

Exam 3 (December 6, 2001)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (10 pts):

You are riding a bicycle up a hill. Your speed is decreasing.

- a) From your point of view, what is the direction of the angular velocity vector of the front wheel of the bicycle? (Your choices are up, down, toward the right, toward the left, toward the front or toward the back.)

Toward the left

- b) From your point of view, what is the direction of the angular acceleration vector of the front wheel of the bicycle? (choices are the same as in part (a))

Toward the Right

- c) From you point of view, what is the direction of the angular momentum vector of the front wheel of the bicycle? (choices are the same as in part (a))

Toward the left

Problem 2 (15 pts):

Circle the correct answer. Show your work. The center of mass of the system of particles shown in the diagram is at point

- a) 1
- b) 2**
- c) 3
- d) 4
- e) 5

let 7 kg mass be at (0,0)

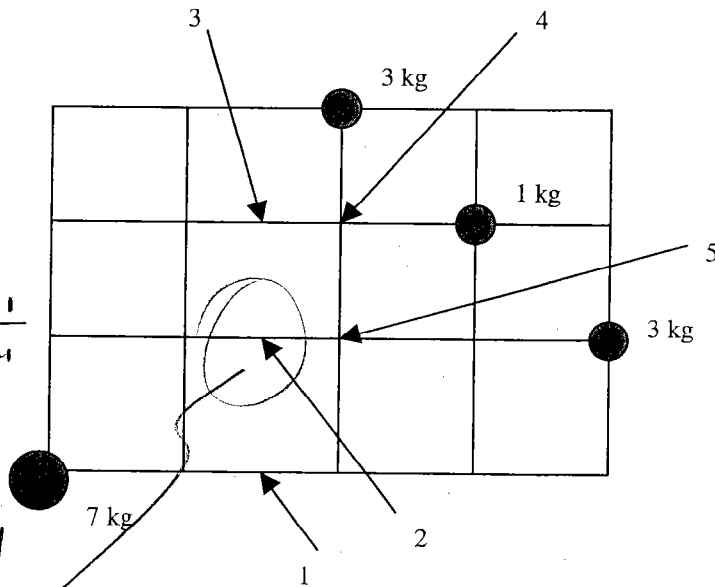
$$X_{cm} = \frac{(0)(7) + (3)(2) + (1)(3) + (3)(4)}{7 + 3 + 1 + 3} = \frac{21}{14}$$

$$X_{cm} = 1.5$$

$$Y_{cm} = \frac{(0)(7) + (3)(3) + (1)(2) + (3)(1)}{14} = 1$$

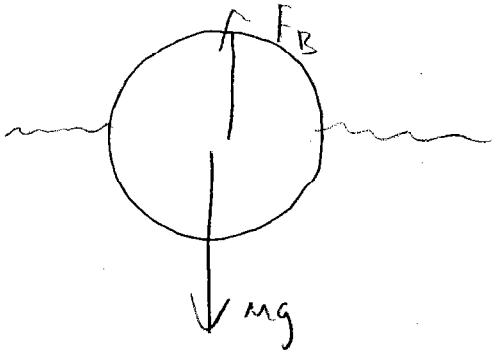
$$Y_{cm} = 1$$

(1.5, 1.0)



Problem 3 (15 pts):

A plastic sphere floats in water with 50.0% of its volume submerged. This same sphere floats in glycerin with 40.0% of its volume submerged. Determine the densities of the glycerin and the plastic in the sphere.



Floats

$$F_B = mg$$

1)	/10
2)	/15
3)	/15
4)	/15
5)	/15
6)	/15
7)	/15
<hr/>	
tot	/100

Water

$$\rho_{\text{plastic}} V_{\text{Ball}} g = (0.5 V_{\text{Ball}}) \rho_{\text{water}} g$$

$$\rightarrow \rho_{\text{plastic}} = 0.5 \rho_{\text{water}}$$

Glycerin

$$\rho_{\text{plastic}} V_{\text{Ball}} g = (0.4 V_{\text{Ball}}) \rho_{\text{gly}} g$$

