Name_____ Date: _____ Course number: _____
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Laboratory Section: _____   Partners' Names: _____

*Last Revised on August 21, 2019*   Grade: _____

EXPERIMENT 4
Moment of Inertia & Oscillations
0. Pre-Laboratory Work [2 pts]

1.

   a) In *Section 3.1*, describe briefly the steps you take to make sure your apparatus setup is done correctly and the timer is working properly. [0.5 pts]

   b) Below is a diagram of how the rotary table in Section 2.1 oscillates. Which number of the picture corresponds to when the photogate timer will measure one period of oscillation. Why? [0.5 pts]
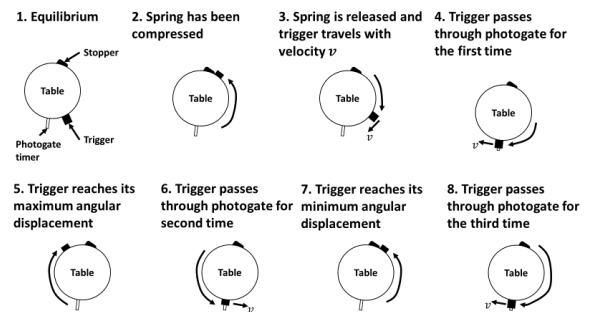
2. In the second section of the lab you will observe the oscillations of a spring loaded with a specific mass. However, you may notice that the spring will oscillate even when there is no mass attached ($M = 0$) in section 2.2.

   a) What is one reason that this can happen? [0.5 pts]

   b) What is one effect that this could have on the experiment? [0.5 pts]

EXPERIMENT 4

Moment of Inertia & Oscillations

1. Purpose

In the first part of this laboratory exercise you will measure the moment of inertia of three different objects about a specified rotational axis and verify the parallel axis theorem. In the second part you will measure the oscillations of a mass on a spring to investigate Hooke's Law.

2. Introduction

2.1 Moment of Inertia and Parallel-Axis Theorem

Consider a rigid body rotating about an axis, as in *Figure 4.1*. If the angular velocity is ω, each point in the body will move with linear speed $r\omega$ where r is the perpendicular distance of the point from the rotational axis. The total angular momentum L of the rotating body points along the axis and is equal in magnitude to,

$$L = \int_{Body} rvdm = \int_{Body} r^2 \omega dm = \omega \int_{Body} r^2 dm = I\omega, \tag{4.1}$$

where

$$I = \int r^2 dm, \tag{4.2}$$

is called the *moment of inertia* of the body about the axis of rotation. In the MKS system of units, the unit of I is kgm^2. If the axis of rotation is chosen to be through the center of mass of the object, then the moment of inertia about the center of mass axis is called I_{CM}. For example, $I_{CM} = \frac{1}{2}MR^2$ for a solid disk of mass M and radius R (see *Figure 4.1*). *Table 4. 1* gives examples of I_{CM} for some objects with different mass distributions.
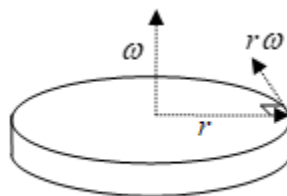

Figure 4.1

The *parallel-axis theorem* relates the moment of inertia about an axis through the center of mass I_{CM} to the moment of inertia I about a parallel axis through some other point. The theorem states that,

$$I = I_{CM} + Md^2, \tag{4.3}$$

where M is the total mass of the body and d is the perpendicular distance between the two axes. This implies I_{CM} is always less than I about any other parallel axis.

Object	Rotational Axis	I_{CM}
Thin Ring	Symmetry Axis	MR^2
Thick Ring	Symmetry Axis	$\frac{1}{2}M(R_1^2 + R_2^2)$
Solid Disk	Symmetry Axis	$\frac{1}{2}MR^2$
Thin Spherical Shell	About a Diameter	$\frac{2}{3}MR^2$
Solid Sphere	About a Diameter	$\frac{2}{5}MR^2$

Table 4.1

When working with rotational motion for rigid bodies, many of the equations are similar to the equations of motion for linear motion. The angular velocity is used instead of linear velocity, and the moment of inertia is used instead of the mass. *Table 4.2* summarizes the correspondence between linear and rotational kinematics for rigid bodies.

