Classical Mechanics.

(LECTURER: ARIJIT BOSE)

All lectures and homeworks can be found at the course webpage http://www.pas.rochester.edu/~arijit.

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Office : Gorgen Building, 5th Floor, room 513.

Office hours : Monday 1pm - 2pm.

Workshops : Tuesday or Thursday (1pm - 3pm)

We will discuss and work on solving problems

NOTE: Make sure that you are registered for the workshop.

PREREQUISITES : (i) Basic Algebra [example: quadratic eqn solving solving from an equation

(ii) Basic calculus → nature of functions / function plots
→ basic differentiation
→ basic integration

(iii) geometry & trigonometry.

Take a look at the "Useful Mathematical Formulas" posted online

COURSE OUTLINE : PHY 121: Mechanics :

(i) Measurement & Estimation
(ii) Motion in one dim.
(iii) Kinematics in 2-3 Dim.
(iv) Newtons Laws
(v) Friction
(vi) Gravitation
(vii) Work, Energy & energy conservation
(viii) linear momentum
(ix) Circular motion, rotation, rigid body dynamics
(x) Fluids
(xi) Oscillations / waves
(x) Thermodynamics basics.

ATTENDANCE : Importance of class & workshop attendance

The Nature of Science.

Science is the study and description of patterns of nature. In nature there is a large variety of phenomenon that happen around us. What we do in science is to seek for regular patterns to describe these diverse phenomenon.

For an instance, an apple falling from a tree and the earth rotating around the Sun can appear to be (completely different) diverse pheomenon, but both of these can actually be explained by a single regularity ("law of nature"), in this case the law of gravitation.

So, science is the search for the 'regularity' (or uniform pattern) behind the different random phenomenon that happen in nature.

The next question is, what is Physics ?

Physics is the most basic of the sciences. It deals with the behaviour and structure of matter. In physics we think in terms of motion (distance, velocity, acceleration, momentum), forces (and force fields) and energy.

What we do in physics is to observe nature and explain (using models) i.e. try to figure out the "laws of nature". In other words we try to point at regularities that we happen to find around us. But, nature is an extremely complicated model and hence our attempts to understand nature becomes more and more involved.

What we do in this course is to understand and explain mathematically (or logically) the Mechanics of what we see around us. That is we try to understand events like the mechanism of skiing or how to bowl in a baseball game etc.

Measurement and Uncertainty; Significant figures.

In the attempt to understand the world around us we need to measure and compare different physical quantities.

A common example is the speedometer of your car that measures and lets you know the speed at which you are driving.

What is uncertainty?

Whenever we make a measurement (for example let us try to measure the length of a sheet of paper with a centimeter ruler) we always involve some uncertainty in the measurement (because the centimeter ruler can measure precisely upto 0.1 cm, the smallest division on the ruler). If the actual width of the paper is between 21.6 cm and 21.7 cm (i.e. slightly more than 21.6 cm and slightly less than 21.7 cm) Then using a centimeter ruler would involve an uncertainty of 0.1 cm which is the <u>least count</u> of the ruler. And would write the width of the paper as $(21.6 \pm \underbrace{0.1}_{\text{uncertainty}})$ cm

And we can calculate the percent uncertainty as

$$\text{percent uncertainty} = \frac{0.1}{21.6} \times 100\% \approx 0.5\%$$

$$\left(\%\,\text{uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%\right)$$

[NOTE: When you do experiments in the lab you will need to find the uncertainty when you use different measuring instruments. In most cases (unless specified) the uncertainty of a measurement using an instrument is taken to be the least count of that instrument. For example: if you measure temperature using a thermometer that has divisions of 0.1°F (= its least count) then the uncertainty is ±0.1°F]

Significant figures.

Let us consider the width of the paper example once again. We see that the width is actually between 21·6 cm and 21·7 cm. So, we can write it as $21\cdot 6 \pm 0\cdot 1$ cm or $21\cdot 7 \pm 0\cdot 1$ cm; both of them are correct!! Hence, we see that when we use a centimeter ruler the last digit (circled digit) is the least significant digit and the first digit is the most significant one (we don't have any uncertainty with the fact that the first digit is 2)

Thus, the numbers of significant figures in this result $(21\cdot 6 \pm 0\cdot 1\text{cm})$ is 3. Now, suppose we use a ruler that has 1 cm as its least count to measure the width; we will find that the width lies between 21 cm and 22 cm. Therefore we can write the width as $(\underline{2}\,\underline{1} \pm 1)$cm. What is the number of significant figures in this result ?? It is 2.

Thus, we see that the number of significant figures depend upon the least count of the instrument we use to measure.

If someone says that the distance between two cities is roughly 80 km, then we understand that 8 is the only significant digit and if we say that the distance is 80 km that means it is 80 ± 1 km, now there are 2 significant digits. If the distance is written as 80·0 ± 0·1 km, this means that there are 3 significant digits.

When we add, subtract, multiply or divide two numbers the result should have only as many significant figures as the number of significant figures used in the calculation with the least number of significant figures.

For example: $\underbrace{10\cdot 0 \text{ cm}}_{3 \text{ sig fig}} \div \underbrace{3\cdot 0 \text{ cm}}_{2 \text{ sig fig}} = \underbrace{3\cdot 3 \text{ cm}^2}_{2 \text{ sig fig only}}$ [on a calculator it will show 3·3333.... but you cannot keep all the extra terms because they are insignificant.]

