

Julie's Guide to Double Problems*

What to do when Physics 113 unveils its most *interesting* weapon
- Julie Langenbrunner (Fall 2003)

Disclaimer: This is a discussion of a systematic approach to problem-solving. It will not always work (though when applicable and implemented correctly it will never give a wrong answer), and it helps to include other aspects of analysis in your Bag of Problem-Solving Techniques: intuition, figures, graphs, and discussion (except at exam time).

It might be best to look through this and look through a provided example at the same time.

Reference Problems:

Ch 2: #78 (police car and speeder)

Wks 1: #3 (Prof Manly: egged)

Ch 3: #81 (nine-iron on the moon)

Wks 2: #3 (football)

#6 (physics euphoria - the depth of the well)

Ch 4: #23 (standing jump - force analysis)

Wks 3: #6 (swinging ball on cable)

Step 1: Recognize the Double Problem as such.

Usually a Double Problem will have 2 situations, each with its own constant acceleration (and constant force). This means you can use equations to model the first one, and equations to model the second one. Often they are the same equations, only applying to each situation separately (it is helpful to use subscripts to show which situation you are referring to).

Step 2: Find the connection or connections.

There will always be at least one value that is common for both situations. It might be a known, it might be an unknown, but it will always be there. Usually it helps to envision what is going on. For example, in the case of the speeder and the policeman, it helps to envision the police car catching up. He won't catch up until he is at the location of the speeder at the same time the speeder is there. Therefore, the final position of the speeder is the same as the final position of the police car, and the time it takes the speeder to get there is the same as the time it takes the police car to get there.

Step 3: List off knowns (givens) and unknowns.

Remember that knowns that are not necessarily stated, such as $v_y = 0$ at the top of a trajectory. Also, there are plenty of unknowns we don't care about. List only those unknowns that the problem asks you to find.

Note: sometimes a known is not an actual value. You might be given a relationship between two values corresponding to the two situations. For example, in the depth-of-the-well problem, $t_1 = t_2 - 2.5$ s., and in the nine-iron-on-the-moon problem, v_{om} (initial velocity on the moon) = v_{oE} (initial velocity on the earth.) It is helpful to consider such a relationship as a known, while the actual values are unknown.

* Called "Double" to distinguish from multiple-part problems, i.e. problems involving part a, part b, part c, etc.

Step 4: Pick an equation (or equations).

Often this is the hardest part. Your criteria for this equation should be "involves some knowns, the connection, and the unknown, and nothing (or little) else." Often there is just one equation (used separately in each situation) that will work. Some harder problems may require you to combine two different equations, but usually you can use one.

(Remember to treat the x and y components separately!)

Step 5: Combine and solve.

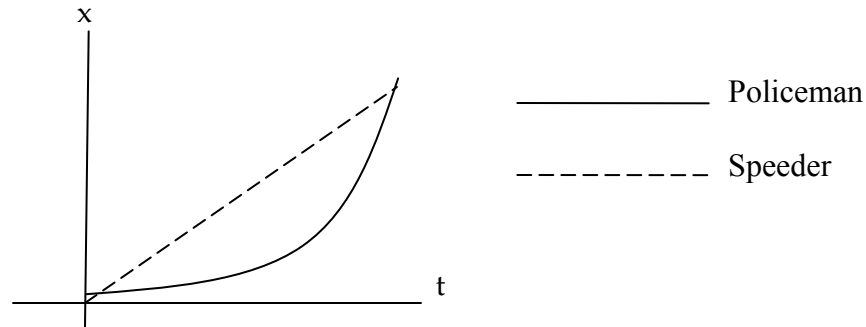
Often, you must combine the equations chosen in step 4 in such a way as to leave only your knowns and your unknown. This can usually be done by substitution. Do this if necessary, and then solve for your unknown.

I included some examples of problems that can be approached this way. Hopefully they will be helpful.

