Critical Behavior of the Meissner Transition in the Lattice London Superconductor

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We carry out Monte Carlo simulations of the three dimensional (3D) lattice London superconductor in zero applied magnetic field, making a detailed finite size scaling analysis of the Meissner transition. We find that the magnetic penetration length λ and the correlation length ξ scale as $\lambda \sim \xi \sim |t|^{-\nu}$, with $\nu = 0.66 \pm 0.03$, consistent with ordinary 3D *XY* universality, $\nu_{XY} \approx 2/3$. Our results confirm the anomalous scaling dimension of magnetic field correlations at T_c. [S0031-9007(98)05449-0]

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The discovery of the high temperature superconductors has revived interest in the effects of fluctuations on the critical behavior of the superconducting transition. The Meissner transition of a bulk type II superconductor in zero applied magnetic field is the most basic case that can be considered. While it was originally thought that this transition was weakly first order [1], it is now generally believed to be in the same universality class as the ordinary three dimensional (3D) *XY* model, except with the temperature scale inverted [2,3]. The argument is based [2–6] on two observations: (i) the ordinary 3D *XY* model can be mapped onto a system of sterically interacting loops with inverted temperature scale $T_{\mathrm{loop}} \propto 1/T$, and (ii) the vortex loops of a fluctuating superconductor interact with a *screened* Coulomb interaction, with screening length equal to the bare magnetic penetration length λ_0. Assuming that the finite interaction length λ_0 of the vortex loops is not a relevant modification of steric (on site) interactions, the universality of the fluctuating Meissner transition and the 3D *XY* model follows [7]. Early Monte Carlo (MC) simulations by Dasgupta and Halperin [3] of a lattice London superconductor model strongly supported this picture by making a qualitative comparison of the shape of specific heat peaks in the two models.

Recently there has been renewed interest in, and controversy concerning, the nature of this transition. Kiometzis *et al.* [8], considering a dual formulation of the fluctuating Ginzburg-Landau (GL) model, have argued that while the correlation length $\xi \sim |t|^{-\nu}$ diverges with the same exponent ν as the ordinary 3D *XY* model, $\nu_{XY} \approx 2/3$, the renormalized magnetic penetration length should diverge as $\lambda \sim |t|^{-\nu'}$ with $\nu' = 1/2$ the mean field exponent. Herbut and Tešanović [9], however, using an analysis of the GL model exact to all orders in perturbation, have argued that, due to the presence of an *anomalous dimension*, $\eta_A = 1$, for fluctuations of the magnetic field, one must have $\nu = \nu'$. Using a one-loop renormalization group (RG) scheme, they further suggested the possibility that $\nu < \nu_{XY}$ [10]. Bergerhoff *et al.* [11], using a nonperturbative RG flow analysis of the GL model, similarly find $\nu < \nu_{XY}$. Herbut [12], however, has argued that for the lattice London limit of the GL model, $\nu = \nu' = \nu_{XY}$.

To investigate this controversy we present here the results of new MC simulations of the 3D isotropic lattice London superconductor (LLS) in zero external magnetic field. Carrying out the first detailed finite size scaling analysis of this model, we find results consistent with a *single* diverging length scale, hence $\xi \sim \lambda$. We find $\nu \approx \nu_{XY}$ consistent with the universality of the ordinary 3D *XY* model. We find clear evidence for the anomalous dimension of magnetic field correlations predicted by Herbut and Tešanović [9].

The Hamiltonian of our model [3] is

$$\mathcal{H} = \sum_{i\mu}\left\{U(\theta_{i+\hat{\mu}} - \theta_i - A_{i\mu}) + \frac{1}{2}J\lambda_0^2[\mathbf{D}\times\mathbf{A}]_{i\mu}^2\right\}. \tag{1}$$

The sum is over all bonds of a 3D simple cubic lattice of unit grid spacing. θ_i is the phase angle of the superconducting wave function on site i, $\psi_i = e^{i\theta_i}$, where the amplitude of ψ_i has been taken constant (the London approximation). $A_{i\mu}$ is the discretized vector potential on the bond at site i in direction $\hat{\mu} = \hat{x}, \hat{y}, \hat{z}$, and if μ, ν, σ is a cyclic permutation of x, y, z, then

$$[\mathbf{D}\times\mathbf{A}]_{i\mu} = A_{i\nu} + A_{i+\hat{\nu},\sigma} - A_{i+\hat{\sigma},\nu} - A_{i\sigma} \equiv 2\pi b_{i\mu} \tag{2}$$

is the counterclockwise circulation of the $A_{i\mu}$ around the plaquette at site i with normal in direction $\hat{\mu}$. $b_{i\mu}$ is the number of flux quanta ϕ_0 of total magnetic field through this plaquette. The coupling is $J = \phi_0^2/16\pi^3\lambda_0^2$, with λ_0 the *bare* magnetic penetration length, and $U(\varphi)$ is the Villain function [13]

$$e^{-U(\varphi)/T} = \sum_{m=-\infty}^{\infty} e^{-(1/2)J(\varphi-2\pi m)^2/T}.$$

