

Laboratory Section: _____ Partners' Names: _____

Last Revised on January 8, 2015

Grade: _____

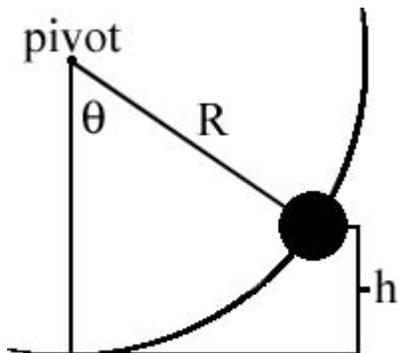
EXPERIMENT 3

Conservation of Momentum and Energy

0. Pre-Laboratory Work [2pts]

1. In *Section 3.1 & 4.1, The Linear Track*, what is the purpose of measuring the **Average Fraction Lost** for the **Momentum** and **Energy**? (1pt)

2. For *Section 3.2, Velocity of a Projectile*, derive the formula for the height, h , in terms of only R and θ , for the Ballistic Pendulum experiment. Identify any quantities you use. (1pt)



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EXPERIMENT 3

Conservation of Momentum and Energy

1. Purpose

The purpose is to experimentally verify the laws of conservation of momentum and energy. In particular you will do the following:

I. The Linear Track

1. Newton's First Law
2. Elastic Collisions
3. Inelastic Collisions

II. Velocity of a Projectile

1. Ballistic Pendulum
2. Projectile Motion

2. Introduction

The laws of conservation of energy and momentum are among the most fundamental and useful laws of physics. They aid in the solution of many mechanics problems and come up frequently in many fields of science. These laws say that if there are no net forces on a system, then that system will have the same momentum,

$$\vec{p} = m\vec{v}, \quad \text{Equation 3.1}$$

at all times. In addition, if there are no external or internal forces acting in or on a system, then the energy of that system will remain constant. Newton's First Law is hidden in these conservation laws. Newton's First Law states that bodies at rest will remain at rest as long as no forces act upon them, and bodies in constant motion will remain in constant motion as long as no forces act upon them. As one can see, Newton's First Law is a statement about conservation of momentum and energy. Objects stay the same, as long as they are left alone.

Despite their fundamental nature, the conservation laws are often difficult to observe in ordinary experiences, primarily because of the presence of friction. Friction between moving bodies and their surroundings means there are external forces acting on the system, therefore, the

conservation laws do not apply. In order to observe the conservation laws, friction must be eliminated as much as possible.

This lab will deal primarily with the conservation laws as they apply to collisions between objects. Collisions can be divided into two different classes: elastic collisions and inelastic collisions. If the kinetic energy,

$$K = \frac{1}{2} m v^2, \quad \text{Equation 3.2}$$

of particles is conserved before and after the collision, then the collision is said to be *elastic*. On the other hand, if the kinetic energy is not conserved, then the collision is said to be *inelastic*. Notice the reference to the phrase kinetic energy. In determining whether the collision is elastic or inelastic, one takes into account *only* the kinetic energy of the system, not the total energy. The *kinetic* energy of a system *may not* be conserved depending on the collision type—elastic or inelastic. However, the *total* energy—kinetic energy plus potential energy plus heat energy plus etc.—of a system is *always* conserved regardless of the collision type and regardless of anything else. *The total energy is always conserved in all cases.* In this experiment, we take the liberty of presuming that only kinetic and heat energies are present. Also, notice the reference to the word “particles”. Solid bodies are not particles, but have structure. However, if the collision leaves their structure unchanged, they can be treated as particles. Note, in the absence of friction the momentum will be conserved in both types of collisions.

In one dimension, the conditions before and after an elastic collision between two bodies of masses m_1 and m_2 , initial velocities v_{1i} and v_{2i} , and final velocities v_{1f} and v_{2f} , are given by

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \text{Equation 3.3}$$

for kinetic energy and

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{Equation 3.4}$$

for momentum. For an inelastic collision, only the momentum equation is valid.