$$\rightarrow \rho_{\text{plastic}} = 0.4 \rho_{\text{gly}}$$

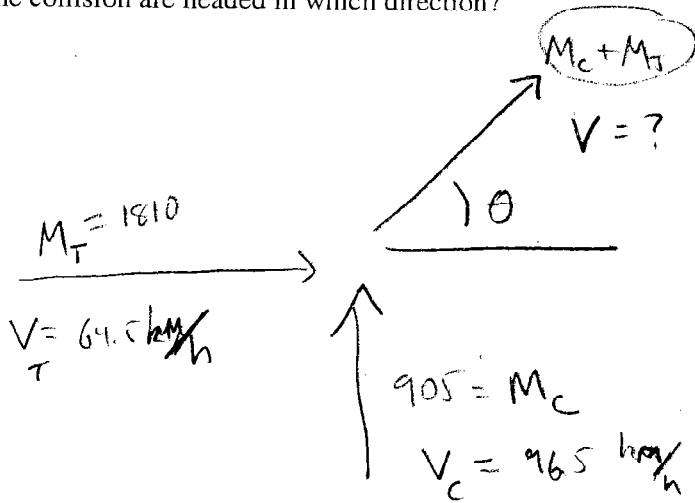
$$\rho_{\text{plastic}} = 0.5 \cdot 1000 \text{ kg/m}^3 = 500 \text{ kg/m}^3$$

$$500 \frac{\text{kg}}{\text{m}^3} = 0.4 \rho_{\text{gly}}$$

$$\rho_{\text{gly}} = 1250 \text{ kg/m}^3$$

Problem 4 (15 pts):

An 1810 kg truck traveling eastward at 64.4 km/h collides at an intersection with a 905 kg automobile traveling northward at 96.5 km/h. The vehicles lock together and immediately after the collision are headed in which direction?



Use momentum conservation in 2 dimensions

x:

$$M_T V_T = (M_C + M_T) V \cos \theta$$

y:

$$M_C V_C = (M_C + M_T) V \sin \theta$$

solve 2 eqns
for 2 unknowns

$$\frac{M_C V_C}{M_T V_T} = \tan \theta = \frac{(905)(96.5)}{(1810)(64.4)}$$

$$\Rightarrow \theta = 36.8^\circ$$

$$\frac{M_T V_T}{(M_C + M_T) \cos \theta} = V = \frac{(1810)(64.4)}{(1810 + 905) \cos 36.8}$$

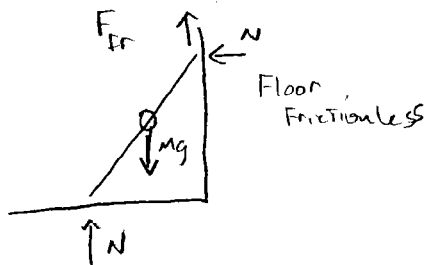
$$= 53.6 \text{ km/h} = v$$

$$53.6 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1000}{1 \text{ km}}$$

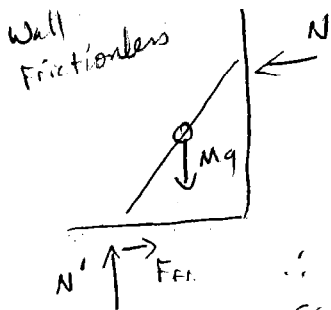
$$V = 14.8 \text{ m/s}$$

Problem 5 (15 pts):

(a) A ladder rests inclined against a wall. Would you feel safer climbing up the ladder if you were told that the floor is frictionless but the wall is rough (can provide friction), or that the wall is frictionless and the floor is rough? Use diagrams, text and equations as necessary to justify your answer.



