P113 University of Rochester NAME Soln Key - SM
S. Manly Fall 2012

Exam 3 (December 4, 2012)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

Problem 1 (10 pts):

a) Consider three identical wheel rims with the three configurations of spokes sketched below. For each wheel, the spokes have a uniform mass density and the same mass. The only difference is that the spokes are different in shape (as shown). If the three wheels roll simultaneously without slipping down an inclined plane from the same height, in what order will they reach the bottom of the inclined plane. *Provide a sentence or two justifying your answer.*

i. A first, then B, then C
ii. A first, then C, then B
iii. C first, then B, then A
iv. C first, then A, then B
v. B first, then A, then C
vi. B first, then C, then A
vii. All three reach the bottom at the same time

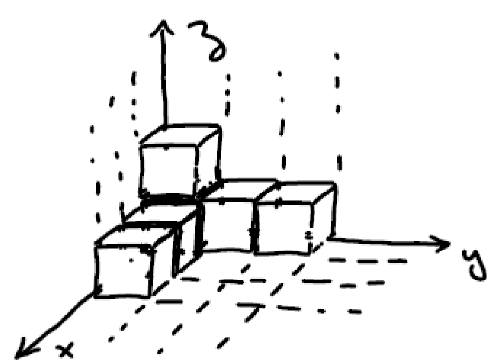
I is largest for C, then A and smallest for B so B first then A then C
large I object will have smaller α and a

b) Consider the same physical situation where the inclined plane has a Teflon surface which removes all friction between the wheels and the plane. In this case , in what order will they reach the bottom of the inclined plane (choose from above). *Provide a sentence justifying your answer.*

if No friction - all wheels slide and do not rotate
So Answer is vii — all three reach bottom at the same time

Problem 2 (15 pts, show work):

Determine the position of the ~~center of~~ mass of the arrangement of the uniform identical cubes shown in the figure. Assume the cubes have sides of 1 meter. Note that one cube sits with a corner at (x,y,z)=(0,0,0) and is obscured by the other five cubes.



$$x_{cm} = \frac{(4M)\frac{1}{2} + (1M)\frac{3}{2} + (1M)\frac{5}{2}}{6M} = 1$$

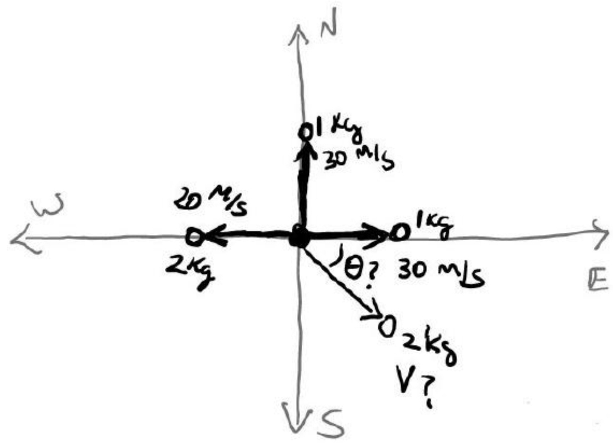
$y_{cm} = 1$ (by symmetry w/ x)

$$z_{cm} = \frac{(5M)\frac{1}{2} + (1M)\frac{3}{2}}{6} = 0.67$$

CM is at (1, 1, .67)

Problem 3 (14 pts, show your work):

One day you take your date out for a fine meal at Al's Steakhouse and Explosive Ordinance Testing Ground. While enjoying your meal by the romantic light of mortar explosions, your mind wanders to physics, as it is prone to do. In particular, you see one explosion that gets you to thinking. Fortunately, you know the owner of the establishment and quickly manage to get the data on that explosion delivered to you so that you can do a calculation.



In the explosion of interest to you, a 6 kg mortar shell explodes into four fragments moving in the horizontal plane, as shown in the sketch. After the explosion, a 1 kg fragment moves in the east direction at 30 m/s, a 1 kg fragment moves north at 30 m/s and a 2 kg fragment moves west at 20 m/s. Unfortunately the proving ground technicians failed to record the direction and speed of the fourth fragment, though they were able to determine its mass as 2 kg. To satisfy your curiosity and to exercise your amazing dexterity in physics, you determine the direction and speed of the fourth fragment … (please do/repeat that calculation below). *You might find it useful to know the trig identitity* $(\sin\theta)^2 + (\cos\theta)^2 = 1$.

$\vec{P}_{initial} = 0$

x/ $0 = (1)(30) + (2)v\cos\theta - (2)(20)$

y/ $0 = (1)(30) - (2)v\sin\theta$

$v\cos\theta = \frac{40-30}{2} = 5$

$15 = v\sin\theta$

$v^2\cos^2\theta + v^2\sin^2\theta = 25 + 225$

$v^2(1) = 250$

$v = 15.8$ m/s

$15 = 15.8\sin\theta$

$\theta = 71.7°$

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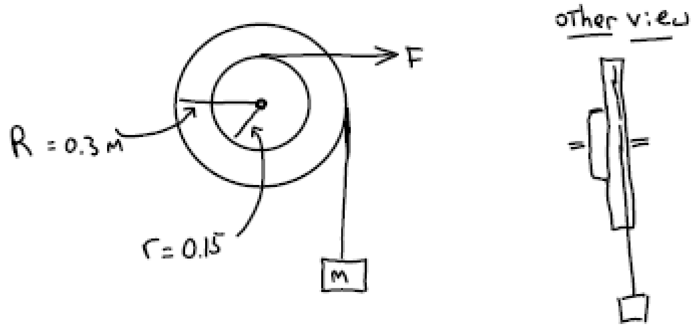
Problem 4 (25 pts, show your work):

A mass of 3 kg hangs from a massless rope that is wound around a large uniform disk of radius R= 0.3 m and mass 2 kg. The disk is free to rotate (frictionlessly) about a horizontal axis. Another uniform disk, coaxial to the first, with radius r= 0.15 m and mass 0.5 kg is glued to the first disk. A constant horizontal force, F=10 N is applied to the outer edge of the second, smaller disk (as shown). The moment of inertia for a uniform disk is (1/2)MR^2.



(a) (2 pts) If the system starts from rest and is released, a few moments later, what is the direction of the angular velocity vector? For parts (a)-(c) Indicate your answer in words as one of the following: left, right, up, down, into paper, out of paper.

into paper

(b) (2 pts) What is the direction of the angular acceleration of the disks?

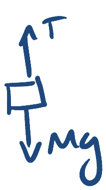
into paper

(c) (2 pts)What is the direction of the angular momentum of the disks?

into paper

1)	6	/10
2)		/15
3)		/14
4)		/25
5)		/16
6)		/20
tot		/100

(d) (15 pts) What is the acceleration of the mass?



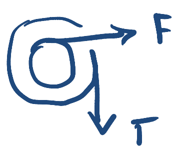
$$ma = mg - T$$

$$I = \tfrac{1}{2}M_1R^2 + \tfrac{1}{2}M_2r^2$$

$$I\alpha = I\frac{a}{R} = TR + Fr$$

$$I = \tfrac{1}{2}2(.3)^2 + \tfrac{1}{2}(.5)(0.15)^2$$

$$I = 0.096 \text{ kg-m}^2$$



$$Ia = mgR^2 - maR^2 + FrR$$

$$a = \frac{mgR^2 + FrR}{I + mR^2} = \frac{(3)(9.8)(0.3)^2 + (10)(.15)(.3)}{.096 + (3)(.3)^2}$$

$$= 8.45 \text{ m/s}^2$$

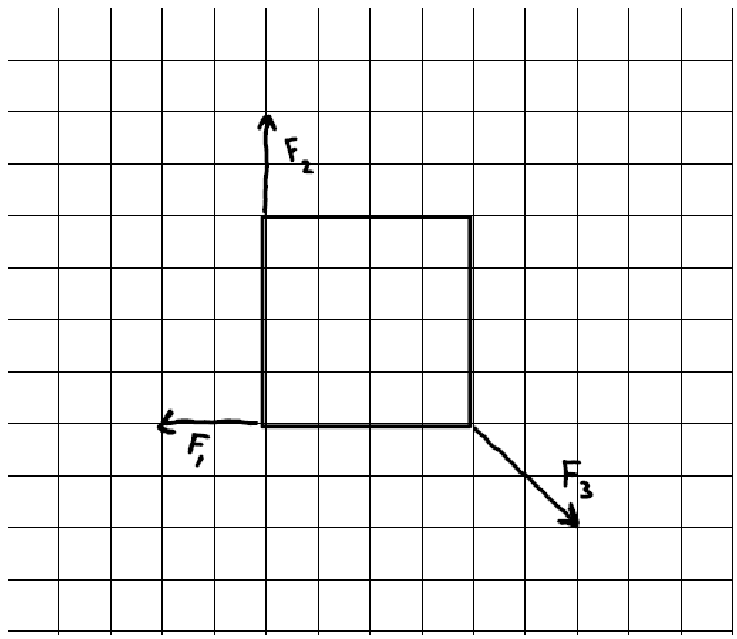
(e) (4 pts) After 3 seconds, what is the magnitude of the angular velocity of the disks?

α ω αt ω $= \omega_0 + \frac{a}{R}t$

$+ \omega = \frac{8.45}{0.3}(3) = 85 \text{ rad/s}$

Problem 5 (16 pts, show your work):

(a) (8 pts) Consider the rigid square object in the plane of the paper. Ignore the third dimension (in and out of the paper) for this problem. Three forces are shown to act on the object. Is the object in static equilibrium? If not, is it possible to put the object into equilibrium with a fourth force? If so, please draw an appropriate fourth force on the sketch.



object is NOT in static equilibrium

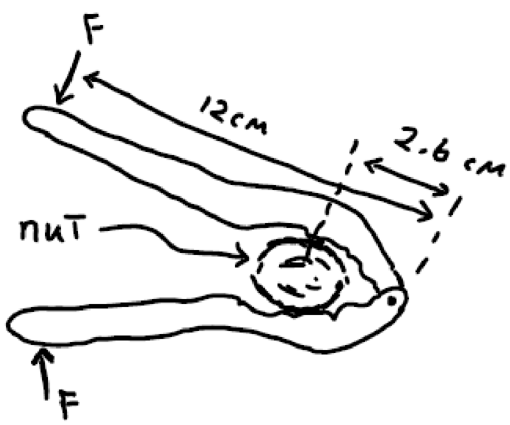
$\sum \vec{F} = 0$

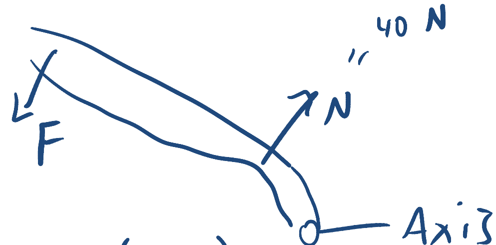
but $\sum \vec{\tau} \neq 0$

can add 4th force to make $\sum \vec{\tau} = 0$ but then $\sum \vec{F} \neq 0$

so – NOT possible to put in static equil.

(b) (8 pts) Tis the season! Consider the nutcracker sketched below. The nut that you see requires a force of 40 N acting on both sides in order to crack. What forces F will be required to crack this nut?



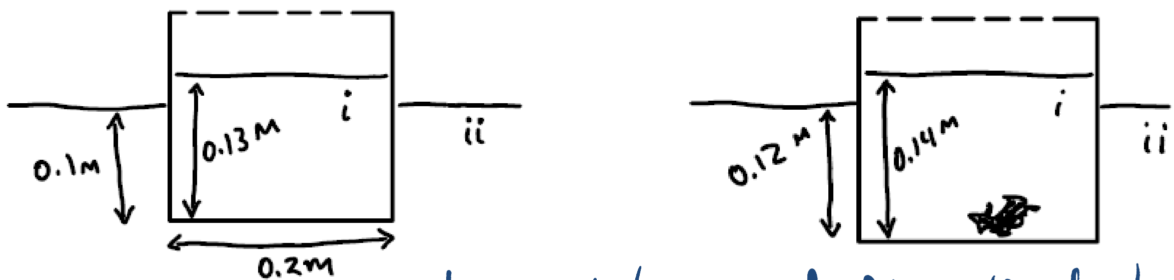


$\sum \tau = 0 \quad F(0.12) - 40(.026) = 0$

$$\boxed{F = \frac{(40)(.026)}{(0.12)} = 8.7\ \text{N}}$$

Problem 6 (20 pts, show your work):

Consider a plastic cup of cubic shape and unknown mass containing methanol (i) floating in a pool of water (ii), as shown in the sketches. Let the sides of the cube be 0.2 m in length and of negligible thickness. Initially, the hollow cube is filled with methanol up to a height of 0.13 m. The water comes up to a height of 0.1 m on the side of the methanol-containing cup. This situation is shown on the sketch at the left. Then an object of unknown density and an irregular shape is placed in the methanol and it sinks to the bottom of the cup (as shown in the sketch to the right). With the object in the methanol, the height of the methanol in the cup is 0.14 m and the water comes up to a height of 0.12 m on the side of the floating cup. What is the mass density of the irregular object placed in the methanol? The density of the water is 1000 kg/m^3 and the density of methanol is 791 kg/m^3.



F_B supports cup + methanol both before and after object placed in the methanol — so only need to use differences in height

ΔF_B supports the object

so $\underbrace{(0.12-0.1)\overbrace{(.2)(.2)}^{\text{Area of bottom of cube}}}_{\text{volume of additionally displaced water}} \rho_w g = M_{object}\, g$

$\Delta F_B =$

$\boxed{M_{object} = 0.8\ kg}$

Volume of object $= (0.14-0.13)(0.2)(0.2) = 4\times10^{-4}\ m^3$

$\rho_{object} = \dfrac{M_{object}}{Volume} = \dfrac{.8\ kg}{4\times10^{-4}} = 2000\ kg/m^3$

As an aside ... does object sink?

WT methanol $= (.13)(.2)(.2)(791)(9.8) = 40.3\ N$

largest Bouyant force possible $= (.2)(.2)(.2)(9.8)(1000) = 78.4\ N$

Would NOT sink unless cup is heavy ... assume it is NOT ∴ NOT needed to solve

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$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$v = v_o + at$$

$$x = x_o + v_o t + \frac{1}{2}at^2$$

$$x = x_o + \left(\frac{v_o + v}{2}\right)t$$

$$v^2 = v_o^2 + 2a(x - x_o)$$

$$x - x_o = \int_{t_o}^{t} v\,dt$$

$$v - v_o = \int_{t_o}^{t} a\,dt$$

$$\sum \vec{F} = m\vec{a}$$

$$F_{\text{friction}} = \mu_k N$$

$$F_{\text{friction}} = \mu_s N$$

$$F_{\text{centripetal}} = \frac{mv^2}{r}$$

$$\vec{F}_{\text{spring}} = -k(\vec{x} - \vec{x}_o)$$

$$\text{work} = \int F \cdot ds$$

$$\text{power} = \frac{dw}{dt}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$s = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

$$P = P_o + \rho g h$$

$$AV = \text{constant}$$

$$\omega = \omega_o + \alpha t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\theta = \theta_o + \left(\frac{\omega + \omega_o}{2}\right)t$$

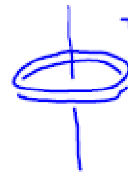
$$\omega = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$KE_{\text{translation}} = \frac{1}{2}MV^2$$

$$KE_{\text{rotation}} = \frac{1}{2}I\omega^2$$

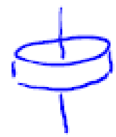
$$I = \sum m_i r_i^2 = \int r^2 dm = \int r^2 \rho\, dv$$

$$X_{cm} = \frac{\sum x_i m_i}{M} = \frac{\int x\,dm}{M}$$



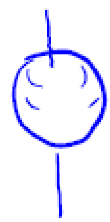
Thin hoop radius R

$I = MR^2$



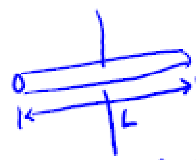
Solid cylinder Radius R

$I = \frac{1}{2}MR^2$



uniform Sphere Radius R

$I = \frac{2}{5}MR^2$



uniform rod of length L

$I = \frac{1}{12}ML^2$

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$I = I_{cm} + mh^2$$

$$\vec{\tau} = \vec{r} \times \vec{F} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$\text{circumference of circle} = 2\pi r$$

$$\text{area of circle} = \pi r^2$$

$$\text{quadratic equation} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\vec{F}_{grav} = -\frac{Gm_1 m_2}{r^2}\hat{r}$$