Spring   Table

Figure 4.1

Linear Kinematics	Rotational Kinematics
Velocity = v	Angular Velocity = ω
Mass = M	Moment of Inertia = I
Momentum = $p = Mv$	Angular Momentum = $L = I\omega$
Kinetic Energy = $K = \frac{1}{2}Mv^2$	Kinetic Energy = $K = \frac{1}{2}M\omega^2$

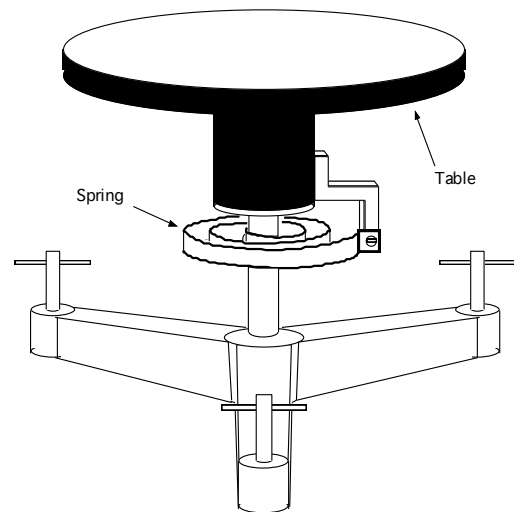**Table 4.2**

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The apparatus in this experiment in *Figure 4.2* consists of a rotary table on which you can mount an object in order to measure its moment of inertia. A torsion spring restricts the motion of the table and provides a restoring torque. If the table is rotated by an angle θ then the torque acting on it will be equal to

$$\tau = -K\theta, \tag{4.4}$$

where K is the "force constant" for a torsional spring, which must be measured. If the sum of the moment of inertia of the table I_{Table} and that of the mounted object I is equal to $I + I_{Table}$, the table will perform a rotary oscillation with the frequency,

$$\omega = \sqrt{\frac{K}{I + I_{Table}}}, \tag{4.5}$$

which corresponds to a period of oscillation,

$$T = 2\pi \sqrt{\frac{I + I_{Table}}{K}}, \tag{4.6}$$

Note that there are two unknown parameters of the apparatus: I_{Table} and K. To determine these parameters, you need to measure the period T_{Table} of the table alone and the period $T_{Table+Object}$ of the table with an object of a known moment of inertia I_0 placed on the table. From *Equation 4.6*, one finds that

$$T_{Table} = 2\pi \sqrt{\frac{I_{Table}}{K}}, \tag{4.7}$$

$$T_{Table+Object} = 2\pi \sqrt{\frac{I_0 + I_{Table}}{K}}, \tag{4.8}$$

Using the technique of solving sets of equations, you square both equations and add them to obtain,

$$\left(T_{Table+Object}\right)^2 + (T_{Table})^2 = 4\pi^2 \frac{2I_{Table} + I_0}{K}, \tag{4.9}$$

and by squaring them and subtracting one from the other you get,

$$\left(T_{Table+Object}\right)^2 - (T_{Table})^2 = 4\pi^2 \frac{I_0}{K}, \tag{4.10}$$

This allows you to solve for one of the unknown constants,

$$K = 4\pi^2 \frac{I_0}{\left(T_{Table+Object}\right)^2 - (T_{Table})^2}, \tag{4.11}$$

The other unknown, I_{Table}, can be obtained first by dividing *Equation 4.9* by *Equation 4.10* to get the following,

$$\frac{\left(T_{Table+Object}\right)^2 + (T_{Table})^2}{\left(T_{Table+Object}\right)^2 - (T_{Table})^2} = \frac{2I_{Table}}{I_0} + 1, \tag{4.12}$$

and then simplifying and solving for I_{Table},

$$I_{Table} = I_0 \frac{(T_{Table})^2}{\left(T_{Table+Object}\right)^2 - (T_{Table})^2}, \tag{4.13}$$

By these means the unknown constants of the table, I_{Table} and K, can be determined if a body of known moment of inertia is available. Now knowing the values of I_{Table} and K, you can find the

moment of inertia I_x of an unknown object on the table by using the equation below,

$$I_x = I_{Table}\left(\left(\frac{T_{Table+x}}{T_{Table}}\right)^2 - 1\right),$$

(4.14)

where $T_{Table+x}$ is the measured period of the table with the unknown object.