The first term in Eq. (1) is the kinetic energy of flowing supercurrents; the second term is the magnetic field energy.

We focus here on the calculation of the magnetic field correlation function

$$F(q) \equiv \frac{4\pi^2 J}{TL^3}\langle b_\mu(q\hat{\nu})b_\mu(-q\hat{\nu})\rangle, \tag{3}$$

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where $b_\mu(q\hat{\nu}) \equiv \sum_i e^{-iq\hat{\nu}\cdot\mathbf{r}_i} b_{i\mu}$ is the Fourier transform of the total magnetic field, and $\hat{\mu} \perp \hat{\nu}$. $F(q)$ is just the wave-vector-dependent magnetic permeability, with $\lim_{q\to 0} F(q) = \partial B/\partial H$ [14]. Our goal is to show that the singular part of $F(q)$ is consistent with the scaling ansatz

$$F(t, Q, L) = \ell^{-1} F(t\ell^{1/\nu}, Q\ell, L/\ell), \tag{4}$$

where $t = T - T_c$, $Q = 2\sin(q/2)$, L is the system length, and ℓ is an arbitrary length rescaling factor. Note that we choose Q rather than q as our scaling variable, since the vortex line interaction that arises from the Hamiltonian (1) is a function of q_μ only through the combinations Q_μ [14]. Since $Q \to q$ as $q \to 0$, this does not affect the long length scaling; our hope is that by using Q we may succeed to slightly extend the scaling region to shorter length scales. Verification of the scaling Eq. (4) will demonstrate that there is only a *single* diverging length scale in the model that describes both the critical behavior of global thermodynamic variables, as well as the spatial variation of magnetic field fluctuations. Since the former is determined by the correlation length ξ, while the latter is determined by the magnetic penetration length λ, we conclude that $\xi \sim \lambda \sim |t|^{-\nu}$.

We carry out standard Metropolis MC on the Hamiltonian (1) for cubic lattices of lengths $L = 8$ to 32, using periodic boundary conditions. We use the particular value of $\lambda_0 = 0.3$ (in units of the grid spacing) [15]. Temperatures will be measured in units of J. In one MC "pass" we first update A_{ix}, A_{iy}, and A_{iz} at each site i, going sequentially through the entire lattice, then follow this by a sequential update of the θ_i. The $A_{i\mu}$ are allowed to fluctuate without constraint. For our largest system size, $L = 32$, we use at each temperature typically 32 000 passes to equilibrate, followed by 1.7×10^7 passes for computing averages.

First we consider the scaling behavior of the magnetic permeability. Evaluating Eq. (4) at the smallest wave vector in our system, $q_{\min} = 2\pi/L$, using $Q_{\min} \approx q_{\min}$, and choosing the rescale factor $\ell = L$, we arrive at

$$LF(t, Q_{\min}, L) = F(tL^{1/\nu}, 2\pi, 1). \tag{5}$$

Exactly at T_c (i.e., $t = 0$), $LF(q_{\min})$ should thus be a constant independent of L. In Fig. 1(a) we plot our data for $LF(q_{\min})$ vs T, for $L = 8$–32. To a very good accuracy, the curves for different L do indeed intersect at a single point, $T_c \approx 0.8$. To further verify the scaling relation Eq. (5), we fit our data for $LF(q_{\min})$ near T_c to a low order polynomial expansion in $(T - T_c)L^{1/\nu}$. We determine the values of $T_c = 0.8000 \pm 0.0002$ and $\nu = 0.66 \pm 0.03$ from a fifth order polynomial fit, restricting data to the ranges $|t| \le t_{\max} = 0.006$ and $L \ge L_{\min} = 12$. Increasing either the order of the polynomial, $L_{\min}$, or decreasing $t_{\max}$ resulted in no change in these fitted values, within the estimated statistical error. In Fig. 1(b) we use these fitted parameters to plot $LF(q_{\min})$ vs $tL^{1/\nu}$, for data in the range $|t| \le 0.01$. The resulting data collapse is *very* good. Our value of ν is thus completely consistent with

