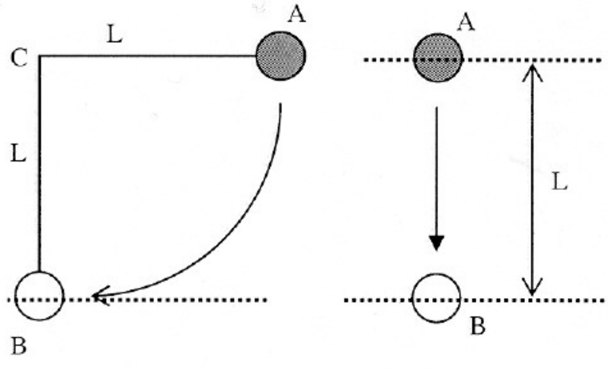
How much work is done by gravity in each case?

$$+mgL = 0.2(9.8)(1.2) \text{ Joules} = 2.3 \text{ J}$$

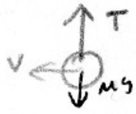
What is the period of the pendulum (*make the assumption that this is a "simple" pendulum and one can assume the amplitude of the motion is small even though it is not*)?

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.2}{9.8}} = 2.2 \text{ s}$$

What is the tension in the string when the pendulum first reaches position B?



$V = \sqrt{2gL}$



Circular Motion

$\cancel{T = \frac{mv^2}{L} = \frac{m2gL}{L}} = 2mg$

$$T - mg = \frac{mv^2}{L}$$

$$T - Mg = 2Mg \cancel{= 3Mg}$$

$$T = 3Mg = 5.88 \text{ N}$$

Problem 7 (7 pts):

Flash forward a few years: Having graduated from Rochester with honors, you attended law school and are now a famous defense lawyer in California. You have a spouse with perfect teeth and you live in a mansion next to Courtney Love's sister. Clearly, you have arrived! Your current job is to defend the famous rap artist Archimedes Ice who is accused in the "notorious" hip-hop pool murder of the well known Southern California DJ known as Decibel Smith.

The prosecution says that Archimedes dropped a bowling ball on Decibel while Decibel floated peacefully in the pool below.

a) If the ball left Archimedes' hand at a height of 5m above the water (and Decibel's head) with an initial velocity of 2 m/s downward, with what velocity did the ball strike Decibel?

Const Accel $V^2 = V_0^2 + 2a(Y - Y_0)$

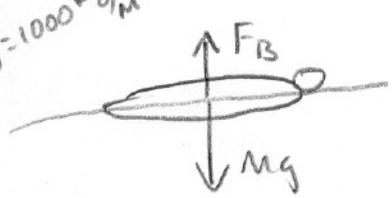
$V^2 = (2)^2 + 2(9.8)(5) = 102$

$V = 10\ ^{m}/_{s}$

* → Give Decibel's specific gravity as 0.9

b) If Decibel weighed 1000 Newtons before his death, calculate what fraction of his body was under water when the bowling ball struck, assuming he was floating peacefully at the time. ***(Assume Decibel's body has a constant density.)***

$\rho_w = 1000\ ^{kg}/_{m^3}$



Floating $\Rightarrow F_B = Mg$ X = fraction under water

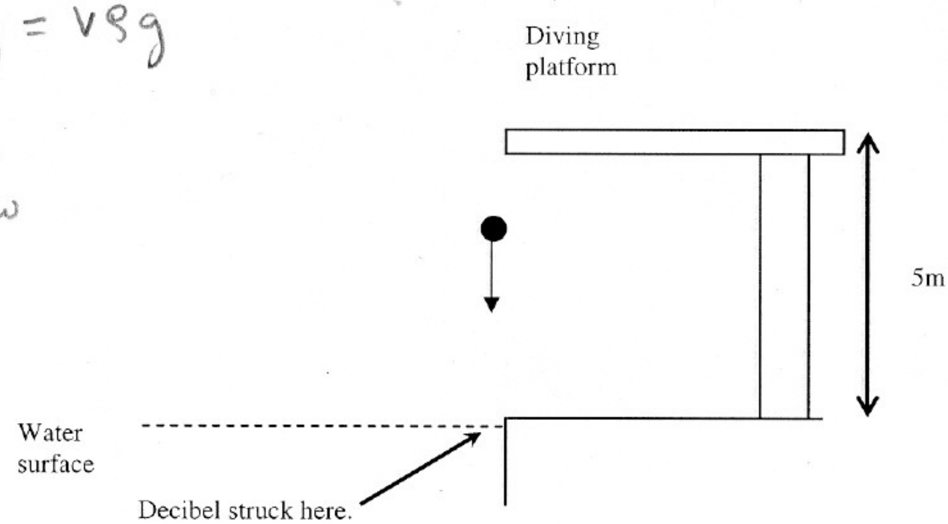
Assume Decibel has density ρ and Volume V.

