Lecture 14
PHY 100. Binding Energy, Chain Reactions
26 = # protons (Z)
Fe = Atomic element symbol
Iron = Element name
55.845 = Average atomic mass per atom
2 8 14 2 = Electron orbital configuration

Average over isotopes: Different number of neutrons in nucleus
Different isotopes can be found in nature
What holds the nucleus together:

**Strong Nuclear Force**
- Much stronger than E.M. and gravity
- Limited to short ranges: \(10^{-15}\) m

Pop this new force into Schrödinger Eq.

The solutions are discrete energy levels for the nucleons very similar to orbitals for \(e^-\) in atom.

\[ E_4 - E_2 = h\gamma \approx 10^6\text{ eV} \]

\(1\) proton, ground state

\(\text{1 proton}\) ground state

The strong force keeps protons glued to each other inside nucleus, but E.M. repulsion still exists.

Most nuclei are stable:

Some are unstable: they naturally decay (spontaneously) through \(\alpha,\beta,\gamma\) radiation.
INSIDE X:
\[ n \rightarrow p + e^- \]
CAUSED BY WEAK FORCE
(A NEW KIND OF FORCE)

\[ \beta: \quad e^- \quad e^- = \beta \]

\[ \alpha: \quad \text{the nucleus} \]
\[ \alpha = \frac{2p}{2n} \]
Time of decay is characterized by \( T_{1/2} \).

Time it takes for 50\% of nuclei in a given sample to decay.

\[
\left( \frac{t}{T_{1/2}} \right) \quad \text{less than 1}
\]

\[
N = N_0 \cdot 2
\]

\# of nuclei at time \( t \)
\# of nuclei at time 0 (initial)

Don't worry about the formula!

Radioactive dating

\( ^{12}C \) in atmosphere: normal carbon in atm.

\( ^{14}C \) in atmosphere: when cosmic rays hit atm.

\( ^{14}C \) is a naturally radioactive \( \beta \) emitter.

\( T_{1/2} = 6000 \text{ years} \)

\( ^{14}C \) incorporated into living tissue.

When you die, \( ^{14}C \) begins decaying, \( ^{12}C \) remains constant.
$^{14}\text{C}/^{12}\text{C}$ ratio gives an estimate of time of death.

$$\frac{N_{^{14}\text{C}}}{N_{^{14}\text{C} \text{(alive)}}} = 2^{\frac{t}{T_{1/2}}} \Rightarrow t = T_{1/2} \times \log \left( \frac{N_{^{14}\text{C}}}{N_{^{14}\text{C} \text{(alive)}}} \right)$$

We don't know $N_{^{14}\text{C} \text{(alive)}}$: we use $^{14}\text{C}/^{12}\text{C}$ in the formula.

We know $^{14}\text{C}/^{12}\text{C}$ from calibrations with tree rings, ...

Example: what is the age of a bone found in an archaeological dig, if the measured $^{14}\text{C}$ is $1/4$ of the normal (in a living material).

1. Half-time = initial amount drops to $1/2$
2. 2 half-times = "" "" "" $1/4$ $(1/2 \times 1/2)$

$$t = 2 \times 6,000 \text{ years} = 12,000 \text{ years}.$$
Binding energy per nucleon is the energy needed to break free one nucleon from the nucleus. It is a measure of how easy it is to break up the nucleus.
Example of fission process:

\[
{^{235}_{92}U + n \rightarrow ^{93}_{37}Rb + ^{146}_{58}Cs + 2n + E}
\]

\(\Sigma\text{masses} = m_f\)  \(\Sigma\text{masses} = m_i\)

\(m_f - m_i > 0 \Rightarrow E = mc^2\)

The resulting 2\(n\) produce a cascade of the same reaction.

For each \(n\) in the initial state, you get two in the final state.

Chain reaction

Critical: 1 split per split
Subcritical: \(< 1\) “” “”
Supercritical: \(> 1\) “” “”
DEMONSTRATED IN 1942 (FERMI)
\[ {^{235}}U \sim 0.7\% \]
\[ {^{238}}U \sim 99.3\% \]
\text{NATURALLY OCCURRING}
\[ \text{NEPTUNIUM} \]
\[ \text{NEUTRINO} \]
\[ 238 \text{U} + n \rightarrow 239 \text{U} \rightarrow ^{239} \text{NP} + e^- + \nu \]
\[ ^{239} \text{Pu} = \text{PLUTONIUM} \]

\[ ^{239} \text{Pu} \text{ CAN BE USED AS NUCLEAR FUEL BECAUSE IT ALSO PRODUCES A CHAIN REACTION } \]

\text{WEAPONS:}

\text{URANIUM: NEEDS TO BE ENRICHED: 90\% \text{U} + 10\% \text{U} + \text{CENTRIFUGES} } \]

\text{PLUTONIUM: EASY TO MAKE IN REACTORS: } ^{238} \text{U} + n \rightarrow ^{239} \text{Pu} \]

\text{CRITICAL MASS: IN ORDER TO SUSTAIN A CHAIN REACTION, THERE HAS TO BE ENOUGH FISSILE MATERIAL SO THAT THE N ACTUALLY ESCAPE ARE EFFICIENT.}

\[ ^{235} \text{U BOMB: 40 kg (BASKETBALL)} \]
\[ ^{239} \text{Pu BOMB: 5 kg (ORANGE)} \]
DECEMBER 1945

Hiroshima (U)

Nagasaki (Pu)

Conventional explosive

Gun barrel

Hollow Uranium "bullet"

Cylinder target

Fast explosive

Slow explosive

Tamper/Pusher

Neutron initiator

Plutonium core

Spherical shockwave compresses core