PHY 121: MECHANICS

FINAL EXAMINATION SOLUTIONS

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[Monday, June 20, 2011.
Duration 3 hours.]

1 (a) $p(x,t) = 0.75 \cos \frac{\pi}{2}(x - 340t)$

part (1) $p_{max} = 0.75$ pascals

part (2) $\lambda = \frac{2\pi}{k}$ $\quad k = \frac{\pi}{2}\ m^{-1}$ $\quad \therefore\ \lambda = \frac{2\cancel{\pi}}{\cancel{\pi}}2 = 4\,m$ (Answer)

part (3) $\omega = \frac{\pi}{2}340\ rad\,s^{-1} \Rightarrow f = \left(\frac{\omega}{2\pi}\right) s^{-1} = \frac{\cancel{\pi}}{2}\frac{340}{2\cancel{\pi}}\ Hz = 85\ Hz$

part (4) $v = f\lambda = \left(\frac{\omega}{2\pi}\right)\left(\frac{2\pi}{k}\right) = \frac{\omega}{k} = \frac{\cancel{\pi}}{\cancel{2}}\frac{340 \times 2}{\pi} = 340\ ms^{-1}$

(b) $f = 400\ Hz$, $u_s = 34\ m/s$, $u_r = 0$, $v = 340\ m/s$.

part (1) & (2) $f' = f\,\frac{v \pm u_r}{v \pm u_s}$

$\because$ Car moves toward the receiver, $f' > f$

$f' = f\,\frac{v}{v - u_s} = (400)\,\frac{340}{340-34}\ Hz = \underline{\underline{444\ Hz}}$. Answer (2)

$\lambda = \frac{v}{f'} = \frac{340}{444}\ m = \underline{\underline{0.765\ m}}$ Answer (1) part (3) $u_s = 0$, $u_r = 34$ m/s towards car.

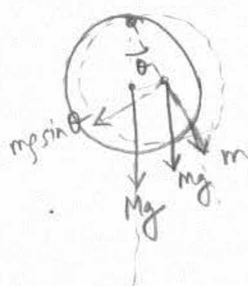
$$f' = f \, \frac{v \pm u_r}{v \pm u_s}$$

$\because$ moving toward $\therefore$ $f' > f$.

$$f' = f \, \frac{v + u_r}{v} = 400 \, \frac{340+34}{340} = \underline{\underline{440 \text{ Hz}}}$$

part (4) Yes, the frequencies are different.

2]



$T = 2.0$ s $R = ?$ (radius of hoop) , m = mass of hoop (not given)

Restoring torque,

$$\tau_R = -(mg \sin\theta) R$$

$$I \frac{d^2\theta}{dt^2} = -(mgR)\,\theta$$

$\sin\theta \approx \theta$
small oscillations

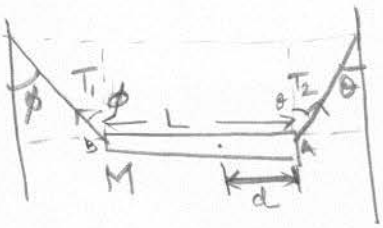
where $\boxed{I = mR^2 + mR^2 = 2mR^2}$ $\boxed{\frac{d^2\theta}{dt^2} = -\left(\frac{mgR}{I}\right)\theta.}$

Therefore, $\omega^2 = \frac{\cancel{m} g R}{2\cancel{m} R^2} = \frac{g}{2R} \Rightarrow \omega = \sqrt{g/2R}$

$T = \frac{2\pi}{\omega} \Rightarrow 2 \text{ sec} = 2\pi \sqrt{\frac{2R}{g}}$ $\therefore$ $\frac{T^2}{4\pi^2} = \frac{2R}{g}$

and, $\boxed{R = \frac{gT^2}{8\pi^2}} = 0.5$ m (Answer)

(3)



Vertical force balance

$$T_1 \cos\phi + T_2 \cos\theta = Mg \quad -(i)$$

$$T_1 \sin\phi = T_2 \sin\theta \quad -(ii)$$

horizontal force balance.