2.2 Hooke's Law and Spiral Spring Oscillations

It is often assumed that a long spiral spring obeys Hooke's law if it is not stretched too far. If the spring is hung vertically from a fixed support and a mass is attached to its free end, the mass can then oscillate vertically in a simple harmonic motion pattern by stretching and releasing it. The period of an oscillation is the time it takes the attached mass to return to its initial starting position. The period T depends upon the attached mass M, the spring force constant k, and the spring mass m. The period is given by,

$$T = 2\pi\sqrt{\frac{M + bm}{k}},$$

(4.15)

where b is the dimensionless and is called the spring mass coefficient. You will calculate it in the lab and compare your measurement to the theoretical value of $b = 1/3$. During the lab, you will measure T for different values of M. To make it easy to estimate k knowing only T and M, we can write Equation 4.15 in the following form:

$$T^2 = \frac{4\pi^2}{k}M + \frac{4\pi^2}{k}bm,$$

(4.16)

We see then that if we were to make a plot with T^2 along the y-axis and M along the x-axis Equation 4.16 is in slope-intercept form: $y = (slope)x + (y - intercept)$. In question 10 you will make just such a plot, and you'll see if your data has the expected linear form.

3. Laboratory Work

3.1 Moment of Inertia and Parallel-Axis Theorem

Procedure

1. After the table reaches its equilibrium position, make sure that the table top is level. Place a level on it and adjust the table apparatus' feet if necessary.
2. Set the photogate timer to pendulum mode and arrange the photogate so that the table's trigger (see Fig. 4.3) will pass through the photogate when the table rotates.
3. Wind the table either clockwise or counter-clockwise such that *the trigger does not pass the timer while winding*. Stop when the table hits the block and can't be turned any further.
4. Release the table. Count the number of times that the trigger passes through the photogate before the timer stops counting. It should be three passes before the timer stops. The displayed time is the period of one oscillation.
5. Measure the mass, and inner and outer radii of the *brass ring*. Record these values in the
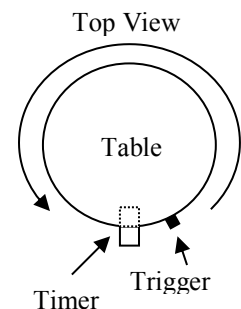

Top View

Table

Timer

Trigger

Figure 4.2

Section 4.1.

6. Measure the period of one oscillation T_{Table} of the *table alone* five (5) times. Record the data in *Table 4.3*. Don't forget to reset the timer after each trial.

7. After making sure the brass ring is mounted down with screws and centered on the table's axis of rotation, measure the period of one oscillation $T_{Table+Ring}$ of *the table/ring combination* five (5) times. Record the data in *Table 4.3*.

8. Measure the mass and radius of the solid disk. Record the data in *Section 4.1*.

9. To measure the period of one oscillation for the table/disk combination, place and screw down the solid disk at one of five different equally spaced positions along the table radius at a distance d from the table's axis of rotation (see *Figure 4.4*). One of the positions must be the center ($d = 0.000$m). Recommended positions are 0.000m, 0.015m, 0.030m, 0.045m and 0.060m (i.e. the spacing of the holes in the table).

10. Measure the periods oscillation at those five different positions. Measure each period five times. Record your data in *Table 4.4*.