$F_B = xV\rho_w g$ $M = V\rho$

$\therefore$

$xV\rho_w g = V\rho g$

$x = \dfrac{\rho}{\rho_w}$



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Problem 8 (8 pts, show your work):

Useful information: $I_{disk}=(1/2)MR^2$ $I_{ring}=MR^2$

Starting from rest at the same time, a coin and a ring roll down an incline without slipping. Which reaches the bottom of the incline first?

a) The ring reaches the bottom first.
b) The coin reaches the bottom first.
c) They arrive at the bottom simultaneously.
d) The winner depends on the relative masses of the two.
e) The winner depends on the relative diameters of the two.

See Below

* change spacing

For the coin above, when it reaches the bottom, which is greater, its translational or rotational kinetic energy?

a) Its translational energy is greater.
b) Its rotational energy is greater
c) They are equal.
d) The answer depends on the radius of the coin.
e) The answer depends on the mass of the coin.

$s = r\theta$
$v = r\omega$

$E_{cons} \Rightarrow \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ ✓

Coin

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{1}{2}MR^2\frac{v^2}{R^2}$

TRANSLATIONAL TERM

$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$

ROTATIONAL term

$mgh = \frac{3}{4}mv^2$

$v = \sqrt{\frac{4}{3}gh}$

Ring

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}MR^2\frac{v^2}{R^2}$

$mgh = Mv^2$

$v = \sqrt{gh}$

coin (disk) goes faster
No dependence on M or R

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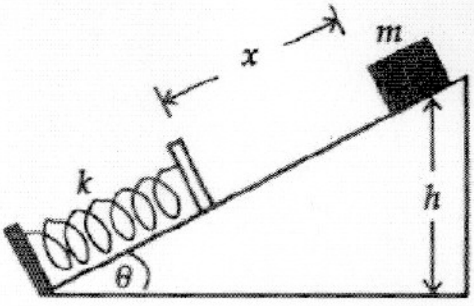
Problem 9 (6 pts, no partial credit):

Assuming the incline in the figure to be frictionless and the zero of gravitational potential energy to be at the elevation of the bottom of the incline,

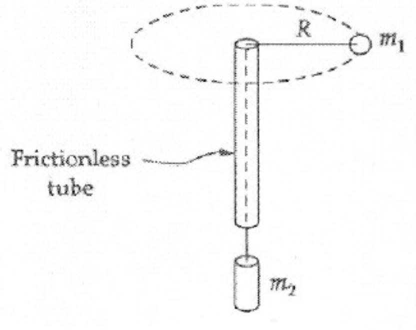
a) **the kinetic energy of the block just before it collides with the spring will be equal to mgh.**
b) **The kinetic energy of the block when it has fully compressed the spring will be equal to mgh.**
c) **The potential energy of the block when it has fully compressed the spring will be zero.**
d) **The energy stored in the spring plus the gravitational potential energy of the block when it has fully compressed the spring will be equal to mgh.**
e) **None of the above statements will be true.**



Problem 10 (7 pts, show your work):