3. Laboratory Work

3.1 The Linear Track

Introduction

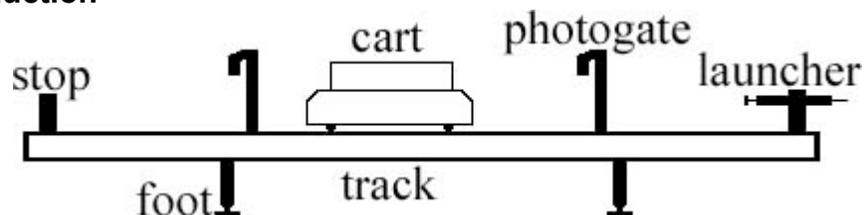


Figure 3.1

For these experiments you will be using a track with carts that have very low friction wheels. The track setup with a cart used in the experiments is shown in *Figure 3.1*. You will be able to measure the time intervals of a cart using the photogate, from which you can calculate the velocity of the cart. The photogate timer is an electronic timer

controlled by the interruption of an invisible infrared light beam when the cart passes the gate. If an object of length, L , interrupts the beam for a time interval, Δt , while passing through the gate, then the average velocity of the object during that time is given by

$$\bar{v} = \frac{L}{\Delta t}. \quad \text{Equation 3.5}$$

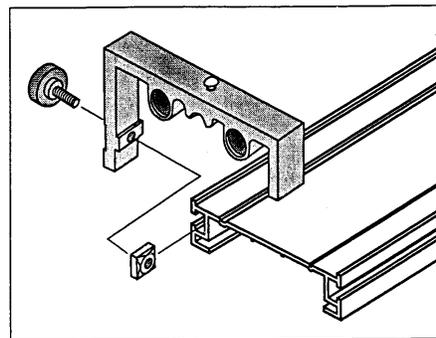
This is the basic measurement you will make in this experiment. By measuring the velocity of the carts before and after various collisions, as well as weighing the carts, you will be able to calculate the energy and momentum and test the conservation laws.

Instructions on Equipment Operation

Timers

Please note that, although you are usually using the two timers on the track at the same time, the timers work *independently*. In order to calculate the speed at a point on the track, use only the measurement from the timer at that position.

Track



Attaching Adjustable End Stop to Track

Figure 3.2

There are several things that must be done in preparation for the experiments. Most importantly is to check if the track is level. Levels are provided for this purpose. Take one, and place it over the track's feet at one end of the track. First, place it along the length of the track and observe the level's "bubble." You want this bubble to be exactly in the middle. If it is not, adjust the feet of the track by turning the screws at the base of the feet. When the bubble is in the middle of the level, repeat this procedure at the opposite end of the track with the level again length-wise. Once both ends have been adjusted, repeat this again at both ends with the level along the width of the track. Make sure that there is an end stop at one of the end of the track and the launcher on the other end. To move the end stop and the launcher, loosen the screws on the side and slide them into position (see *Figure 3.2*). Lastly, align the two photogates so that they are about two and a half cart-lengths apart.

Carts

One end of the cart has a magnet inside. The other end has a plunger and Velcro strips labeled in *Figure 3.3* as "Hook-and-pile Pads". You will use the plunger for elastic collisions and the Velcro for inelastic collisions. The Velcro may not stick together well. To test this, push the plunger in all the way on both carts. Put the two

carts together by their Velcro ends. If they are not sticking well, you can roughen the Velcro by repeatedly putting the carts together and pulling them apart.

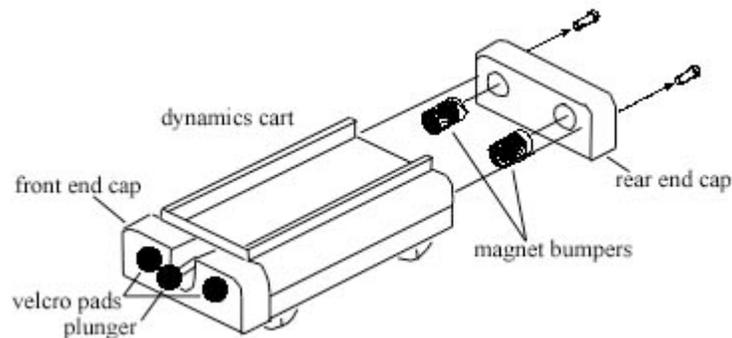


Figure 3.3

Check that the carts have no irregularities by testing them on the track. Place each cart on the track, making sure that the wheels are in the grooves. Give each cart a small push and see how well it moves on the track. There should be minimal loss in velocity along the track. If there is a noticeable loss in velocity, check the level of the track again. If the level is fine, check the wheels of the cart and ask the TA for assistance.