Top View
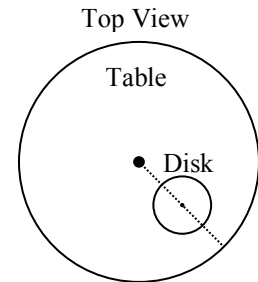
Figure 4.3

3.2 Hooke's Law and Spiral Spring Oscillations

Introduction
According to Hooke's law, the extension of a spring should be proportional to the force exerted by the spring on an attached mass- $F = -kx$, where F is the force exerted by the spring, x is the extension of the spring, and k is the *force constant* of the spring. You will test the validity of Hooke's law by making measurements of spring extension as increasingly greater amounts of mass are attached. Then, you will measure the oscillation period of several different hanging masses to measure the spring mass coefficient needed to take into account the spring's own mass m.

Procedure
1. Weigh the Slinky Jr.™ and mass holder to determine the mass of the "spring" system, m. Record it in Section 4.2.

2. Mount the spring by clipping one end of it to the flat metal piece mounted to the stand.

3. For each of the attached masses, M, in Table 4.6, calculate the force of gravity acting on those masses, $F = Mg$ (where $g = 9.8$ m/s²).

4. Let the spring come to equilibrium. If possible, move the ruler/spring vertically so that the ruler's zero is at the bottom of the spring (or the bottom of the mass holder, if you prefer). Record the position of the spring's bottom end (i.e. its equilibrium length) on the ruler.

5. Measure the change in the extension of the spring x as you attach different amounts of mass to the end of the spring. Keep in mind that the "extension of the spring" is the length of spring extended *beyond the spring's equilibrium length*. Use masses of 5g, 10g, 15g, 20g and 25g. Attach the masses gently. Record the data on *Table 4.6*.

6. Measure the period of *ten* oscillations for the same five different attached masses. Set the spring and mass into motion by stretching the mass **gently** by about 15cm to 20cm and then releasing it. While in motion, the mass should not touch the floor or anything else. If you pulled the mass down, then one oscillation occurs every time the mass returns to its initial position (i.e. the lowest part of its oscillation). **Divide the total time by 10 to get the average period of one oscillation.** Record the one-oscillation periods on *Table 4.6*.

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EXPERIMENT 4

Moment of Inertia & Oscillations

4. Post-Laboratory Work [20pts]

4.1 Moment of Inertia and Parallel-Axis Theorem [10 pts]

Mass of Ring (kg) = _____
Ring's Inner Radius (m) = _____
Ring's Outer Radius (m) = _____

Trial	T_{Table}	$T_{Table+Ring}$
1		
2		
3		
4		
5		
Average		

Table 4.3

Mass of Solid Disk (kg) = _____
Radius of Solid Disk (m) = _____

Distance (m)	$d_{x_1} = $ _0.000m_	$d_{x_2} = $_____	$d_{x_3} = $_____	$d_{x_4} = $_____	$d_{x_5} = $_____
Trial	T_{x_1}	T_{x_2}	T_{x_3}	T_{x_4}	T_{x_5}
1					
2					
3					
4					
5					
Average (sec)					

Table 4.4

TA or TI Signature _____ 7 of 13

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1. Calculate all the quantities in *Table 4.5*. As for the values of the measured periods, use the average values from *Tables 4.3* and *4.4*. Write the formula for each quantity except from I_{x_2} to I_{x_5} and the Error. Remember to include units! You do not have to show your calculations. [3pts]

Equation: $I_{Ring} =$	Theoretical Value of the Moment of Inertia of the Brass Ring (*Table 4.1*) Note that we would consider it a thick ring. Also a heavy ring, incidentally.
Equation: $K =$	Spring Constant of the Table (*Equation 4.11*, where $I_0 = I_{Ring}$ and $T_{Table+Object} = T_{Table+Ring}$)
Equation: $I_{Table} =$	Calculated Moment of Inertia of the Table (*Equation 4.13*, where $I_0 = I_{Ring}$)
Equation: $I_{Disk} =$	Theoretical Value of the Moment of Inertia of the Brass Disk (*Table 4.1*)
Equation: $I_{x_1} =$	Calculated Moment of Inertia when the Brass Disk is placed at d_1 (*Equation 4.14*, where $T_{Table+x} = T_{x_1}$)
$I_{x_2} =$	Calculated Moment of Inertia when the Brass Disk is placed at d_2
$I_{x_3} =$	Calculated Moment of Inertia when the Brass Disk is placed at d_3
$I_{x_4} =$	Calculated Moment of Inertia when the Brass Disk is placed at d_4
$I_{x_5} =$	Calculated Moment of Inertia when the Brass Disk is placed at d_5
$Error = |I_{Disk} - I_{x1}| =$	Calculated Error in Moment of Inertia of the Solid Disk

