The Conditions for Equilibrium.

first condition.

$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0.$

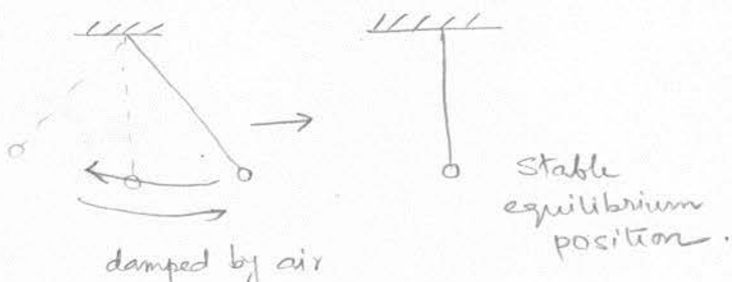
Second Condition

$\sum \tau = 0$

example: The stationary stick problem we did in last lecture.

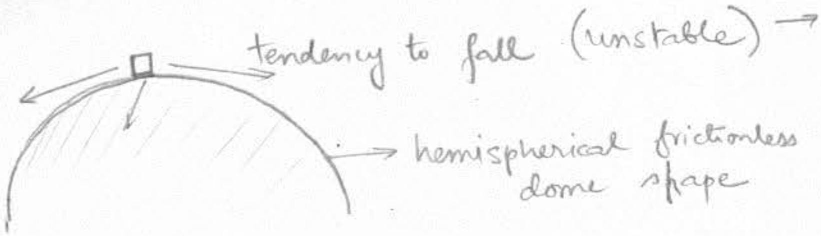
Stability and Equilibrium

(i) Stable equilibrium



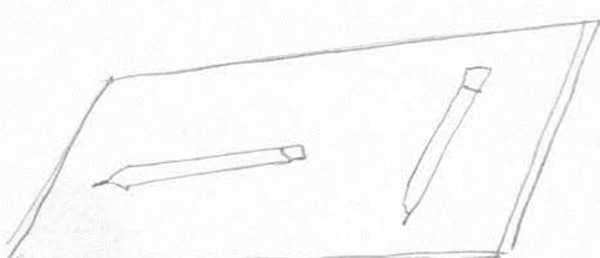
If the pendulum is slightly disturbed from equilibrium position, it will come back to it.

(ii) Unstable equilibrium



if it is slightly disturbed if will never get back the equilibrium position

(iii) Neutral equilibrium

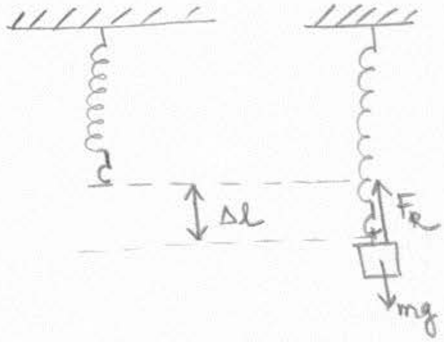


It doesn't matter how you place the penuil on a table it will remain at its position.

If disturbed from equilibrium, it attains equilibrium at the new position.

Elasticity → Stress and Strain

Hook's Law.



$F_R \propto \Delta l$ (Hook's Law).

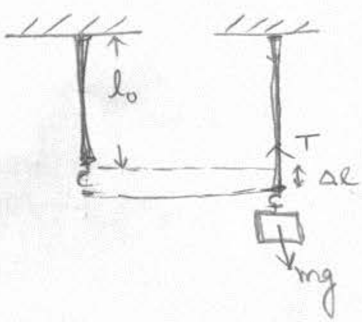
For equilibrium

$$mg = F_R \left(= \underbrace{k}_{\text{spring constant}} \Delta l\right)$$

$$\therefore mg = k \, \Delta l$$

$$\boxed{\Delta l = \frac{mg}{k}}$$

Think about a string instead of a spring.



Area of c.s. of string be A.