The track will be used to create various collisions between the carts. The collisions will be arranged so that an initially moving cart will pass through a photogate before hitting another object, so as to measure the initial velocity. After the collision, the cart will again pass through a photogate, so as to measure the final velocity. With these measured velocities, as well as the measured masses, the momentum and kinetic energy of the carts before and after the collisions can be calculated and compared.

Steps will be taken to deal with systematic errors in this experiment. The approximate effects of friction will be measured. Launchers are used so the initial conditions of consecutive trials can be reproduced.

3.1.1 Newton's First Law

In this part of the experiment, one cart at a time will be rolled down the track, past two timers. The time for the cart to roll past each timer will be recorded.

Newton's First Law states that an object will remain in its state of motion so long as no forces act upon it. In this case, a cart moving with no friction would not change its velocity as it moves along the track.

We are *not* testing the conservation laws here. The data from this section of the experiment will be used to estimate the *systemic error due to friction*.

The goal in all of the trials is to compare the initial momentum and energy of the carts with the final momentum and energy of the carts so as to make a judgment about the Conservation Laws. This will be done by dividing the final value of the quantity by the initial value in each trial, yielding the fraction of the quantity that is conserved,

$$\frac{p_f}{p_i} = \text{Fraction of Momentum Conserved.} \quad \text{Equation 3.6}$$

$$\frac{K_f}{K_i} = \text{Fraction of Kinetic Energy Conserved.} \quad \text{Equation 3.7}$$

If the conservation laws hold, the **Fractions of Momentum and Kinetic Energy Conserved** should be very close to one. Results much less than one indicate that momentum or kinetic energy were lost (see the **Key Idea** below).

The purpose of *Section 3.1.1 Newton's First Law* is not to test the conservation laws, but rather to get an estimate of the energy and momentum lost due to *external* forces - mainly friction. The presence of any external forces creates a *systemic* error in all measurements. We can estimate that error by calculating the average fraction lost of both kinetic energy and momentum:

$$\text{Average Fraction of Momentum Lost} = \frac{\overline{\Delta p}}{p} = 1 - \overline{\left(\frac{p_f}{p_i}\right)} \quad \text{Equation 3.8}$$

$$\text{Average Fraction of Energy Lost} = \frac{\overline{\Delta K}}{K} = 1 - \overline{\left(\frac{K_f}{K_i}\right)} \quad \text{Equation 3.9}$$

KEY IDEA: The **Average Fraction Lost** gives a rough estimate of the amount of momentum and energy lost due to friction. You will be asked to judge whether a collision verifies the Conservation Laws. As long as your results differ by at most 2 or 3 times the **Average Fraction Lost** found from *Section 4.1.1*, you can assume those discrepancies are **only** due to friction, and not an invalidation of the Conservation Laws.

Procedure

1. Answer question 1 in 4.1.1.
2. Check that the track is level in both directions (along its length and across its length). Test each direction at a few locations along the length of the track.
3. Place a wooden block on top of one of the cars. Place a plastic board between the two masses length-wise so that it fits firmly.
4. Fix the height of the photogates, so the board cuts the beam (a red LED on the side of the photogate lights up when the beam is cut). You can adjust the height of the photogates by loosening the screw on the post and moving the bracket up or down. Make sure you adjust both of the photogates. Place the photogates about 2.5 cart-lengths apart.
5. Adjust the cart launcher so the scale reads around 1 cm to 2 cm when it is cocked. You can adjust the compression by loosening the screw on the latching clamp and sliding it into position.

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6. Take the cart that you've set up and bring it up to the launcher. Make sure the launcher is aimed at the center of the cart. Make sure that the cart's wheels are in the grooves of the track.
7. Launch the cart by pulling the string on the launcher. Once it has passed through both of the photogates, stop the cart and record the times displayed on the photogates.
8. Repeat *Steps 4 – 6* again for a total of 2 trials. Record your data in Table 3.1.