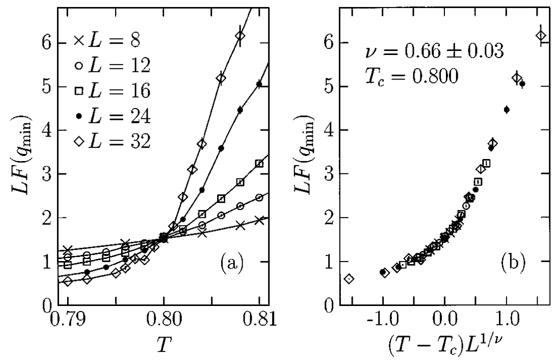

FIG. 1. (a) $LF(q_{\min})$ vs T for system sizes $L = 8$–32. The common intersection of all curves locates T_c. (b) Scaling collapse of $LF(q_{\min})$ vs $(T - T_c)L^{1/\nu}$ using fitted values of T_c and ν.

$\nu_{XY} \approx 2/3$. Note that by taking $q = q_{\min}$, $L \to \infty$, and $\ell = t^{-\nu}$ in Eq. (4), our results imply that the magnetic permeability vanishes, as $T \to T_c^+$, as $\partial B/\partial H \sim t^\nu$.

We now consider the q dependence of $F(q)$. In Fig. 2 we plot $F(q)$ vs Q for $L = 8$–32, exactly at T_c and for one representative temperature above and below T_c. We see, as expected, that for $T > T_c$, $F(q)$ approaches a constant as $Q \to 0$, while for $T < T_c$, $F(q)$ vanishes as Q^2. Exactly at T_c, however, $F(q)$ appears to vanish linearly as Q. This is a clear suggestion of the anomalous dimension of magnetic field correlations predicted by Herbut and Tešanović, according to which at T_c, $F(q) \sim q^{\eta_A}$ with $\eta_A = 4 - D$ in D dimensions [9]. It is interesting to note that, while there is a considerable finite size effect for $T > T_c$, finite size effects at a fixed value of Q appear negligible for all $T \le T_c$.

To further verify the anomalous scaling dimension of magnetic field correlations, we can apply Eq. (4) at $t = 0$, taking as the rescaling factor $\ell = L$, to get

$$LF(0, Q, L) = F(0, QL, 1). \tag{6}$$

In Fig. 3 we plot $LF(q)$ exactly at T_c vs LQ, for $L = 8$–32 and $q \le \pi/2$. We find a good collapse of the data to a single curve that vanishes linearly as $LQ \to 0$.

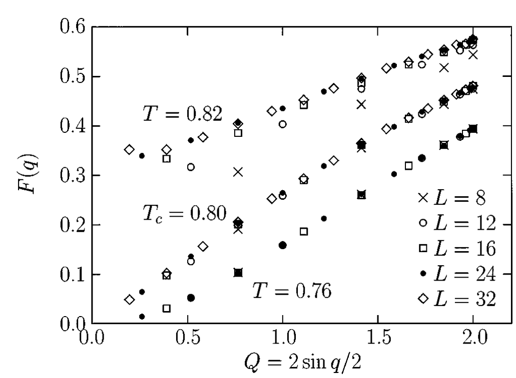

FIG. 2. $F(q)$ vs Q for system sizes $L = 8$–32, at $T = 0.76 < T_c$, $T_c = 0.80$, and $T = 0.82 > T_c$. Note the virtual absence of finite size effects for $T \le T_c$.

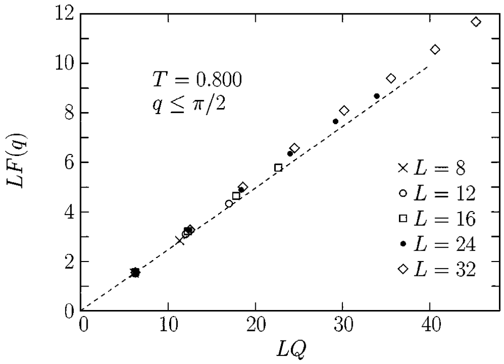

FIG. 3. Scaling collapse of $LF(q)$ vs LQ at T_c, for $L = 8-32$, $q < \pi/2$. $LF(q)$ vanishes *linearly* as $LQ \to 0$.