Define: $\text{stress} = \dfrac{F}{A} = \left(\dfrac{mg}{A}, \text{ in this case}\right)$ $\dfrac{\text{force}}{\text{area}}$

$$\text{strain} = \frac{\Delta l}{l_0} \quad (\text{no units}).$$

Young's modulus, is a constant for the material of the string.

$$Y = \frac{\text{stress}}{\text{strain}}$$

[Actually, $\text{strain}\left(\dfrac{\Delta l}{l}\right) \propto \text{stress}\left(\dfrac{F}{A}\right)$

$\text{stress} = \underbrace{Y}_{\text{const.}} \text{ strain}$]

$$\boxed{Y = \frac{F/A}{\Delta l / l}} \qquad [Y] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2} = \underline{ML^{-1}T^{-2}} \quad \left(\frac{N}{m^2} \text{ units}\right)$$

$$[\text{stress}] = \frac{[F]}{[A]} = ML^{-1}T^{-2} \quad ; \quad [\text{strain}] = \frac{[\Delta l]}{[l]} = \underline{1}.$$

In this example:

$$\therefore \quad \frac{\Delta l}{l} = \frac{F}{AY} \quad \Rightarrow \quad \Delta l = \frac{mg\,l}{A\,Y} = mg\left(\frac{l}{AY}\right)$$

Compare with Hook's Law: $\Delta l = \dfrac{mg}{k} = mg\left(\dfrac{l}{AY}\right)$

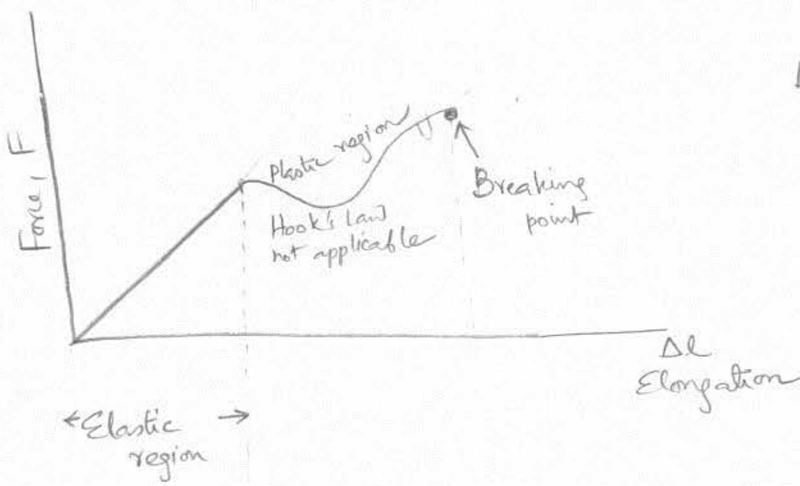
$$\therefore \quad \boxed{k = \frac{AY}{l}}$$

constant for a string (coefficient of expansion)

Applied force vs. elongation for a typical metal under tension

We saw, $\Delta l \propto F$ [Within Elastic region]

elongation $\propto$ applied force.



$$\Delta l = F\left(\frac{l}{AY}\right)$$

slope

example: $Y_{iron} = 1 \times 10^{11}\ \frac{N}{m^2}$

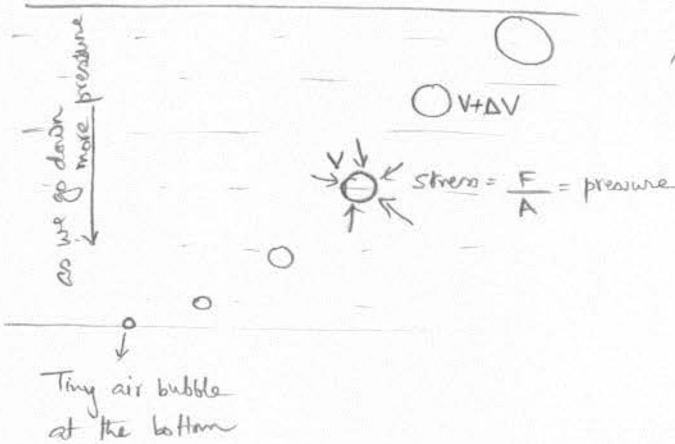
$Y_{nylon} = 5 \times 10^{9}\ \frac{N}{m^2}$

↑ More rigid

Y greater means harder to deform/elongate.