3.1.2 Elastic Collisions

Now you will be testing the conservation laws. During this part of the experiment, you will use two collision carts to create elastic collisions. Place the **magnetic ends** of two carts facing each other.

The launcher setting and the positions of the timers are recommended only. Feel free to make any necessary changes.

When positioning the timers, keep in mind that we want a measurement of the *speed* and *direction* of each of the two carts, both before and after the collisions.

Also remember that energy and momentum are lost continuously as the carts travel. Therefore, it is advisable to position the timers so that the measurements are taken immediately before and after the collision.

You will set up the following situations:

I. Carts of Equal Mass: (use data Table 3.2)

Use the wooden masses on each cart. Set the launcher scale to around 1 cm.

II. Lighter Mass Colliding with Heavier Mass

Use the wooden mass on the launched cart and metal masses on the stationary cart. Set the launcher scale to around 1 cm.

Procedure

1. Answer question 3 in 4.1.2.
2. Place the timers apart by about 2.5 times the length of the cart. See the list above for the cart combinations and the launcher setting.
3. Launch a cart into a stationary one between the two timers. The carts will collide, and if both carts are moving after the collision, there will be two times recorded on one of the timers. The total time for that timer is obtained by pressing the memory switch. Subtract the first time from the total time on the same timer to get the second time, i.e., the time it took for the second cart to clear the photogate.
4. Repeat *Steps 1 – 3*, with for the second combination of carts listed above.

3.1.3 Inelastic Collisions

Use the same technique as in *Section 3.1.2 Elastic Collisions* to test the conservation laws in inelastic collisions. This section is purely qualitative, so you **may remove the photogates** if they get in the way. Place the **Velcro ends** of two carts facing each other.

You will set up the following situation:

I. Carts of Equal Mass

Use two metal masses on each cart. Set the launcher scale to around 2 cm.

II. Other situations of your own choosing

3.2 Velocity of a Projectile

Introduction

In this experiment, the velocity of a projectile as it leaves a spring gun will be measured using two methods. First, the conservation laws will be used to determine the velocity. For comparison, it will also be measured using projectile motion.

Instructions on Equipment Operation

Ballistic Pendulum

Insure that the apparatus is firmly fastened down so that it will not move during the experiment. Make sure the base is level and adjust it if necessary. The ball mounts on the gun's push rod via a hole through its diameter. Also insure this fits smoothly on your apparatus, and that the ball catcher is aligned with the spring gun.

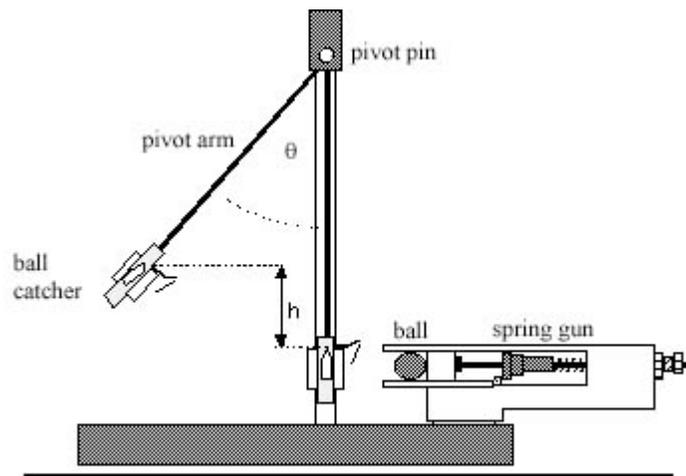


Figure 3.4

To remove the ball from the catcher, lift the latch spring with your finger while pushing out from the rear. The ball should come out easily.

To weigh the pendulum, carefully unscrew the pivot pin (at the top of the pendulum—it has a small knob that allows it to be unscrewed), slide out the pin and remove the pendulum. Place the pin in some safe place, so that it is not lost while you

are weighing the pendulum. Replace the pendulum by reversing the disassembly sequence.