Balance torque about A,

$$Mg\, d = T_1 \cos\phi\, L \quad -(iii)$$

from (i) & (iii) $\quad T_1 \cos\phi + T_2 \cos\theta = \frac{T_1 \cos\phi\, L}{d}$

$$\therefore \quad T_1 \cos\phi \left(1 - \frac{L}{d}\right) = - T_2 \cos\theta$$

$$T_1 \sin\phi = T_2 \sin\theta$$

divide $\quad \frac{\tan\phi}{\left(1 - \frac{L}{d}\right)} = - \tan\theta \Rightarrow \tan\phi = - \tan\theta + \frac{L}{d}\tan\theta.$

$$\therefore \quad \frac{L}{d}\tan\theta = \tan\phi + \tan\theta \quad \Rightarrow \quad \boxed{d = \frac{L \tan\theta}{\tan\phi + \tan\theta}}$$ Answer.

$L = 6.10\,m$

$\theta = 36.9^\circ \quad \phi = 53.1^\circ$

$d = 2.199\,m$

$$\therefore \boxed{d = 2.20\,m} \text{ (3 sig fig)}$$

Answer.

(4) (a) $l = 1000\,m$ $\Delta l = ?$ $\Delta T = 30°C$

$\alpha = 11 \times 10^{-6}\ K^{-1}$

$\Delta l = l \alpha \Delta T = (1000)(11 \times 10^{-6})\, 30 = \underline{\underline{0.33\ m}}$ Answer

$\text{Strain} = \frac{\Delta l}{l} = \frac{0.33}{1000}$

$Y = 200 \times 10^{9}\ \frac{N}{m^2}$, $\text{stress} = Y\ \text{strain} = \left(200 \times 10^{9} \times \frac{0.33}{1000}\right) \frac{N}{m^2}$

$\boxed{\text{stress} = 6.6 \times 10^{7}\ \frac{N}{m^2}}$ Answer

(b) $T_1 = 10°C$ $T_2 = 30°C$ $\therefore \Delta T = 20°C$

$V = 1L$ $\beta_w = 1.1 \times 10^{-3}\ K^{-1}$, $\alpha_g = 9 \times 10^{-6}\ K^{-1} \therefore \beta_g = 3\alpha_g$

$\Delta V_{water} = V\beta_w \Delta T = (1)(0.207 \times 10^{-3})\, 20\ L = 4.14 \times 10^{-3}\ L$

$\Delta V_{flask} = V\beta_g\, \Delta T = 1 \times (3 \times 9 \times 10^{-6}) \times 20\ L = 540 \times 10^{-6}\ L$

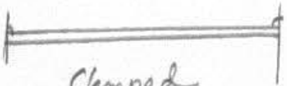
Volume of water spilling $= \Delta V_{water} - \Delta V_{flask} = (\beta_w - 3\alpha_g)V(\Delta T)$

$= (0.207 \times 10^{-3} - 3 \times 9 \times 10^{-6})(1)(20)\ L$

$= 3.6 \times 10^{-3}\ L$ $\therefore$ $\boxed{V_{spill} = 3.6\ mL}$

Answer.

(5)



Clamped

Let length of heated bar be L

When the copper breaks, it length would have contracted by ΔL

$$\Delta L = L\alpha(\Delta T)$$

$$\text{strain}_{max} = \left(\frac{\Delta L}{L}\right)_{max} = \alpha(\Delta T)$$

breaking stress $= 230\ MN/m^2$

breaking strain $= \dfrac{\text{stress}}{Y}$

$$\left(\frac{\Delta L}{L}\right)_{max} = \frac{230\times 10^6}{110\times 10^9}$$

$$\left(\frac{\Delta L}{L}\right)_{max} = \frac{23}{11}\times 10^{-3}$$

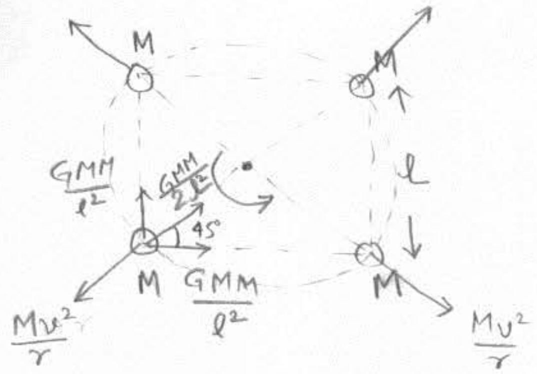
$$\therefore \quad \alpha(\Delta T) = \frac{23}{11}\times 10^{-3}$$

$$\Delta T = \frac{23}{11}\times 10^{-3}\ \frac{1}{17\times 10^{-6}}\ {}^\circ C = 122.9\ {}^\circ C$$