Finally, in the thermodynamic limit $L \to \infty$, we can use Eq. (4) with $\ell = |t|^{-\nu} \equiv \xi$ to get

$$F(t, Q, \infty)/Q = (Q\xi)^{-1} F_{\pm}(1, Q\xi, \infty), \tag{7}$$

where $F_{\pm}$ refers to distinct branches for $T > T_c$ and $T < T_c$. Using the values of T_c and ν found in the fit of Fig. 1(b) to determine $\xi = |T - T_c|^{-\nu}$, we plot in Fig. 4 our data for $F(q)/Q$ vs ξQ on a log-log scale. We use only data for which finite size effects appear to be small, and which are in the scaling region. We see an excellent collapse of the data. Figure 4 clearly demonstrates that there is only a single diverging length scale for the spatial variation of magnetic field correlations, and that this length scale is ξ. For the $T > T_c$ branch, we see that $F(q)/Q$ diverges as $1/\xi Q$ as $\xi Q \to 0$, indicating that $F(q)$ approaches a finite constant $\propto \xi^{-1}$. For the $T < T_c$ branch, we see that $F(q)/Q$ vanishes as ξQ as $\xi Q \to 0$, indicating that $F(q)$ vanishes as ξQ^2. However, for both branches $F(q)/Q$ approaches the same constant as $\xi Q \to \infty$, indicating that $F(q)$ vanishes linearly in Q exactly at T_c. Figure 4 thus gives another demonstration of the anomalous dimension of magnetic field scaling at T_c.

To get a better physical understanding of the effects of this anomalous dimension of magnetic field scaling, consider applying a small external magnetic field given by

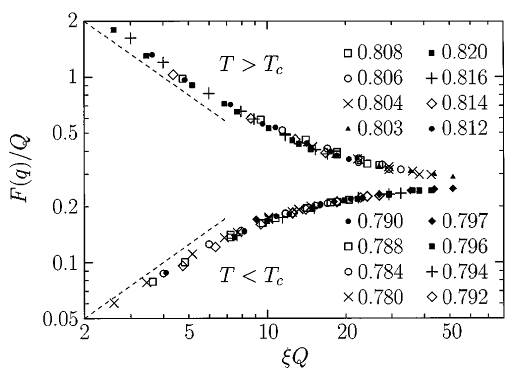

FIG. 4. Log-log scaling collapse of $F(q)/Q$ vs ξQ. The dashed lines at small ξQ have slopes of ± 1 to indicate the asymptotic behavior as $\xi Q \to 0$.

$\mathbf{A}^{\text{ext}}$. The London equation, describing the total screening of the Meissner state, $T < T_c$, gives for the induced supercurrent [16]

$$\langle j_\mu^{\text{ind}}(q\hat{\nu})\rangle = \frac{J\lambda_0^2}{\alpha(q)} \langle A_\mu(q\hat{\nu})\rangle, \tag{8}$$

where the vector potential of the total magnetic field is the sum of the applied and induced fields, $\langle A_\mu(q\hat{\nu})\rangle = A_\mu^{\text{ext}}(q\hat{\nu}) + \langle A_\mu^{\text{ind}}(q\hat{\nu})\rangle$, and

$$\lambda_0^2/\alpha(q = 0) = n_s(T)/n_s(T = 0) \tag{9}$$

is determined by the density of superconducting electrons n_s.

The induced supercurrent is also related to $\langle A_\mu^{\text{ind}}(q\hat{\nu})\rangle$ by Ampère's Law, which for the gauge $\mathbf{Q} \cdot \mathbf{A} = 0$ can be written as

$$\langle j_\mu^{\text{ind}}(q\hat{\nu})\rangle = -J\lambda_0^2 Q^2 \langle A_\mu^{\text{ind}}(q\hat{\nu})\rangle. \tag{10}$$

Noting that $F(q)$ is the magnetic permeability, we have

$$F(q) = \frac{\langle A_\mu(q\hat{\nu})\rangle}{A_\mu^{\text{ext}}(q\hat{\nu})} = \frac{1}{1 - \langle A_\mu^{\text{ind}}(q\hat{\nu})\rangle/\langle A_\mu(q\hat{\nu})\rangle}. \tag{11}$$

Combining this with Eqs. (8) and (10) gives

$$F(q) = \frac{\alpha(q)Q^2}{1 + \alpha(q)Q^2}. \tag{12}$$

Comparing with the results of Fig. 4, we see that for finite ξ at $T < T_c$, we have $\lim_{q\to 0} \alpha(q) \sim \xi$. Equation (9) thus implies that the superconducting electron density vanishes as $n_s \sim \xi^{-1} \sim |t|^\nu$.