A ball of mass m_1, connected to another mass m_2 by a string, is whirled at a constant speed in a horizontal circle of radius R equal to 0.8 m. If the mass m_2=5.0 kg, the kinetic energy of the ball is

a) **0.98 J**
b) **2.45 J**
c) **4.90 J**
d) **19.6 J**
e) **39.2 J**



$T = \frac{m_1 v^2}{R} \rightarrow \frac{m_1 v^2}{R} = m_2 g$

$T = m_2 g$

$v^2 = \frac{m_2 g R}{m_1}$

$KE_{Ball} = \frac{1}{2} m_1 v^2 = \frac{1}{2} m_1 \frac{m_2 g R}{m_1} = \frac{1}{2} m_2 g R = \frac{1}{2}(5)(9.8)(0.8)$

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Problem 11 (8 pts):

A 45 kg rock climber is climbing a "chimney" between two rock slabs as shown in the figure below. The statis coefficient of friction between her shoes and the rock is 1.2 and betweent he rock and her back it is 0.80. She has reduced her push against the rock until her back and her shoes are on the verge of slipping.

a) What is the normal force of her shoes against the rock?

$F_1 \equiv$ Force of friction at feet $= \mu_s^{feet} N_1$

$F_2 \equiv$ " " " " back $= \mu_s^{BACK} N_2$

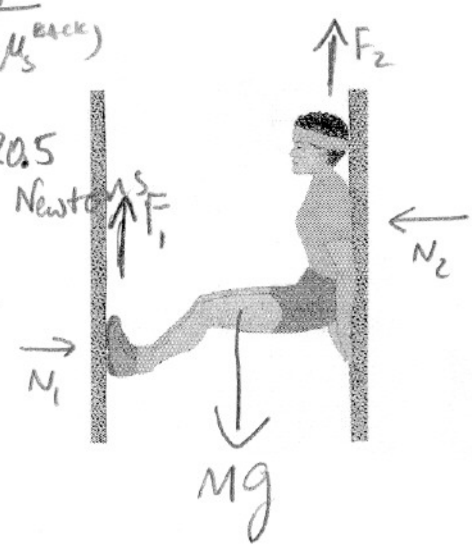
$\Sigma F_y = 0 \Rightarrow N_1 = N_2 = N$

$\Sigma F_x = 0 \Rightarrow F_1 + F_2 - Mg = 0$

$\mu_s^{feet} N + \mu_s^{back} N = Mg$

$N = \dfrac{Mg}{(\mu_s^{feet} + \mu_s^{BACK})}$

$N = \dfrac{(45)(9.8)}{(1.2 + 0.8)} = 220.5$ Newton



*

Move Down

b) How does the normal force of her shoes against the rock compare to the normal force of her back against the rock?

The SAME

c) What fraction of her weight is supported by the friction force on her shoes?

Fraction WT supported by shoes $= \dfrac{\mu_s^{feet} N}{Mg} = \dfrac{(1.2)(220.5)}{(45)(9.8)} = 0.6$

60%

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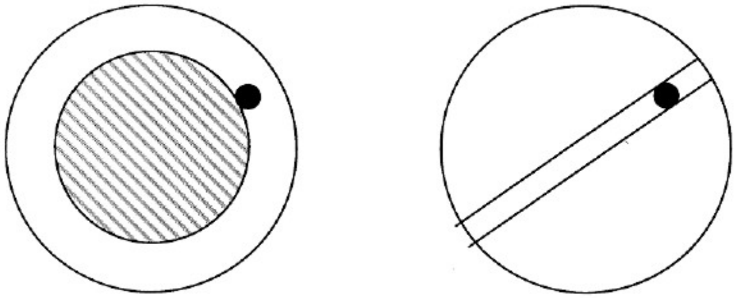
Problem 12 (10 pts):

Fact:* *The gravitational field at a certain radius within a solid object with spherical symmetry is determined entirely by the mass that is at a smaller radius. To make this concept more clear, consider the left-hand sketch below. Let the outer circle represent the radius of an object with spherical symmetry. The gravitational force on the object at a position given by the black dot comes only from the shaded region of the spherically symmetric object. One can prove that the shell of mass outside that radius makes no net contribution to the gravitational field.



Now, imagine that a 10 cm hole is drilled all the way through the earth. You drop a ball of mass m into the hole as shown in the right-hand sketch above. Assume the Earth has a constant mass density (which is not true in reality) and assume no form a friction acts on the ball.