Ch. 2 #78

A police at rest, passed by a speeder traveling at a constant 110 km/h, takes off in hot pursuit. The police officer catches up to the speeder in 700 m, maintaining a constant acceleration.

(a) Qualitatively plot the position versus time graph for both cars from the police car's start to the catch-up point.



(b) Calculate how long it took the police officer to overtake the speeder.

Step 1: This is a double problem because we have two situations, each with its own constant acceleration.

$$a_{\text{speeder}} = 0$$

$a_{\text{policeman}}$ is constant, but nonzero.

Step 2: The initial and final positions of the speeder are the same as those of the policeman.

$$x_{\text{os}} = x_{\text{op}} = x_0 \quad x_s = x_p = x \quad (\text{this can also be seen from the graph.})$$

Also, the time it takes for the speeder to get from x_0 to x is the same as the time it takes for the policeman.

$$t_s = t_p = t$$

Step 3:

Knowns

$$a_s = 0$$

a_p is constant

$$x_{\text{os}} = x_{\text{op}} = x_0$$

$$x_s = x_p = x = 700 \text{ m}$$

$$v_{\text{os}} = v_s = 110 \text{ km/h}$$

$$t_s = t_p = t$$

Unknowns

$$t$$

$$a_p$$

$$v_p$$

looking ahead to (c) and (d)

Step 4: We want to find t first, and there are a few ways to do this. We know all we need to know about the speeder to find t_s , so that will give us t_p too. 2-12a will not work, (it will yield $0=0$, which we already know) and 2-12c will not work either (it does not contain t , our desired value). We are left with 2-12b: $x = x_0 + v_0 t + \frac{1}{2} a t^2$

Step 5: There is no need to combine anything here, so we can skip that part. Using equation 2-12b (with $x_{\text{os}} = 0$ and $a_s = 0$), we have

$$x_s = v_{\text{os}} t_s$$

$$t_s = 22.9 \text{ s}$$

$$t = 22.9 \text{ s}$$

(c) calculate the required police car acceleration

Step 4: (We need not repeat the other steps. Although, it should be noted that we can now add another value, $t = 22.9\text{s}$, to our list of knowns.) We want to find a_p now, so we would like an equation relating that with some of our knowns. In this case also, we know enough about the policeman to calculate his acceleration without considering the speeder problem. We can throw out equation 2-12c, and 2-12a, because they include v_p , value that we are not yet concerned with. This leaves us with 2-12b. $x = x_o + v_o t + \frac{1}{2} a t^2$

Step 5: With $x_{op} = 0$ and $v_{op} = 0$, we have

$$x_p = 0.5 * a t^2$$

$$a_p = 2.7 \text{ m/s}^2$$

(d) calculate the speed of the police car at the overtaking point.

Step 4: Now we can block out 2-12b, because it does not contain our desired value v_p . We could use either 2-12a or 2-12c, but 2-12a looks easier. $v = v_o + a t$

Step 5: With $v_{op} = 0$, we have

$$v_p = a_p t$$

$$v_p = 61.8 \text{ m/s}$$

Ch 3: #81

Apollo astronauts took a "nine iron" to the Moon and hit a golf ball about 180 m. Assuming that the swing, launch angle, and so on, were the same as on Earth where the same astronaut could hit it only 30 m, estimate the acceleration due to gravity on the surface of the Moon. (We neglect air resistance in both cases, but on the Moon there is none.)



Step 1: We should recognize this as a double problem because it has two different situations, each with constant acceleration. On the Earth, the acceleration is a constant g , and on the moon, the acceleration is a constant a_m (or g_m if you prefer).