Breaking temperature, $T_B = (300 - 122.9)\ {}^\circ C \approx \underline{177\ {}^\circ C}$ (3 sig figs)

Answer

(6)



length of diagonal, $d = \sqrt{\ell^2+\ell^2} = \sqrt{2}\,\ell$

$$2\frac{GM^2}{\ell^2}\cos 45^\circ + \frac{GM^2}{2\ell^2} = \frac{\cancel{M}v^2}{\sqrt{2}\ell/2}$$

$$2\frac{GM}{\ell^2}\frac{1}{\sqrt{2}} + \frac{GM}{2\ell^2} = \frac{\sqrt{2}v^2}{\ell}$$

$$\sqrt{2}\frac{GM}{\ell} + \frac{GM}{2L} = \sqrt{2}\,v^2$$

$$v^2 = \frac{1}{\sqrt{2}}\frac{GM}{\ell}\left(\sqrt{2}+\frac{1}{2}\right) \Rightarrow \boxed{v = \sqrt{\frac{1}{\sqrt{2}}\frac{GM}{\ell}\left(\sqrt{2}+\frac{1}{2}\right)}}$$

$$v = \sqrt{\frac{GM}{\ell}\left(1+\frac{1}{2\sqrt{2}}\right)}$$ Answer

$$I = 4M\left(\frac{\sqrt{2}\ell}{2}\right)^2 = \cancel{4}^{2}M\frac{\ell^2}{\cancel{2}} = 2M\ell^2$$ Answer.

(7)

$L = 3\,m$, $\quad \mu = 0.0025\ kg/m$

$f_n = 252\ Hz \qquad f_{n+1} = 336\ Hz$

let v be The velocity of wave in the string.

$$v = f_1 \lambda_1 = f_n \lambda_n$$

$v = \sqrt{\frac{F}{\mu}}$ (also) $\qquad \frac{\lambda_1}{2} = L \Rightarrow \lambda_1 = 2L$

$$n\left(\frac{\lambda_n}{2}\right) = L$$

$$\lambda_n = \frac{2L}{n}$$

Thus, $\frac{f_n}{f_{n+1}} = \frac{252}{336} = \frac{3}{4} = \frac{\cancel{v}n/2L}{\cancel{v}(n+1)/2L} = \left(\frac{n}{n+1}\right) \longleftarrow f_n = \frac{v}{\lambda_n} = \frac{vn}{2L}$

Thus, $\boxed{n = 3}$ and $(n+1) = 4$

Now,

$f_1 \lambda_1 = f_n \lambda_n \Rightarrow f_1 = \frac{f_n \lambda_n}{\lambda_1} = \frac{f_3 \lambda_3}{\lambda_1} = 252 \times \frac{(2L/3)}{(2L)} = \frac{252}{3}$

(for $n = 3$) $\qquad [\therefore 3f_1 = f_3]$

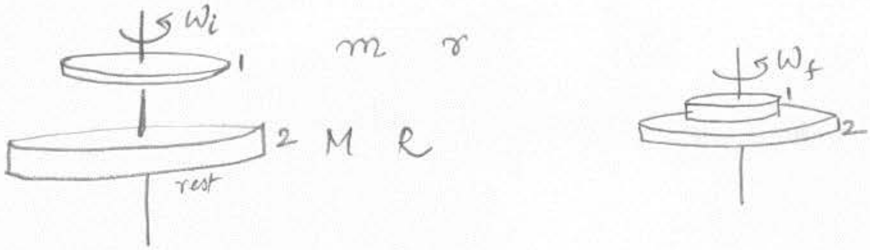
$$\boxed{f_1 = 84\ Hz}$$

$v = \sqrt{\frac{F}{\mu}} = f_1 \lambda_1 = 84 \times 2L = 84 \times 2 \times 3 = 504.$

$\therefore F = (504^2)\,\mu = 504^2 \times 0.0025 = \underline{\underline{635\ N}}$

$\therefore$ $(635\,N < 700\,N)$ the tension is $< 700\,N$ so the wire is a good choice.