Table 4.5

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2. On the graph below,
   a) Plot the moment of inertia of the *solid disk* (I_{x_i}) on the y-axis vs. the *square* of the distance between the center of the table and the center of the solid disk ($d_{x_i}^2$) on the x-axis. [1.25pts]
   b) Include error bars on each data point. Each error bar should have the same length: $|I_{Disk} - I_{x1}|$ from Table 4.5. [0.25 pts]
   c) Add a title to the graph, label the axes, and make sure you include units! [0.5pts]
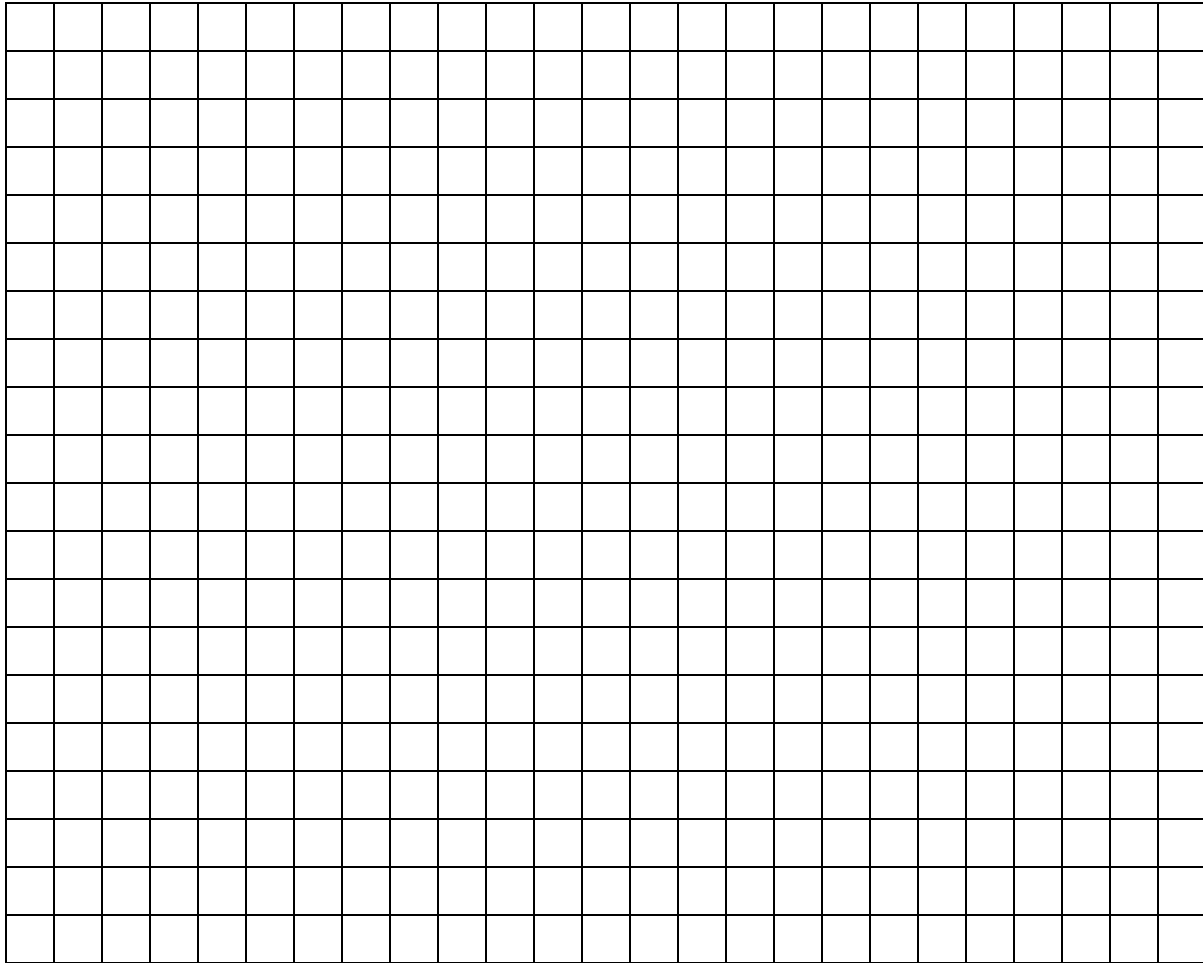
3.
   a) Draw a best-fit straight line for your data (it need not go through the origin). [0.5pts]
   b) Find the y-intercept value of your best-fit line and circle it on the graph: [0.25pts]
   $b_{exp} = y_{intercept} =$