Bulk Modulus.

Example.



↑ Air bubble grows in size as it rises up through liquid; why?

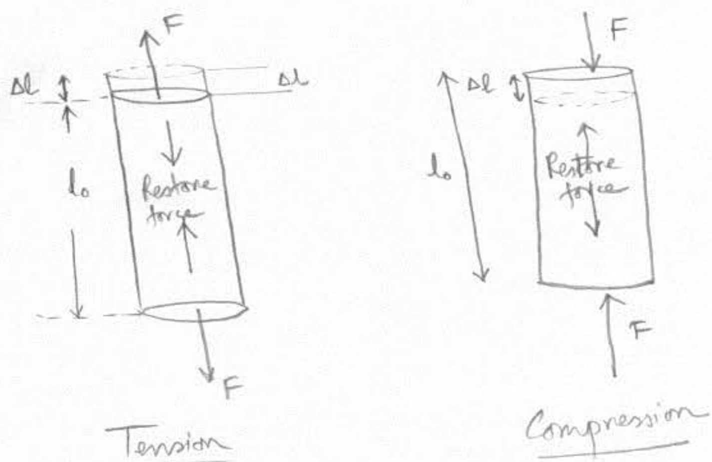
$$\text{stress} = \frac{F}{A}$$

$$\text{strain} = -\frac{\Delta V}{V} \quad (\text{-ive, compression})$$

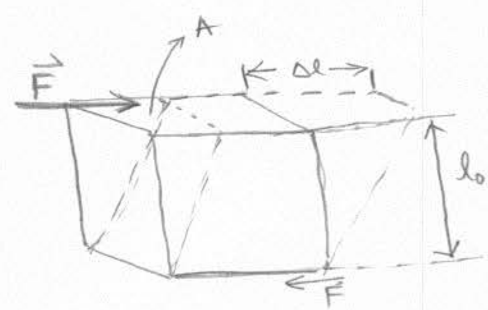
$$\text{Bulk modulus, } B = \frac{\text{volume stress}}{\text{volume strain}} = \frac{F/A}{-\Delta V/V}$$

$$\boxed{B = \frac{-P}{(\Delta V/V)}}$$

B is +ive quantity; the -ive to take care of ΔV being negative.



Restore force manages to balance applied F after the expansion | compression is over.

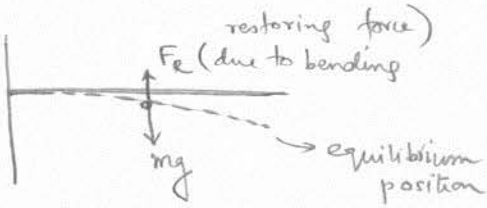


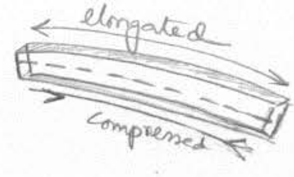
$$\text{stress} = F/A$$

$$\text{strain} = \frac{\Delta l}{l_0}$$

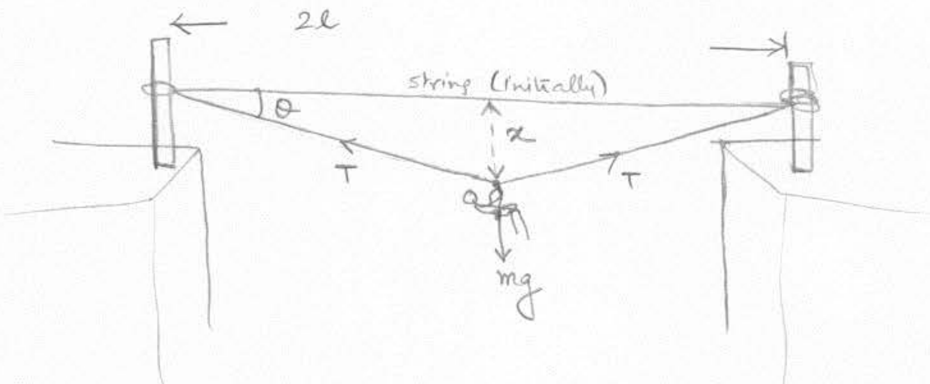
$$\sigma = \frac{\text{stress}}{\text{strain}} = \left(\frac{F}{A}\right)\left(\frac{l_0}{\Delta l}\right)$$