NOTE: $5\cdot 0\text{cm} \times 500\text{cm} = \underbrace{2\cdot 5}_{} \times 10^2 \text{ cm}^2$

2 significant figures only → YOU can see the importance of Scientific Notation.

An idea you will find useful in labs.

(a) Divide 2·0 by 3·0 the calculator show 0·6666... but the answer that you write can have only 2 sig. fig., thus, 0·67

Suppose you need to calculate, $\frac{2.0}{3.0} \times 11 =$ (7.3333.... on calculator) 7·3 (2 sig fig.)

but if you round off $2.0 \times 3.0 = 0.67$
and then calculate $0.67 \times 11 = (7.37)$ 7·4 (upto 2 sig. fig.)

The idea is, if you are using a calculator; round off only at the last step.

(b) $2.5 \times 3.2 =$ (8 on a calculator)
but you should write it as 8·0 [since, it has 2 significant figures]

scientific notation

We commonly write numbers as powers of 10 in scientific notation.
For example 36,900 is written as 3.69×10^4 or 0.00015 is 1.5×10^{-4}

[One advantage of scientific notation is it allows us to clearly express the number of significant figures.

eg: 36,900 has three, four or five significant figures!!
With scientific notation the ambiguity can be avoided.
i.e. if it is known that the number has 3 sig. fig. then we write it as 3.69×10^4
if we know that it has 4 sig. fig then write as $\underbrace{3.690}_{} \times 10^4$

general: Number of digits here give the no. of significant fig.

Accuracy and precision

Precision means repeatability of the measurement.
Accuracy refers to how close a measurement is to the true value.

System of units, standard SI system

The measurement of any quantity is made with respect to a particular standard of unit which is internationally accepted and used.

The SI system uses meter for Length, seconds for Time and kilogram for mass.

What length exactly is 1 metre is accepted internationally and it is the standard of length, there is a platinum iridium rod in the French Academy of sciences that represents this length.

1 meter is defined as the length of path travelled by light in vacuum during a time interval of (1/299,792,458) second

Similar standards for time (seconds) and mass (kilogram) exists.

Converting Units.

If a car is moving at 20 miles per hour (mi/h) what is its speed in

(a) m/sec (b) km/hr

Conversion factor: 1 inch = 2.54 cm and 5280 ft = 1 mile.
and 12 inch = 1 ft.

$$\therefore \quad 1\ mi = (5280\ \cancel{ft})\left(12\ \frac{\cancel{in}}{\cancel{ft}}\right)\left(2.54\ \frac{\cancel{cm}}{\cancel{in}}\right)\left(\frac{1\ m}{100\ \cancel{cm}}\right) = 1609\ m$$

$$1\ hour = (60\ \cancel{min})\left(60\ \frac{sec}{\cancel{min}}\right) = 3600\ sec$$

$$20\ \frac{mi}{hr} = \left(20\ \frac{mi}{hr}\right)\left(1609\ \frac{m}{mi}\right)\left(\frac{1}{3600}\ \frac{h}{sec}\right) = 8.9\ \frac{m}{s} \quad \text{(2 sig fig's)}$$

(b) $$20\ \frac{mi}{hr} = \left(20\ \frac{\cancel{mi}}{hr}\right)\left(1609\ \frac{\cancel{m}}{\cancel{mi}}\right)\left(\frac{1}{1000}\ \frac{km}{\cancel{m}}\right) = 32\ \frac{km}{hr} \quad \text{(2 sig fig's)}$$

Order of magnitude estimate

eg: Estimate the thickness of a page of a book.

We use a 1000 page book and measure its thickness and find it to be 3.55 cm.

1000 page book has 500 sheets, front and back.

$$\therefore \text{thickness of page} = \frac{3.55\,\text{cm}}{500} = 7.1 \times 10^{-3}\,\text{cm} \approx 10^{-2}\,\text{cm}$$

(order estimate)

Dimensions and Dimensional Analysis

When we talk about dimensions of a quantity, we refer to the type of base units that make it up.

Three basic units:

1. Length $[L]$
2. Time $[T]$
3. Mass $[M]$

What is the dimension of area? $[L^2]$

velocity? $[L]/[T] = LT^{-1}$

Use of dimentional analysis:

(i) It tells you whether the formulae you wrote is incorrect or not (dimentionally incorrect)

i.e. dimention on both side of equation must match

suppose we write down $v = u + at$

dimension on LHS $= LT^{-1}$

for RHS, $u + at = [LT^{-1}] + [LT^{-2}][T] = LT^{-1} + LT^{-1}$

both terms have same dimension and matches with LHS

so formulae is correct.

(ii) Suppose you don't know the exact formulae for the time period of a pendulum. [Time period means, time taken for one complete oscillation i.e. to & fro motion]

What can the Time period (T) depend on ?

(a) Length of the string, l

(b) acceleration due to gravity, g

(c) mass of pendulum, m

write T as,

$$T = l^a g^b m^c$$

, where a, b, c are powers of the dependant quantities

LHS, dimension T

RHS, $L^a [LT^{-2}]^b M^c$

$$L^{a+b} T^{-2b} M^c = T$$

$\therefore$ $a+b=0 \Rightarrow a=-b$

$-2b = 1 \Rightarrow b = -\frac{1}{2}$

$c = 0$

$\therefore a = \frac{1}{2}$

$b = -\frac{1}{2}$

$$T \approx \frac{l^{1/2}}{g^{1/2}} = \underset{\downarrow}{c}\sqrt{l/g}$$

some dimensionless constant which we can't find using dimensional analysis