Step 2: The connection is stated in the problem. The launch angle and the initial velocity is the same in both systems.

$$v_{0m} = v_{0E} = v_0$$

$$\theta_m = \theta_E$$

This also means that each component of the initial velocity is the same on the moon and on the Earth.

$$v_{0xm} = v_{0xE} = v_{0x}$$

$$v_{0ym} = v_{0yE} = v_{0y}$$

Step 3:

Knowns

$$a_{xm} = a_{xE} = 0$$

$$a_{yE} = g_E$$

$$\text{at top, } v_{Ey} = 0$$

$$\text{at top, } v_{my} = 0$$

$$x_m = 180 \text{ m}$$

$$x_E = 30 \text{ m}$$

$$v_{0xm} = v_{0xE} = v_{0x}$$

$$v_{0ym} = v_{0yE} = v_{0y}$$

Unknowns

g_m

Step 4: The equation worthy of this problem should definitely include our unknown, and our connecting values (v_0 and θ). It will probably also include x , because the problem gave us the x -value for each part. (This is not always the case. Sometimes problems give you information you just don't need. Don't be surprised if this happens, but do check your work and make sure you really didn't need this information.)

Looking at our equations, we should draw away from 2-12a and 2-12b, because those include time, which we were not given and do not necessarily want to have in the end. 2-12c remains: $v^2 = v_0^2 + 2a(x-x_0)$ This equation, when dealt with in the x -coordinate ($a_x = 0$), would give us $v_x = v_x$, which does not help, but if dealt with in the y -coordinate, it includes our unknown, our connecting values (in the form of v_{0y}), and the x -value that the problem gave. The only other value left in the equation is v_y , which we don't know for the end of each trajectory, but we do know that at the top of each trajectory (on the moon and on the earth), $v_y = 0$.

Step 5: The best thing to do in a problem like this, after the equation has been chosen, is to concentrate on one of the situations at a time, and then try to combine them in the end. First, let's look at our Moon situation.

Using 2-12c, we have

$$0 = v_{oy}^2 + g_m 2 (180 \text{ m})$$

Solving for g_m

$$\text{(Eq. 1)} \quad g_m = - v_{oy}^2 / (2 * 180 \text{ m})$$

Now, our Earth situation...

$$\text{(Eq. 2)} \quad 0 = v_{oy}^2 + g_E 2 (30 \text{ m}) \quad (\text{I have chosen } g_E = - 9.8 \text{ m/s}^2)$$

Now to combination. Notice that we have two equations and two unknowns (g_m and v_{oy}). Solve Eq. 2 for v_{oy} and substitute that into Eq. 1, and you have your answer!

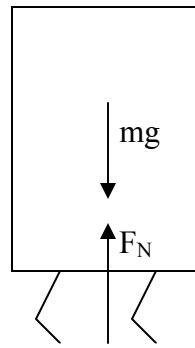
$$g_m = 1/6 * g_E = 1.6 \text{ m/s}^2$$

Ch. 4: #23

An exceptional standing jump would raise a person 0.80 m off the ground. To do this, what force must a 61-kg person exert against the ground? Assume the person crouches a distance of 0.20 m prior to jumping, and thus the upward force has this distance to act over before he leaves the ground.

Step 1: We can recognize this as a Double Problem because this “jumping” actually is a two-phase action. The first phase is where the person is “pushing himself off” the ground, which is actually him pushing on the ground and the ground pushing him back. The second part is when he is in the air. He is no longer applying a force to the ground. The earth is applying a force to him (gravity), but that is the only force left acting. In each phase, there is a constant force acting (therefore constant acceleration). In the first phase, the constant force on the person is a combination of the force applied by the ground (normal), and the force applied by the earth (gravity). In the second phase, the only force left is the gravitational force.

(Extra discussion on jumping: Keep in mind, that any time you are standing on the ground, you are pushing on the ground and the ground is pushing back on you. To actually jump, you must get the ground to push you more than it usually does, so the way to do this is to push it more than you usually do. Newton’s 2nd law forces the ground to push on you more, when you push on it more. That is why people crouch down before they jump; otherwise, it is quite hard to apply more of a force to the ground than you normally do with just your weight.)