To cock the gun, first place a ball in the launcher's barrel. Then push the ball down the barrel with a ramrod until the trigger catches the piston at the desired point. There are three range settings: short, medium or long range. We recommend using the long-range setting. **Never** look into the barrel of the launcher or put your fingers into the barrel.

To launch the ball, pull lightly upward on the trigger assembly.

3.2.1 Ballistic Pendulum Introduction

A ball is fired by a spring gun into a catcher arranged to swing as a pendulum (See *Figure 3.4*). When the ball is caught, the combination of the ball and catcher become the bob of the pendulum. Although the collision between ball and catcher is *inelastic* and energy is *not conserved*, *momentum is*. With the catcher at rest, the initial momentum of the system is provided by the ball shot with velocity, v_b . Just after the ball is caught, the momentum is due to motion of the center of mass of the pendulum assembled from the catcher and ball, having velocity, v_p . Conservation of momentum requires the two momenta be the same

$$mv_b = (M + m)v_p, \quad \text{Equation 3.10}$$

where m and M are the masses of the ball and pendulum respectively.

After the ball is caught, the energy of the combined ball and catcher system is conserved. Initially the center of mass is at the lowest possible point with velocity, v_p . As it moves upward against the force of gravity, the kinetic energy is converted into potential energy. At its highest point all the initial kinetic energy of the pendulum has been used to produce a change in the potential energy,

$$(M + m)gh = \frac{1}{2}(M + m)v_p^2, \quad \text{Equation 3.11}$$

which can be determined by measuring the increase in height, h , of the center of mass of the assembly. In this lab, you will measure h indirectly using the angle indicator located at the pivot of the pendulum and the formula you found for the second pre-lab question. Finally, the value of the initial velocity of the ball can be determined,

$$v_b = \left(1 + \frac{M}{m}\right)\sqrt{2gh}. \quad \text{Equation 3.12}$$

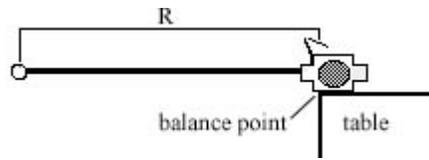


Figure 3.5

Procedure

1. Measure the mass of your ball (use the metal one) and the mass of the pendulum.
2. To find the distance from the pivot pin to the center of mass of the ball-pendulum system, R , place the ball in the catcher, and balance the entire assembly on the edge of the table, so that it balances there, but cannot be moved closer to the edge without falling over. The balance point will be directly under the center of mass. (see Figure 3.5)
3. Starting with the pendulum at rest and the angle indicator reset to zero, fire the ball into the catcher five times and measure the maximum angle to which the pendulum swings each time. It will not always be the same. Make sure the spring gun is at the same setting for each trial. Be careful not to get your hand caught in the gun when you fire the ball. Record your data in Table 3.2.
4. Make sure you reset the angle indicator to zero before each trial.

3.2.2 Projectile Motion

Introduction

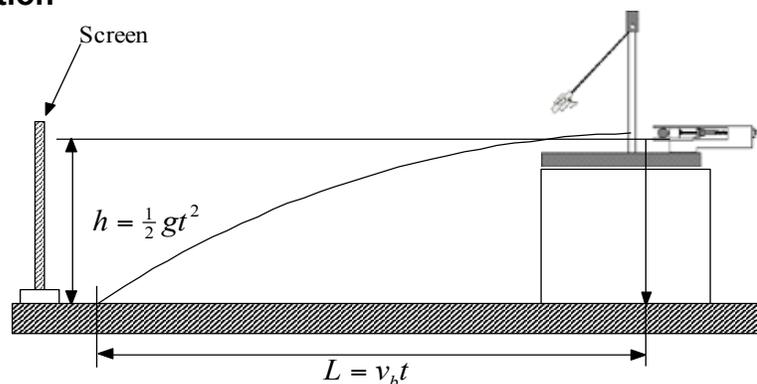


Figure 3.6

Make sure the apparatus is securely fastened and leveled on a block on the table. Latch the pendulum bob out of the way and practice firing the ball from the gun out onto the floor. Be careful of wild shots.

