Exam 2 (October 15, 1999)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (24 pts):

Consider a mass of 2 kg that is moved by one or more forces along the surface of a horizontal, frictionless floor. In the following diagrams, one vector represents one of the forces (of magnitude 3 N) that acts on the mass. The other vector represents the net displacement, S = 4 meters, of the mass due to the motion caused by the forces acting on it. Determine the work performed on the mass by the force shown during the movement.

(a) \[ W = F \cdot \Delta \cos 30^\circ = 1 \cdot 15 \cdot \cos 30^\circ = \frac{104}{2} \text{ N} \cdot \text{m} \]

(b) \[ W = \frac{-12}{\cos 30^\circ} = 10.4 \text{ N} \cdot \text{m} \]

(c) \[ W = -12 \cdot \cos 30^\circ = 10.4 \text{ N} \cdot \text{m} \]

(d) \[ W = -12 \cdot \cos 30^\circ = 10.4 \text{ N} \cdot \text{m} \]

The following are diagrams of systems in equilibrium (static, not moving). They consist of one or more masses of 5 kg, connected to a set of spring scales calibrated in Newtons. Consider the strings and scales to be massless and the pulleys to be massless and frictionless. Assume there is no friction between the inclined plane and the mass. What value is read by the scales in each case?

(f) Scale will read \( M_g \) = 49 N

(g) \[ T = \frac{m g}{\sin \theta} \]

(h) \[ T = \frac{m g}{\sin 270^\circ} \]

NAME: Solution Set 562
Problem 2 (26 pts):

a) Consider the drawing below. Mass 1, is supported by the table. It is attached to mass 2 via a string passing over a massless, frictionless pulley. The coefficient of static friction between the table and mass 1 is 1.50. The coefficient of kinetic friction between the table and mass 1 is 1.03. Let mass 1 = 4 kg and mass 2 = 5 kg. The system starts out at rest.

Describe the movement of mass 1 along the table, i.e., find the acceleration.

\[ N = M_1g \]
\[ M_1a_x = T - F_r = T - \mu_s N \]
\[ a_x = 0 \Rightarrow T = \mu_s N \]
\[ \mu_s = \frac{1.50}{4} = 0.375 \]

**Assume Not Moving for Now**

\[ M_2g = I_k \]

\[ \frac{M_2g}{m_2g} = \frac{5}{4} = 1.25 \]

**This is Threshold \( \mu_s \)**

- If \( \mu_s \leq \mu_s_{th} \), system is moving and must go back and use \( \mu_k \)
- If \( \mu_s > \mu_s_{th} \), system is NOT MOVING.

\[ \mu_s = 1.50 > 1.25 = \mu_s_{th} \]

**System is Not Moving**

\[ a_x = 0 \]

b) Repeat part (a) for a new configuration of masses and pulleys pictured below. Again, assume the pulleys are massless and frictionless and the strings are massless. The other numbers remain the same.

Find \( \mu_s \) threshold to see if system is moving.

\[ T' = F_{Fr} \]

\[ T' = 2T \] \( \text{w/} a_x = 0 \)

\[ 2M_2g = \mu_s M_1g \]

\[ \mu_s = \frac{2M_2g}{M_1g} = \frac{2M_2}{M_1} = \frac{10}{4} = 2.5 \]

**2.5 = \mu_s_{th} > \mu_s = 1.5 \**, so system is Moving

**Note:** String geometry says \( a_m \) down = \( 2a_m \) to right

\[ M_1a_m = T' - F_r = 2T - F_r = 2T - \mu_k M_1g \]

\[ m_2a_m \text{ down} = m_2g - T \]

\[ M_1a_m + m_2a_m \text{ down} = 2M_2g - \mu_k M_1g \]

**NAME Solution - 5N**
Problem 3 (25 pts):

Consider a 10 kg painting held in place by two wires of equal length (0.5 meters). Each wire makes an angle of $\theta$ with the horizontal, as shown in the figure.

a) Find the general equation for the tension in the wires as a function of the weight of the painting and the angle, $\theta$.

\[ \sum F_y = 0 = 2T_y - mg \]
\[ 2T \sin \theta = mg \]
\[ T = \frac{mg}{2 \sin \theta} \]

Tension in each wire is the same by symmetry.

b) For what angle is the tension the least?

\[ \sin 0^\circ < \sin \theta < \sin 90^\circ \]

Tension is least when $\sin \theta$ is greatest.

This happens at $\sin \theta = 1$ or $\theta = 90^\circ$.

\[ \Rightarrow \theta = 90^\circ \]

(c) Assuming the angle in part (b), solve for the tension in each wire using your answer to part (a).

\[ T = \frac{(10 \times 9.8 \times 0.5)}{2 \times \sin 35^\circ} = 85 \text{ N} \]

\[ \text{force} = 1000 \text{ N/m} \]

\[ 85 \text{ N} = \frac{1000 \Delta x}{2} \]

\[ 0.85 \text{ m} = 8.5 \text{ cm} \]

\[ \Delta x = 0.85 \text{ m} - 0.085 \text{ m} = 0.765 \text{ m} \]

\[ \text{Normal length} = 0.5 - 0.085 = 0.415 \text{ m} \]

NAME

Solution Set SM
Problem 4 (25 pts):

My kids have a toy that consists of a rod that is held vertically. Firmly attached to this rod at one end is a wedge with an opening angle from the horizontal of $\theta = 25^\circ$ degrees. On top of this wedge is a thin wire with a bead of mass $m$ on it. The wire is parallel to the surface of the wedge. When the rod is spun at a constant speed, the bead is observed to rise up a distance $L = 0.05$ meters along the wedge surface and remain there. What is the speed of the mass?

![Diagram of the problem setup with a wedge and a bead on a wire.]

For circular motion:

\[ F_{\text{radial}} = \frac{mV^2}{L} \]

To find $N$, note that in the $y$ direction:

\[ \sum F_y = 0 \]

\[ N \cos \theta - Mg = 0 \]

\[ N = \frac{Mg}{\cos \theta} \]

For circular motion:

\[ \frac{Mg \sin \theta}{\cos \theta} = \frac{mV^2}{L \cos \theta} \]

or

\[ V^2 = \frac{Lg \sin \theta}{\cos \theta} \]

\[ V = \sqrt{\frac{Lg \sin \theta}{\cos \theta}} \]

\[ V = \sqrt{(0.05) \times 9.8 \times \sin 25^\circ} \]

\[ V = 0.45 \text{ m/s} \]