The renormalized magnetic penetration length λ is determined by the pole of $F(q)$. If one could ignore the q dependence of $\alpha(q)$, one would then conclude that $\lambda^2 = \alpha(q = 0)$. From this follows $\lambda \sim \sqrt{\xi}$ and $n_s \sim \lambda^{-2}$. These are indeed the expectations from mean field theory [16], as well as the "uncharged" superconductor represented by the ordinary 3D XY model [17] (given by the limit $\lambda_0 \to \infty$). They also hold in the present model, at low temperatures.

However, as $T \to T_c^-$, Eq. (12) and Fig. 4 imply that $\lim_{q\to 0} \alpha(q) \sim 1/q$. This is a consequence of the anomalous scaling dimension of the magnetic field. It is this singular dependence of $\alpha(q)$ on q that shifts the pole of $F(q)$ so that $\lambda \sim \xi$ rather than $\sqrt{\xi}$, in the "charged" superconductor critical region. In this critical region the London relation $n_s \sim \lambda^{-2}$ no longer holds [18].

Note also that $F(q)$ determines the decay of the magnetic field away from a test vortex line, hence it determines the renormalized interaction between vortex lines. The q dependence of $\alpha(q)$ implies that in the critical region, the interaction between two straight and parallel test vortex lines separated by distance r will change from the $\ln r$ of

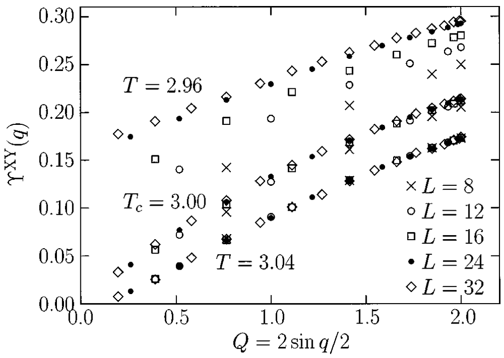

FIG. 5. $\Upsilon^{XY}(q)$ vs Q for the ordinary 3D XY model. Data is for $L = 8-32$ at $T = 2.96 < T_c$, $T_c = 3.0$, and $T = 3.04 > T_c$. Note the virtual absence of finite size effects for $T \geq T_c$.

mean field (MF) theory at $r < \sqrt{\xi} \sim \lambda_{\mathrm{MF}}$, to the faster decay of $1/r$ for $\sqrt{\xi} < r < \xi \sim \lambda$.

Finally, we note that the anomalous scaling of $F(q)$ also has some interesting consequences for the ordinary 3D XY model. One can show that, within the mapping of the 3D XY model to a gas of sterically interacting loops, the helicity modulus of the XY model maps into a loop-loop correlation function. Identifying such loops as the vortex lines of the LLS, which as $q \to 0$ (or $\lambda_0 \to 0$) become identical with magnetic flux, one concludes [19] that the *wave-vector-dependent* helicity modulus [14,20] $\Upsilon^{XY}(q)$ of the ordinary 3D XY model should be the dual of $F(q)$. We have carried out independent MC simulations of the ordinary 3D XY model, in the Villain approximation, calculating $\Upsilon^{XY}(q)$ for an ensemble with "fluctuating twist" boundary conditions (fbc) [20]. We plot our results for $\Upsilon^{XY}(q)$ vs Q in Fig. 5, for $L = 8-32$ and $T = 2.96 < T_c$, $T_c = 3.0$, and $T = 3.04 > T_c$. Note the striking similarity to Fig. 2, only with the temperature scale inverted. Finite size effects are negligible for $T \geq T_c$. As $Q \to 0$, Υ^{XY} approaches a constant for $T < T_c$, $\Upsilon^{XY} \sim Q^2$ for $T > T_c$, and $\Upsilon^{XY} \sim Q$ exactly at T_c. Thus the anomalous scaling of $F(q)$ at T_c shows up as an anomalous scaling of $\Upsilon^{XY}(q)$ at T_c [21].

To conclude, we have presented MC data that verifies the scaling ansatz of Eq. (4). This ansatz implies that there is only a single diverging length scale in the problem, and that the magnetic penetration length scales as $\lambda \sim \xi \sim |t|^{-\nu}$. We find the value of $\nu = 0.66 \pm 0.03$ consistent with $\nu_{XY} \approx 2/3$ of the ordinary 3D XY model, and confirm the predicted anomalous scaling dimension of magnetic field correlations at T_c.

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