   c) Circle two points on your best fit-line: (x_1, y_1) and (x_2, y_2). Then, calculate the slope (don't forget your units) [0.25pts]:
   $$m_{exp} = slope = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} =$$

4. If you take the parallel-axis theorem, $I = Md^2 + I_{CM}$, and you plot I vs. d^2 (y vs. x), the data should be linear. Given that the equation for a line is $y = mx + b$ where m is the slope and b is the y-intercept, what variables in parallel-axis theorem correspond to m and b? [1 pt]

$m_{theory} = slope =$

$b_{theory} = y_{intercept} =$

5. Using your answers to questions 3 and 4, how much do your measured values for the slope and y-intercept differ from the theoretical values? Remember to include units! [2 pts]

$\left| m_{theory} - m_{exp} \right| =$

$\left| b_{theory} - b_{exp} \right| =$

6. Name two sources of experimental error. Give a reason as to how each source of error would affect your values for the moment of inertia. [1 pt]
First error:

Second error:

4.2 Hooke's Law and Spiral Spring Oscillations [10 pts]
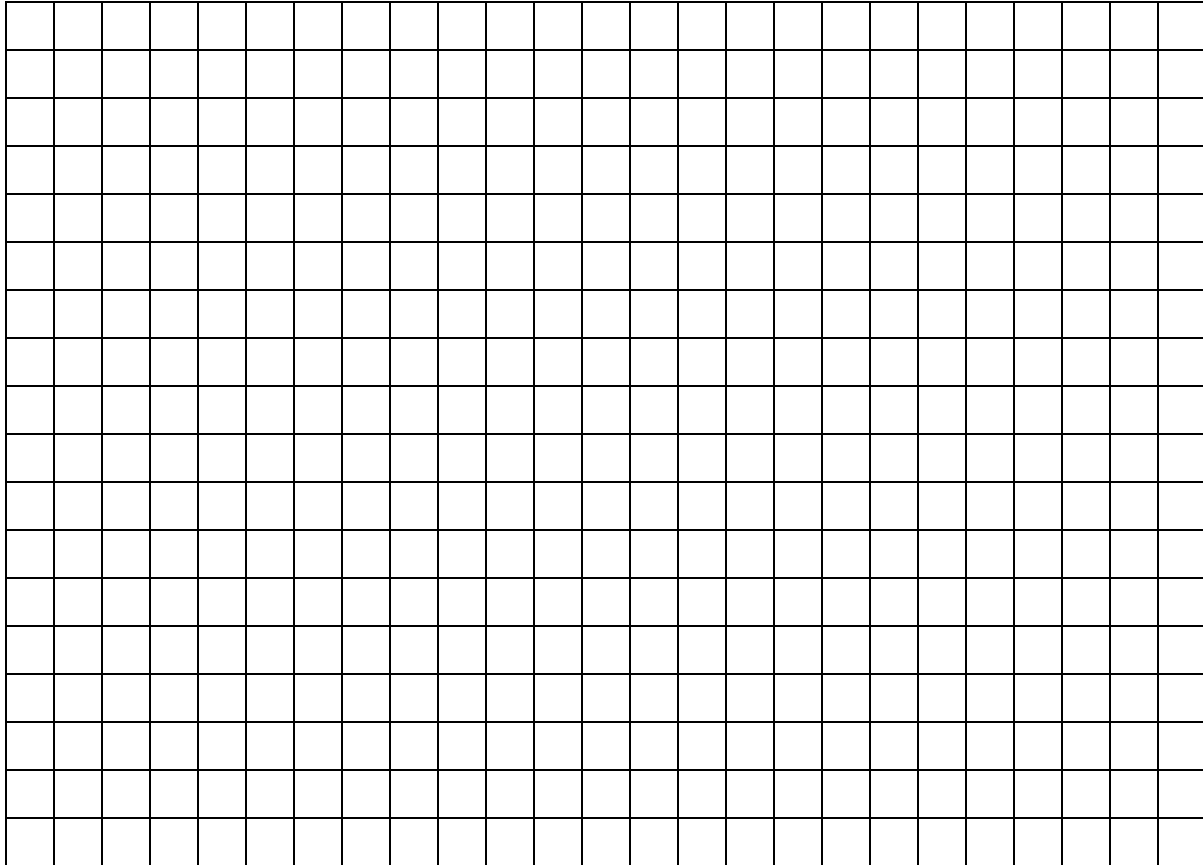
$m =$ mass of spring (kg) = _____