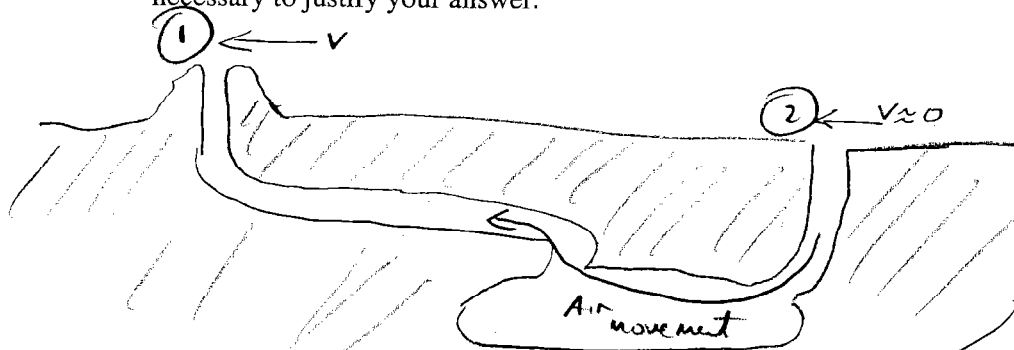
In the case with a frictionless floor there is no force to counteract the normal force of the wall on the ladder. Thus static equilibrium is impossible to attain and the ladder falls.



In the case where the wall is frictionless each force has another force that can act against it in both linear dimensions and for rotation therefore it is possible to achieve static equilibrium.

∴ I would only be comfortable climbing a ladder in the case of a rough floor and frictionless wall.

(b) Prairie dogs ventilate their burrows by building a mound over one entrance, which is open to a stream of air. A second entrance at ground level is open to almost stagnant air. How does this construction create an air flow through the burrow? Use diagrams, text and equations as necessary to justify your answer.



Use Bernoulli's eqn

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{const}$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Assume $h_1 \approx h_2$, $v_2 = 0$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2$$

The pressure at point ① (low entrance) is less than the pressure at point ② due to the difference in air movement.

This pressure difference will cause air to move from the low entrance to the high entrance, ventilating the burrow.

Problem 6 (15 pts):

During the big move out of the dorms at the end of a semester it is common to hear parents telling their kids to pick things up by "bending at the knees". The annoying refrain of "be careful or you'll hurt your back" echoes down the hall along with classics like:

"What do you mean you didn't do laundry all semester?!"

"This slice of pizza is *how* old?!"

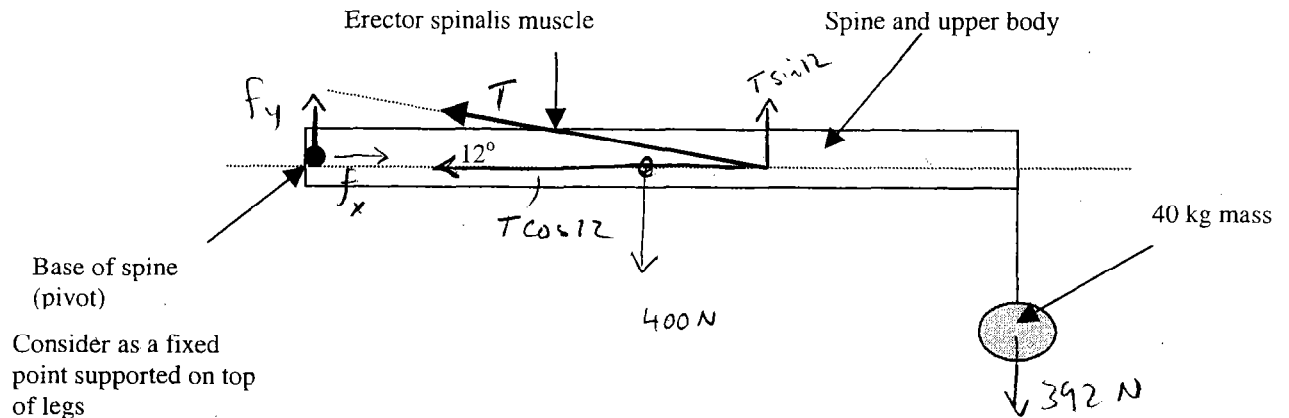
"You're telling me this contraption is a water pipe you made for your Middle Eastern culture class?"

Hmmm."

"Whose underwear is this? No, forget I asked. I don't want to know."

And of course, "Please tell me you had help emptying all these beer cans!"