(8)



Conservation of angular momentum

$$I_1 \omega_i + I_2 (0) = (I_1 + I_2)\,\omega_f$$

$$\cancel{\tfrac{1}{2}} m r^2 \omega_i = \left(\cancel{\tfrac{1}{2}} m r^2 + \cancel{\tfrac{1}{2}} M R^2\right)\omega_f \quad \Rightarrow \quad \boxed{\omega_f = \frac{(m r^2)\,\omega_i}{m r^2 + M R^2}}$$

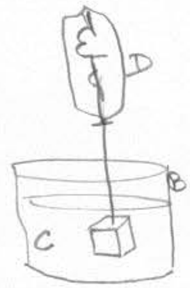
$$K_i = \frac{1}{2} I_1 \omega_i^2 = \frac{1}{2}\left(\frac{1}{2} m r^2\right)\omega_i^2$$

$$K_f = \frac{1}{2}(I_1 + I_2)\,\omega_f^2 = \frac{1}{2}\left(\cancel{\tfrac{1}{2}} m r^2 + \frac{1}{2} M R^2\right)\omega_f^2$$

$$\frac{K_f}{K_i} = \frac{(m r^2 + M R^2)\,\omega_f^2}{m r^2 \omega_i^2} = \frac{\cancel{(m r^2 + M R^2)}\left[(m r^2)^{\cancel{2}}\,\cancel{\omega_i^2} \big/ (m r^2 + M R^2)^{\cancel{2}}\right]}{\cancel{m r^2}\,\cancel{\omega_i^2}}$$

$$\boxed{\frac{K_f}{K_i} = \frac{(m r^2)}{m r^2 + M R^2}}$$ Answer.

(9)



$m_B = 1.00$ kg

$m_c = 1.80$ kg

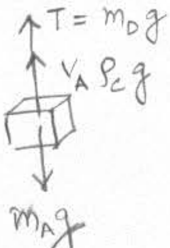
$m_D = 3.50$ kg

$m_E = 7.50$ kg

$V_A = 3.80 \times 10^{-3}\ m^3$

(a) ρ_c ? (b) $m_{Di} = ?$

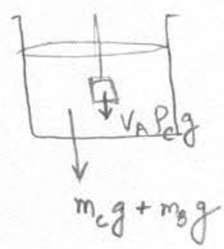
$m_{Ei} = ?$



$$m_D \cancel{g} + V_A \rho_c \cancel{g} = m_A \cancel{g}$$

$$m_D + \underbrace{V_A \rho_c = m_A}_{\text{unknowns.}} \quad \text{— (i)}$$

$m_A = m_{Di}$ (reading of D initially, when block out of liquid)



$$m_E = (m_c + m_B) + V_A \rho_c \quad \text{— (ii)}$$

and $m_{Ei} = m_c + m_B = (1.80 + 1.00)$ kg $= \underline{\underline{2.80 \text{ kg}}}$

from (ii) $\rho_c = \dfrac{m_E - (m_c + m_B)}{V_A} = \dfrac{7.50 - (2.80)}{3.80 \times 10^{-3}} = 1.24 \times 10^3\ kg\,m^{-3}$

(a) density of liquid, $\boxed{\rho_c = 1.24 \times 10^3\ kg\,m^{-3}}$ Answer

(b) $m_{Di} = m_A = m_D + V_A \rho_c = 3.50 + 3.80 \times \cancel{10^{-3}} \times 1.24 \times \cancel{10^{3}} = 8.21$ kg

$m_{Ei} = m_c + m_B = 2.80$ kg

Scale D showed 8.21 kg when block is out of liquid

scale E showed 2.80 kg

Answer.

(10) Conservation of angular momentum.