(Our jumper is a box because it’s easier to put arrows on a box than on a stick figure)

Step 2: Finding the connection here is a matter of imagining the situation. It’s not the acceleration, it’s not the time, it’s not the distance, it’s not the starting velocity, it’s not the ending velocity, what is it? In this case, you know that the second phase begins as soon as the first phase ends, so the velocity must be the same at the end of #1 and the beginning of #2. Using subscripts 1 for the first phase and 2 for the second phase:

$$v_1 = v_{o2}$$

Step 2.5: It is a bit confusing where to set the $y = 0$ point. If we set it at the ground, then the second phase should begin at $y_{2o} = 0$ because the second phase begins when the jumper’s feet are just leaving the ground, but then the first phase occurs below $y = 0$, or below the ground? In this case, it is much more intuitive to speak of simply $y_1 - y_{o1} = 0.2$ m, instead of speaking of y_o and y_{o1} separately. As long as we realize that $y_1 - y_{o1} = 0.2$ m is the distance over which the upward acceleration happens, that’s enough to justify using that value.

Step 3: (no x or y subscripts because this is only in the y direction)

| <u>Knowns</u> | <u>Unknowns</u> |
|--------------------------------|-----------------|
| $y_1 - y_{01} = 0.2 \text{ m}$ | F_1 |
| $v_{o1} = 0$ | |
| $m = 61 \text{ kg}$ | |
| $y_{o2} = 0$ | |
| $y_2 = 0.8 \text{ m}$ | |
| $a_2 = -9.8 \text{ m/s}^2$ | |
| at top, $v_2 = 0$ | |

Step 4: When deciding how to go about this problem, perhaps your mind should do something like this: "Looking at just phase 1, we need the force that this guy needs to apply to the ground. We know $F=ma$, but we don't know acceleration. What's acceleration? (glancing at equation 2-12a) We can find acceleration if we know the time, how can we find the time? All of my equations for time involve acceleration, which we don't know.... Ok, so we can't find time. How else can we find acceleration? We can find acceleration if we know.... (glancing at equation 2-12c) the initial velocity, the final velocity, and the distance. But we don't know the final velocity. How can we find the final velocity? Wait... the final velocity of phase 1 is my connecting value to phase 2... so using phase 2, can we find the final velocity of phase 1 which is the initial velocity of phase 2? Sure! We can use eq. 2-12c! ($v^2 = v_o^2 + 2a(x - x_o)$) We know the velocity at the top, we know the acceleration, and we know the distance!"

So our added unknowns that we need to go through before we can find the force that the jumper needs to apply to the ground are:

- the initial velocity of phase 2 which is the final velocity of phase 1
- the acceleration needed in phase 1 to get to the velocity at the end of phase 1

From here we can find the force necessary to jump.

Step 5: Using 2-12c for the second phase, we have

$$0 = (v_{o2})^2 + 2g(0.8 \text{ m}) \quad \text{I have set } g = -9.8 \text{ m/s}^2$$
$$v_o = 4.0 \text{ m/s}$$

Using 2-12c for the first phase, we have

$$(4.0 \text{ m/s})^2 = 2a_1(0.2 \text{ m})$$
$$a_1 = 39.2 \text{ m/s}^2$$

Using $\Sigma F = ma$

$$F_N + mg = ma \quad (\text{note here that since I have already set } g = -9.8 \text{ m/s}^2, \\ \text{mg will come out negative, which is good!})$$

The normal force is going to be equal but opposite to the force that the jumper applies to the ground, so if we solve for that, we know our answer!

$$F_N = ma - mg = m(a - g)$$
$$= 61 \text{ kg} (39.2 \text{ m/s}^2 - (-9.8 \text{ m/s}^2))$$
$$= 2989 \text{ N}$$

This is the force that the ground must apply to the jumper, so this is the force that the jumper must apply to the ground, only in the opposite direction.