M = attached mass (kg)	$Force = Mg$ (N)	Extension (m)	Period (sec)	Period Squared (sec²)
0.005				
0.010				
0.015				
0.020				
0.025				

Table 4.6

7.

    a) Plot **Force** of gravity versus **Extension** (y vs. x) using the data from *Table 4.6*. [1 pt]

    b) Include title, axis labels and units. [0.5 pts]

    c) Draw a best-fit straight line through the data points. [0.5 pts]

8. Based on the best-fit straight line, does your data agree with Hooke's Law? Explain why. [1pt]

9. As in **Question 3**, calculate the slope of the best-fit straight line. Circle the two points on the line that you will use to get the slope. Find the spring constant, k, and remember units! [1pt]

$$k_{Hooke} = slope = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} =$$

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10. Looking at Equation 4.16, if we were to plot T^2 along the y-axis and M along the x-axis:
   a) What would be an equation for the theoretical value of the slope? Remember the equation for a line: $y = (slope)x + (y_{intercept})$ [0.5pts]
   $$slope_{theory} =$$

   b) What would be an equation for the theoretical value of the y-intercept? [0.5pts]
   $$y_{intercept_{theory}} =$$

11. Using your data for the period squared and the attached mass from Table 4.6:
   a) Plot the period squared versus the attached mass, i.e. T^2 vs. M (y vs. x). [0.5pts]
   b) Draw a best-fit straight line through the data. [0.25pts]
   c) Include a title, axes labels and units. [0.25pts]