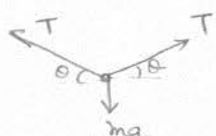
Bending of beams.





Problem : find Y ? (A = area of c.s. of rope)





$$2T\sin\theta = mg$$

$$\Delta l = 2\sqrt{l^2+x^2} - 2l$$

$$\text{Strain} = \frac{\Delta l}{l} = \frac{2\sqrt{l^2+x^2} - 2l}{2l}$$

$$\frac{\Delta l}{l} = \sqrt{1+\left(\frac{x}{l}\right)^2} - 1$$

$$\text{stress} = \frac{T}{A} = \frac{mg}{2\sin\theta}\,\frac{1}{A}$$

$$Y = \frac{mg}{2\sin\theta}\,\frac{1}{A}\,\frac{1}{\sqrt{1+\left(\frac{x}{l}\right)^2}-1} = \frac{mg}{2A\sin\theta\left(\sqrt{1+\frac{x^2}{l^2}}-1\right)}$$

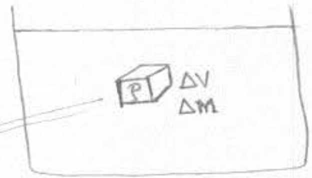
$$\sin\theta = \frac{x}{\sqrt{l^2+x^2}}$$

$$\sin\theta = \frac{x/l}{\sqrt{1+\left(\frac{x}{l}\right)^2}}$$

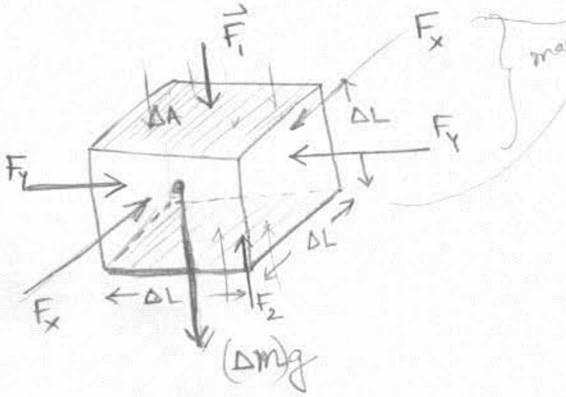
$$\boxed{Y = \frac{mg}{2A\,x/l\left(1-\frac{1}{\sqrt{1-x^2/l^2}}\right)}}$$

FLUIDS.

Density of a fluid : $\rho = \dfrac{\Delta m}{\Delta V}$



pressure, $p = \dfrac{\text{Thrust}}{\text{Area}}$



$F_1 + \Delta m g = F_2$ (for vertical equilibrium)

$\Rightarrow F_2 - F_1 = (\Delta m) g$

$P_1 = \dfrac{F_1}{\Delta A}$ $\quad P_2 = \dfrac{F_2}{\Delta A}$ $\quad P_x = \dfrac{F_x}{\Delta A}$ $\quad P_y = \dfrac{F_y}{\Delta A}$

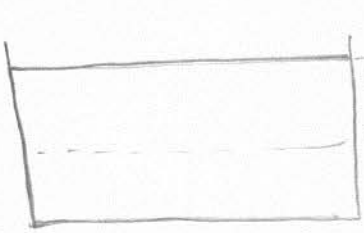
$P_2 \Delta A - P_1 \Delta A = \Delta m \, g$