Anyway, back to picking up heavy things ... The spine pivots mainly at the fifth lumbar vertebra, with the principal supporting force provided by the erector spinalis muscle in the back. To understand why back problems commonly occur during heavy lifting, let us consider the following model for a person bending forward to lift a 40 kg mass. Let the spine and upper body be represented by a uniform horizontal rod of weight 400 N, supported by a pivot at the base of the spine. The erector spinalis muscle, attached at a point two thirds of the way up the spine, maintains the position of the back. The angle between the spine and this muscle is 12 degrees in this model. Find the tension in the back muscle (erector spinalis) and the compressional force in the spine (at the base).



$$\sum F_y = 0 = 392 + 400 - T \sin 12 - F_y$$

$$\sum F_x = 0 = F_x - T \cos 12$$

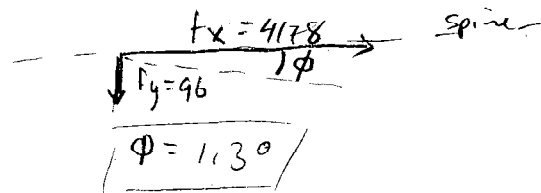
$$\sum \tau = 0 = (400)(\frac{1}{2}L) + (392)(L) - (\frac{2}{3}L)T \sin 12 \Rightarrow 592 - \frac{2}{3}T \sin 12 = 0$$

$$T = \frac{592(3)}{(2) \sin 12} = 4271 \text{ N}$$

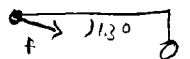
$$F_x = (4271) \cos 12 = 4178 \text{ N}$$

$$F_y = 792 - (4271) \sin 12 = -96 \text{ N}$$

$$F_{\text{TOTAL FORCE on spine}} = \sqrt{(4178)^2 + (96)^2} = 4179 \text{ N}$$

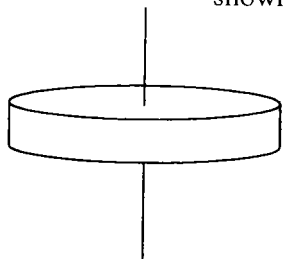


in a direction of 1.3° down from spine



Problem 7 (15 pts):

- (a) A solid, uniform cylinder has a mass of 2 kg and a radius of 10 cm. What is the moment of inertia of this cylinder about an axis passing through the center of the cylinder as shown in the drawing?



$$I = \frac{1}{2} MR^2 = \frac{1}{2} (2)(.1)^2 = 0.01 \text{ kgm}^2$$

- (b) Suppose this cylinder experiences a torque that provides a constant angular acceleration of 1 rad/s^2 that begins at $t=0$. What is the size of this torque and at what time will the cylinder reach a rotational rate of 100 rad/s ?

$$\omega = \omega_0 + \alpha t$$

$$100 = 0 + 1t \quad t = 100 \text{ s}$$

$$\tau = I\alpha \quad |\tau| = (0.01 \text{ kgm}^2) 1 \text{ rad/s}^2$$

$$|\tau| = 0.01 \text{ kg} \frac{\text{m}^2}{\text{s}^2}$$

Final = $\frac{\text{kgm}^2}{\text{s}^2}$
// units ok

- (c) What is the ^{Final} initial kinetic energy of the cylinder ~~in part (b)~~?

$$KE_{\text{rot}} = \frac{1}{2} I \omega^2 = \frac{1}{2} (0.01) (100)^2 = 50 \text{ J}$$

$\frac{\text{kgm}^2}{\text{s}^2}$

- (d) Suppose the cylinder in part (a) rotates about a vertical, frictionless axle with angular speed $100 \text{ radians/second}$. A second cylinder that has a moment of inertia of ~~200~~ 0.005 kg-m^2 and initially is not rotating drops onto the first cylinder. Because of friction between the surfaces, the two eventually reach the same angular speed ω_f . What is the final angular speed?

Use Angular Momentum conservation

$$I_1 \omega_1 = (I_1) \omega_{\text{final}} + (I_2) \omega_{\text{final}}$$

$$(0.01) (100) = \underset{0.01}{(I_1)} + \underset{0.005}{(I_2)} \omega_{\text{final}}$$

$$\omega_{\text{final}} = 67 \frac{\text{rad}}{\text{s}}$$