Prove the subsequent motion of the ball is simple harmonic motion.

At distance r from center, have restoring force due to gravitation.

$$F = -\frac{GMm}{r^2} \qquad M = \rho_{Earth} V = \rho \tfrac{4}{3}\pi r^3 \qquad F = -\frac{G\rho \frac{4}{3}\pi r^3 m}{r^2} = -G\rho\tfrac{4}{3}\pi r m$$

(*) move down

How long will it take the ball to return after you drop it?
(For this calculation, you might find the following information useful: ***$G=6.67x10^{-11}\ m^3/s^2kg$, $M_E=5.98x10^{24}$ kg, $R_E=6.37x10^6$ m)***

$$\boxed{m\frac{d^2r}{dt^2} = -\underbrace{\left[G\rho\tfrac{4}{3}\pi m\right]}_{\text{CONSTANT}} r}$$

differential eqn for SHM
w/ soln
$r = \sin(\omega t + \varphi)$
$\omega^2 = G\rho\frac{4}{3}\pi$

$$\omega = \frac{2\pi}{T} \qquad T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{G\rho\frac{4}{3}\pi}} = 5058 \text{ seconds}$$

$$\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} = \frac{5.98x10^{24}}{\frac{4}{3}\pi(6.37x10^6)^3} = 5522\ kg/m^3$$

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Problem 13 (10 pts):

Consider a uniform rope with total mass M and total length L coiled on the floor as shown below. Starting at t=0, one end of the rope is lifted with a force T such that it rises with a velocity described by $v=Ct^3$, where C is a constant with units of m/s^4.

What is the height of the center of mass of the length of rope in the air as a function of time? (*In this question, you are being asked to ignore the length of the rope that is still sitting on the floor at a given time.*)

$\frac{dx}{dt} = Ct^3 \quad x(t) = \int_0^t Ct^3\,dt = \frac{Ct^4}{4}$ ← Represents the position of the top of the rope as a fn of time. The rope is uniform. So

$$x_{cm}(t) = \left(\frac{1}{2}\right)\frac{Ct^4}{4} = \frac{Ct^4}{8}$$

What is the total work done by the force T from t=0 to the time that the far end of the rope has just left the floor?

M is a fn of time

$$W = \int F\cdot dx = \int T\cdot dx = \int \lambda x(3Ct^2 - g)\,dx$$

Need t^2 in terms of x

$x = \frac{Ct^4}{4}$

$t^2 = \sqrt{\frac{4x}{C}}$

$T - Mg = Ma \qquad T = m(a-g) = \lambda x(3Ct^2 - g)$

What is M?

$M = \lambda x = \frac{M}{L}x$

What is a?

$v = Ct^3$

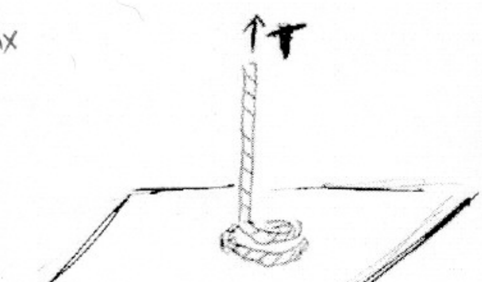
$\frac{dv}{dt} = a = 3Ct^2$

$$W = \int_0^L \lambda x\left(3C\sqrt{\frac{4x}{C}} - g\right)dx$$

$$W = \int_0^L 6\lambda\sqrt{C}\,x^{3/2}\,dx - \int_0^L \lambda x g\,dx$$

$$W = \frac{6\lambda\sqrt{C}\,L^{5/2}}{5/2} - g\lambda\frac{L^2}{2}$$

units ok



Due to late exams, it will be several days before my TA's can grade the finals. Once that happens and I have turned the final grades in to the registrar, I will provide a mechanism for you to see all of the information that contributed to your grade. My goal is to do that by the 22nd or 23rd of December. I will either email your university account with the numbers in my spreadsheet that contributed to your grade or I will notify you of how to find your grades in WEBCT.

For now, forget it, relax, and most of all … Have a great holiday!!