With the catcher removed from its path, the ball will fall freely after it leaves the gun. It will follow a parabolic arc until it hits the table, traveling a horizontal distance,

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L , while falling a vertical distance, h . The equations of projectile motion can be used to analyze the motion in these directions, as shown in *Figure 3.6*. Combining the two equations to eliminate time, the initial velocity of the ball is then

$$v_b = L\sqrt{\frac{g}{2h}}. \quad \text{Equation 3.13}$$

or solving for L ,

$$L = v_b\sqrt{\frac{2h}{g}}. \quad \text{Equation 3.14}$$

Procedure

1. Measure the height, h , from the table to the *bottom* of the ball in the launcher.
2. Answer question 1 in 4.2.2.
3. With the ball in place and the gun released, find the position on the table directly below the center of the ball and mark it with a piece of tape.
4. Fire the ball from the gun out over the table several times. Be careful of wild shots. Tape a piece of white paper over the area of the floor where the ball lands. Cover it with carbon paper, carbon side down, so that the impact point of the ball will produce a mark. Do not tape down the carbon paper.
5. Fire five shots, marking and numbering each of the impacts.
6. Measure the range, L , of each of these shots from your tape from step 3. Record the values in Table 3.3.

Last Revised on January 8, 2015

EXPERIMENT 3

Conservation of Momentum and Energy

4. Post-Laboratory Work [20pts]

4.1 The Linear Track [11pts]

4.1.1 Newton's First Law (from Section 3.1.1) [3pts]

1) What value do you expect for the Fraction of Momentum Lost when the friction is very large or very small? What about for the Fraction of Kinetic Energy Lost? Why?(2pts)

$$\frac{\overline{\Delta p}}{p} = \qquad \qquad \qquad \frac{\overline{\Delta K}}{K} =$$

Table 3. 1 (wooden block)

(t_i is the time, Δt , it took the board on the cart to pass through the first (initial) photogate, and t_f is the time it took the board to pass through the second (final) photogate)

Trial	t_i	t_f	p_f / p_i	E_f / E_i
A				
B				

Averages: _____ $L =$ _____

The average velocity, v , of a cart as it passes through a photogate is given by Equation 3.5, the momentum, p , when passing through the photogate by Equation 3.1, and the kinetic energy, K , by Equation 3.2.

By substituting 3.5 into 3.1, and into 3.2: $\frac{p_f}{p_i} = \frac{m\left(\frac{L}{t_f}\right)}{m\left(\frac{L}{t_i}\right)} = \frac{t_i}{t_f}$ and $\frac{E_f}{E_i} = \frac{\frac{1}{2}m\left(\frac{L}{t_f}\right)^2}{\frac{1}{2}m\left(\frac{L}{t_i}\right)^2} = \frac{t_i^2}{t_f^2}$

You can see that the mass and the length the object cancel **because they did not change**, and calculating the Fractions Conserved reduces to a single division.

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2) Fill in Table 3.1 and calculate the Fractions Lost below. Show your work for the Fractions Conserved and Fractions Lost. How do these compare with your predictions in question 1?(1pt)

$$\frac{\overline{\Delta p}}{p} =$$

$$\frac{\overline{\Delta K}}{K} =$$

4.1.2 Elastic Collisions – Equal Masses (from Section 3.1.2) [5pts]

3) What value do you expect for the Fraction of Momentum Lost? What about for the Fraction of Kinetic Energy Lost? Why? (1pt)

$$\frac{\overline{\Delta p}}{p} =$$

$$\frac{\overline{\Delta K}}{K} =$$

4) Will the simplified equations for Fractions Conserved that you used for the wooden and metal carts by themselves still hold? Why or why not? If not, what changes would you have to make to the equations? Think about what's moving before the collision (initial), and what's moving after the collision (final). Consider the lengths and masses of the two carts to be equal, since they are very close. (1pt)

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Remember that velocity (v) and momentum (p) are vectors (have directions) and make sure the numerical values of the velocities and momenta reflect the direction.

Table 3.2 (Equal Mass Elastic Collision)

Trial	M_1	M_2	t_{1i}	t_{2i}	t_{1f}	t_{2f}
A				N/A		

$$L_1 = \underline{\hspace{2cm}} \quad L_2 = \underline{\hspace{2cm}}$$

Note: The time interval t_{2i} for the second cart is entered as N/A, because the second cart does not pass through any photogates before the collision. This is not a problem, because you know it's initial velocity is zero.