(a) $$I_m \omega_i = (I_m + I_R)\,\omega_f$$

I_m = moment of inertia of man about pivot $= md^2$

$$I_m + I_R = md^2 + \frac{1}{3}ML^2$$

$$I_R = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = ML^2\left(\frac{1}{12}+\frac{1}{4}\right) = \frac{\cancel{1+3}}{\cancel{12}_{\,3}}ML^2 = \frac{1}{3}ML^2$$

$$\omega_f = \frac{(md^2)\,\omega_i}{\left(md^2 + \frac{1}{3}ML^2\right)} \qquad \omega_i = \frac{v}{d}$$

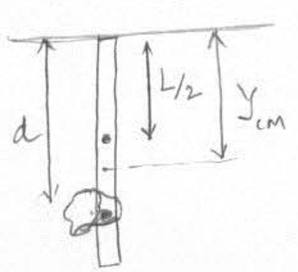
$$\omega_f = \frac{md^{\cancel{2}}\, v/\cancel{d}}{md^2 + \frac{1}{3}ML^2} = \left(\frac{mvd}{md^2 + \frac{1}{3}ML^2}\right)$$

$$\frac{K_i}{K_f} = \frac{\cancel{\frac{1}{2}} I_m \omega_i^2}{\cancel{\frac{1}{2}}(I_m + I_R)\,\omega_f^2} = \frac{md^{\cancel{2}}\, v^2/\cancel{d^2}}{\cancel{\left(md^2 + \frac{1}{3}ML^2\right)}\left(\frac{mvd}{md^2 + \frac{1}{3}ML^2}\right)^2} = \frac{\cancel{m}\cancel{v^2}\left(md^2 + \frac{1}{3}ML^2\right)}{m^{\cancel{2}}\cancel{v^2}d^2}$$

$$\frac{K_i}{K_f} = \frac{md^2 + \frac{1}{3}ML^2}{md^2} = 1 + \frac{1}{3}\left(\frac{M}{m}\right)\left(\frac{L}{d}\right)^2$$

$$\therefore \boxed{\frac{K_f}{K_i} = \frac{1}{1 + \frac{1}{3}\left(\frac{M}{m}\right)\left(\frac{L}{d}\right)^2}}$$ Answer.

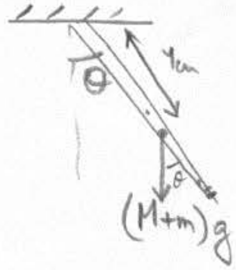
(b)



$$y_{cM} = \frac{\left(\frac{L}{2}\right)M + dm}{(M + m)} = \frac{md + \frac{ML}{2}}{(M + m)}$$

$$\boxed{Y_{cM} = \frac{md + ML/2}{(M + m)}}$$

(c)



$$I \frac{d^2\theta}{dt^2} = -(M+m)g\, y_{cm} \sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{(M+m)g}{\left(md^2+\frac{1}{3}ML^2\right)} y_{cm}\,\theta$$

$$\therefore \omega^2 = \frac{(M+m)g\,y_{cm}}{md^2+\frac{1}{3}ML^2} \Rightarrow \boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{md^2+\frac{1}{3}ML^2}{\cancel{(M+m)}g} \times \frac{\cancel{(M+m)}}{\left(md+\frac{ML}{2}\right)}}}$$

$$T = 2\pi\sqrt{\frac{md^2+\frac{1}{3}ML^2}{g\left(md+ML/2\right)}}$$

(11) at max height, h $v=0$

$$0 = u - gt \Rightarrow u = gt$$

$$0 = u^2 - 2gh \Rightarrow h = \frac{u^2}{2g} = \frac{g^{\cancel{2}}t^2}{2\cancel{g}} = \frac{1}{2}gt^2$$

$$h = \frac{1}{2}g\left(\frac{t_{f1y}}{2}\right)^2 = \frac{1}{2}(9.8)\times 4^2 = \underline{\underline{78.4\ m}}$$ Answer.

(12) $E_i = \frac{1}{2}mv^2$

$E_f = \frac{1}{2}kx_{max}^2$

a) $E_i = E_f \Rightarrow x_{max}^2 = \frac{mv^2}{k} \Rightarrow \boxed{x_{max} = \sqrt{\frac{mv^2}{k}}}$ Answer.

b) $E_i = \frac{1}{2}mv^2$

$E_f = \frac{1}{2}kx^2 + \frac{1}{2}mv_x^2$

$\cancel{\frac{1}{2}}kx^2 + \cancel{\frac{1}{2}}mv_f^2 = \cancel{\frac{1}{2}}mv^2 \Rightarrow v_f^2 = \frac{mv^2 - kx^2}{m}$

$\boxed{v_x^2 = v^2 - \left(\frac{k}{m}\right)x^2}$ Answer