$P_2 - P_1 = \dfrac{\Delta m \, g}{\Delta A} = \dfrac{\Delta m \, g \, \Delta L}{\Delta A \, \Delta L} = \left(\dfrac{\Delta m}{\Delta V}\right) g \, \Delta L$

$P_2 - P_1 = \rho g \Delta L$

if ΔL is a height difference h

$\therefore \boxed{P_2 - P_1 = \rho g h}$

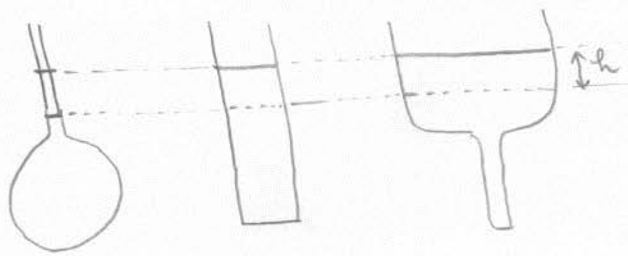


→ pressure on surface = atmospheric pressure (P_0)

→ pressure at depth $h = P_0 + \rho g h$

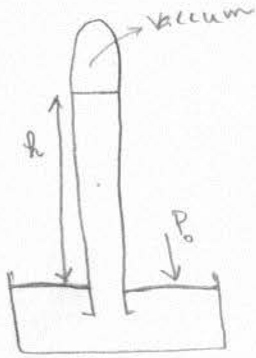
density of liquid

Compare:



→ pressure at depth h is $p = P_0 + \rho g h$ for all the vessels

Mercury Barometer to measure pressure.



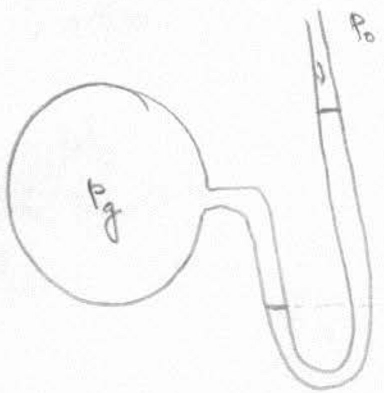
ρ = density of mercury

P_0 = atmospheric pressure

$$P_0 = \rho g h$$

Note: The shape of the tube doesn't matter. We use this simple thin tube to minimize the amount of mercury needed.

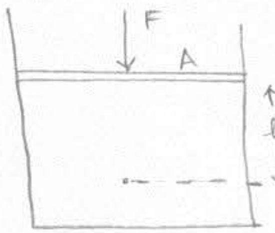
Measuring pressure in a tank.



Manometer

Pascal's Principle.

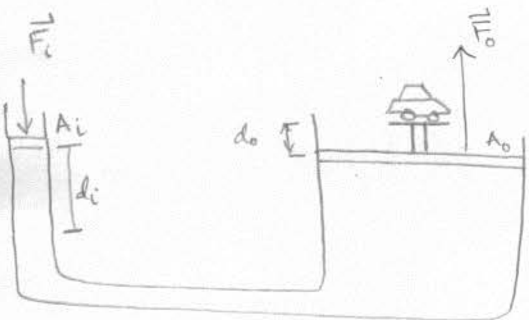
A change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and on the walls of its container.



$P_{ext} = F/A$.

$p = P_{ext} + \rho g h$

Hydraulic lever.



$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o}$$

$$F_o = \left(\frac{F_i}{A_i}\right) A_o \quad > F_i \quad (\because A_o > A_i)$$

May sound counterintuitive

But, Volume of liquid shifted (conserved)

$$V = A_i d_i = A_o d_o \quad \Rightarrow \quad d_o = d_i \left(\frac{A_i}{A_o}\right) < d_i \quad \because A_i < A_o$$

Conservation of energy : Work input = Work output.

$$W = F_o d_o = \left(\frac{F_i}{A_i}\right) A_o \, d_i \left(\frac{A_i}{A_o}\right) = F_i d_i$$

With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance.