Exam 1 (February 12, 2004)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (12 pts, show work/logic to get credit):

In the situation sketched below, the electric field is zero at point A. What is $Q_t$?

- a) $+32 \mu C$
- b) $-32 \mu C$
- c) The electric field cannot be zero at point A.
- d) $+16 \mu C$
- e) $-16 \mu C$

\[
\frac{k Q_1}{(40)^2} - \frac{k(8 \times 10^{-6})}{(20)^2} = 0
\]

\[
Q_t = \frac{(8 \times 10^{-6})(40)^2}{(20)^2} = +32 \mu C
\]

Problem 2 (12 pts, show work/logic to get credit):

The charge on an originally uncharged insulated conductor (shown on the lower left in the sketch below) is separated by induction from a positively charged rod brought near the conductor. For which of the various (3-D, closed) Gaussian surfaces shown below represented by the dashed lines does $\int E \cdot dA = 0$? (The symbol “$\int$” means integral over the entire closed surface.)

- a) S1
- b) S2
- c) S3
- d) S4
- e) S5
- f) All of the surfaces
- g) S1, S2, S3, and S5

\[
\int E \cdot dA = 0
\]

implies no net charge is contained inside the closed Gaussian surface.

S1, S2, S4 obviously contain a non-zero net charge inside.

S5 has a net charge because the right hand rod has a net charge. The left object has charge separated but not removed. So S3 has no net charge enclosed.
Problem 3 (12 pts, show work/logic to get credit):

In the sketches below, are shown three electric field configurations. The straight arrows represent the field. They are so-called lines of force. The points A and B are separated by the same distance in all three cases. A proton (charge of +e) is released at point A in each of the three cases. The momentum of the proton is measured when it reaches point B. Rank the three cases in order of descending proton momentum as it is determined at point B in the three cases.

\( P^{(1)}_{\text{Proton}} > P^{(2)}_{\text{Proton}} > P^{(2)}_{\text{Proton}} \)

Electric field is strongest where field lines most dense. Strong E means large acceleration because \( F = qE \) and \( F = ma \).

Problem 4 (16 pts, show work/logic to get credit):

The electrostatic potential as a function of distance along a certain line in space is shown in graph (1). Which of the curves in graph (2) is most likely to represent the electric field as a function of distance along the same line?

a) 1  
b) 2  
c) 3  
d) 4  
e) 5

\( E = -\frac{dV}{dr} \)

= \( a \) positive constant in this case.
Problem 5 (16 pts):

Flash forward in time: After graduation you enter Freddie Coulomb’s Clown Training School and become a world renowned clown. Now you spend your days entertaining little armies of brats at birthday parties. Your therapist is concerned that you may go psycho if you see another slice of Chuck E. Cheese pizza. Nevertheless you carry on bravely.

One of your favorite stunts is to rub a blown up balloon on your head (okay, if you are bald at this point in the future you rub it on the head of one of the kids) and then place it on the wall, where it sticks (adheres) to the wall. The kids are always amazed! One day, while sweet-talking one of the Moms/Dads you tell them that this clown gig is temporary and that you were a major physics animal in college. Impressed, they ask you to explain why the balloon sticks to the wall. Briefly explain why this is so below. Use sketches, equations and text as you see fit.

The balloon acquires a net static electric charge when it is rubbed on someone’s head. As the balloon approaches (and even touches) the wall, the electrostatic field of the balloon induces charge separation in molecules and structures in the wall. Where the charge is induced, unlike charges will be closer to the balloon than the like charges. This will mean that there will be a net attraction of the balloon to the wall due to the distance scaling of Coulomb’s Law.
Problem 6 (18 pts. show your work):

A thin glass rod is bent into a semicircle of radius $R$ centered at the origin as shown in the sketch below. A charge of $+q$ is distributed uniformly on the left half of the semicircle (shown as a dark line) and a charge of $-q$ is distributed uniformly on the right half of the semicircle (shown as a lighter line). Find the magnitude and direction of the electric field at the origin. (Helpful hint: this is NOT a Gauss’s Law problem.)

\[ E = \int \frac{k dq}{r^2} \]

To solve:

\[ E = 2 \int_0^{\frac{\pi}{2}} \frac{k R}{R^2} \cos \theta (\hat{x}) \]

From symmetry, only the $\cos \theta$ component of $E$ survives the integration. The $\sin \theta$ component of charge on the left cancels with the $\sin \theta$ component of charge on the right. By symmetry $E$ will be along $+x$ axis direction.

\[ dq = 2ds = 2R d\theta \]

Charge sign of $dq$ already handled thru symmetry argument.

\[ E = 2 \frac{k}{R^2} \int_0^{\pi/2} \cos \theta d\theta \]

\[ E = 2 \frac{k}{R^2} \frac{\sin \theta}{\theta} \bigg|_0^{\pi/2} = \frac{2k \pi}{R} \]

\[ E = \hat{x} \cdot \frac{2k \pi}{R} = \hat{x} \frac{2k \pi}{R} \]

\[ E = \hat{x} \cdot \frac{2k \pi}{R \pi R} = \frac{4k \pi^2}{\pi R^2} \hat{x} \]
Problem 7 (18 pts, show your work):

Consider two infinite, parallel wires A and B situated in a plane a distance D apart. Let each wire carry a static electric charge per unit length of +λ coulombs per meter. Determine the electric field at a point P which is a distance Z above the plane containing the two wires. Point P is constrained to be on the plane that passes symmetrically between the two infinite wires and is perpendicular to the plane containing the two wires. (Hint: a whole is the sum of its parts.)

![Diagram of two parallel wires with a point P above them.](image)

The electric field at point P is the superposition of the electric field from each of the two infinite line charges. We can use Gauss's law to determine the field from each line charge.

\[ \oint E \cdot dA = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

\[ \oint E \cdot dA = \frac{\lambda L}{\varepsilon_0} = 1E_1 \frac{2\pi RL}{2\pi R_0} \]  

Curved surface  

\[ 1E_1 = \frac{\lambda}{2\pi \varepsilon_0} \text{ directed radially away from wire.} \]

If \( \lambda \) is positive, \( \cos \theta \) components cancel.

\[ E_P = 2E_{\text{wire}} \cos \theta \hat{z} \]

\[ E_P = 2 \frac{\lambda}{2\pi \varepsilon_0} \frac{\frac{1}{2} D}{R} \hat{z} = \frac{\lambda D}{2\pi \varepsilon_0 R^2} \hat{z} = \frac{\lambda D}{2\pi \varepsilon_0 (Z^2 + D^2/4)} \]