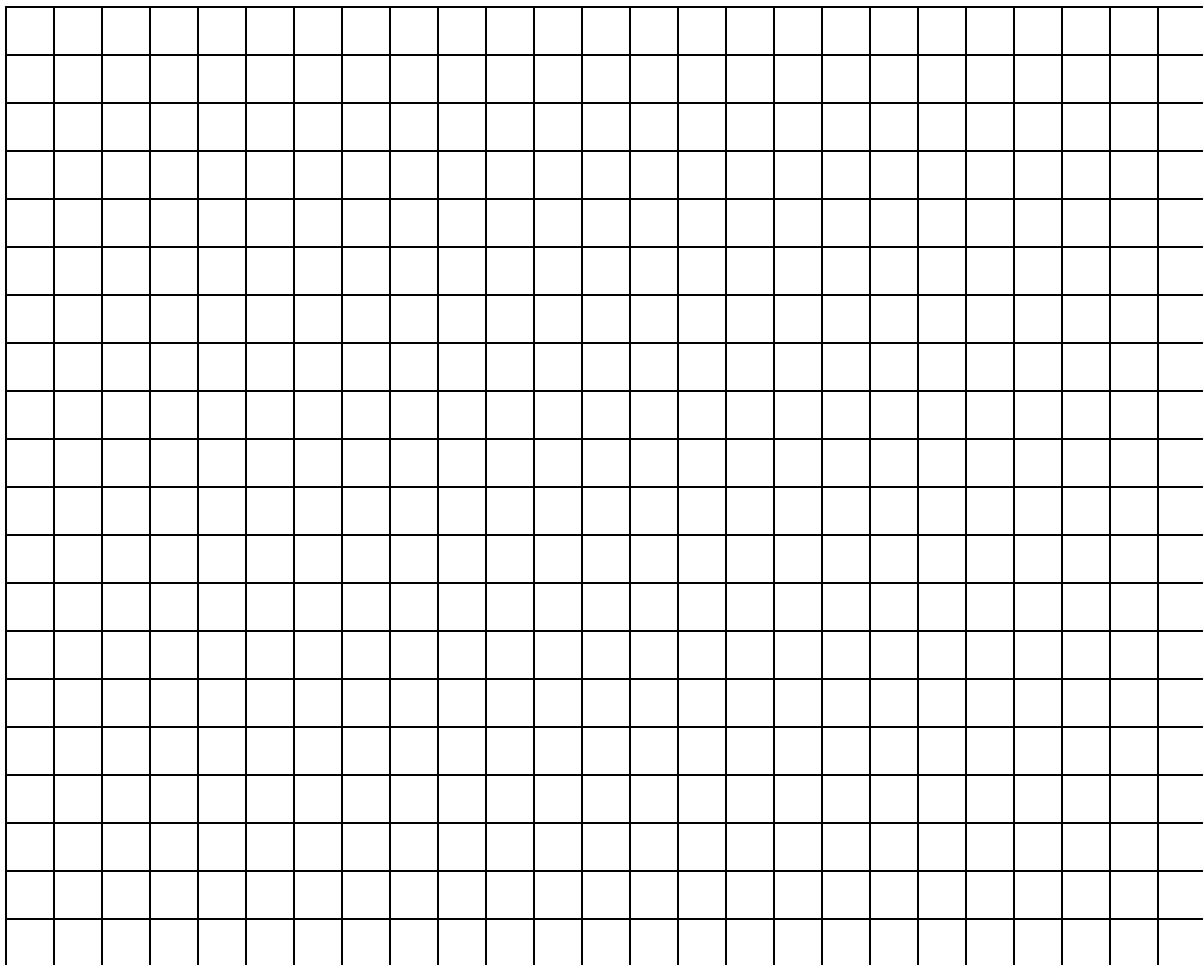
12. Calculate the slope of the best-fit straight line. Circle the two points on the line used in the calculation. Setting the experimental value of the slope equal to the slope equation found in **Question 10(a)**, find the spring constant k. [1pt]

$$slope_{exp} = \frac{y_2 - y_1}{x_2 - x_1} = slope_{theory}$$

$$k_{Harm} =$$

13. Find and circle the y-intercept value of the best-fit straight line. Like **Question 12**, find the spring mass coefficient b by plugging the numeric value of the experimental y-intercept into the equation found in **Question 10(b)**. Use the k_{Harm} value calculated in **Question 12**. [1pt]

$$y_{intercept_{exp}} = y_{intercept_{theory}}$$

14. Let's say that if the effective spring mass, bm, is less than 10% of the smallest attached mass, $M = 0.005$ kg (a seemingly random percentage, I admit), then the spring mass is negligible. Based on your value for b found in **Question 13**, is $\frac{bm}{M} \leq 0.1$ (i.e. is the spring's mass negligible)? [1pt]

15. For the last question, we'll check whether the spring constants, k, calculated in **Questions 9 and 12** are comparable.

a) First, find the difference between the two k values, remember to include units! [0.5pts]
$$|k_{Hooke} - k_{Harm}| =$$

b) Like in question 14, let's arbitrarily say that these k measurements are consistent if they differ by less than 10% of the smaller value. Is $\frac{|k_{Hooke} - k_{Harm}|}{k_{Smaller}} \leq 0.1$? [0.5pts]