5) Using your answer to question 4, find the Fractions of Momentum and Kinetic Energy Lost, and compare them to what you predicted in question 3. (1pt)

$$\frac{\overline{\Delta p}}{p} =$$

$$\frac{\overline{\Delta K}}{K} =$$

6) Using the Average Fractions of Momentum and Kinetic Energy Lost from *Section 4.1.1* and your Fraction Conserved values just calculated, are the Laws of Conservation of Momentum and Kinetic Energy obeyed? Explain. (Remember the **KEY IDEA**) (2pt)

4.1.3 Elastic Collisions – Small Mass into Heavy Mass (from Section 3.1.2) [0pts]

If the wooden cart derails after the collision, turn down the launcher power until it does not.

Think about what happens in this collision, and how it is different from the Equal Mass Elastic Collision. It may be useful to look back at the simplified equations for Fractions Conserved that you used in 4.1.1.

4.1.4 Inelastic Collisions – Equal Mass Collision (from Section 3.1.3) [0pts]

Think about this too. Again, look at the simplified equations for Fractions Conserved. Figure out what part of the reasoning behind them does not apply to this case.

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4.1.5 Inelastic Collisions – Other situations of your own choosing (from Section 3.1.3) [3pts]

Now we are going to observe inelastic collision, when the combined carts keep moving in one direction. Now we will conduct the experiment qualitatively, use your hands as launchers. Launch both carts toward each other with different combination of mass of each cart. Try to find the situation such that the combined cart keeps moving. Try to launch the carts with the same amount of force. You can remove the photogates to make more space for launching.

7) Which of the combinations you tested worked the best? (Weigh each cart from your best trial) Why?(2pts)

$M_1 =$ _____ $M_2 =$ _____

8) What limits the maximum rebound velocity?(1pt)

MAKE SURE TA & TI STAMPS EVERY PAGE BEFORE YOU START**4.2.1 Ballistic Pendulum (from Section 3.2.1) [3pts]****Table 3.2 (Ballistic Pendulum Data)**

Trial	1	2	3	4	5	\bar{h}
<i>Angle, θ</i>						N/A
$h = R(1 - \cos\theta)$						

Pendulum Length, R _____ Mass of ball, $m =$ _____ Mass of Pendulum, $M =$ _____

1. Calculate the average height of the center of mass of the ball, \bar{h} , and its standard deviation, Δh . Show your work (1pt)
2. Using the result of the above calculation, find the initial velocity of the ball, v_b , from Equation 3.12. Show your work. (1pt)
3. Now find the error in the ball's velocity, Δv_b , using the ideas of Error Propagation. We assume there is no error in the measurement of masses. Show your calculation. (1pt)

$$\frac{\Delta v_b}{v_b} = \frac{\Delta h}{2\bar{h}}$$

MAKE SURE TA & TI STAMPS EVERY PAGE BEFORE YOU START**4.2.2 Projectile Motion (from Section 3.2.2) [6pts]**

h = _____ (♦ Notice that this h is different from h in 4.2.1)

Make and record a reasonable estimate of Δh , the error in the height measurement: _____

1. Use Equation 3.14 to predict the length L that the ball will travel. Show your work. Mark the expected location with a piece of tape. (1pt)

Table 3.3 (Projectile Motion)

Trials	1	2	3	4	5	Average
Distance, L						$\bar{L} =$

2. Calculate the average range, \bar{L} , and its error, ΔL , from the standard deviation. (1pt)

3. Calculate v_b from the measurements of h and \bar{L} using Equation 3.13. Show your work. (1pt)

4. Calculate the error in the initial velocity, Δv_b . The equation from propagation of error is:

$$(1pt) \quad \frac{\Delta v_b}{v_b} = \sqrt{\left(\frac{\Delta L}{\bar{L}}\right)^2 + \left(\frac{\Delta h}{2h}\right)^2} .$$

5. Was your predicted L consistent with your measured L? If not, what do you think caused the discrepancy? (2pts)