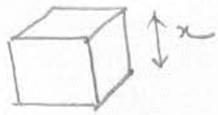
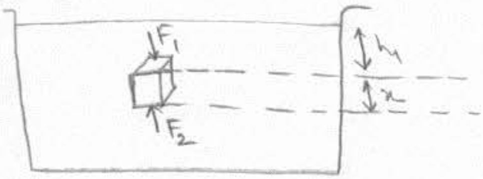
Archimedes's Principle.

Buoyant force, $\vec{F}_b$

eg: Hollow plastic cube



$V = x^3$



$P_1 = P_0 + h_1 \rho g$

$P_2 = P_0 + (h_1 + x)\rho g$

ρ = density of water

pressure diff = $P_2 - P_1 = x \rho g$

$F_2 - F_1$ = Net upward (Bouyant) force = $(P_2 - P_1)\, x^2 = x^3 \rho g = V\rho g$.

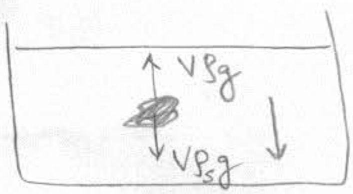
$F_b = V \rho g = m_f g$ → mass of fluid displaced by the body

V → volume of water displaced by the body

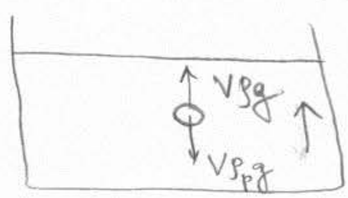
ρ → density of water

Archimedes principle: When a body is partially / completely submerged in a fluid, it experiences an upthrust which is equal to the weight of the fluid displaced by the immersed part of the body.

Now,



ρ = density of liquid



Stone (volume, V)

$V\rho_s = m_s$

stone sinks

Plastic hollowshell (vol, V)

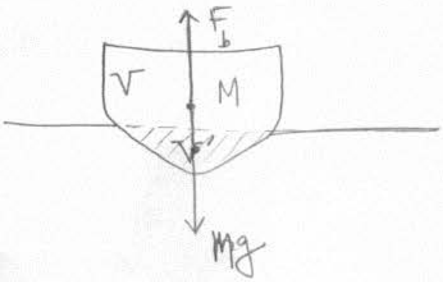
$V\rho_p = m_p$

it rises.

Floating

When a body floats in a fluid, the magnitude F_b of the buoyant force on the body is equal to the magnitude F_g of the gravitational force on the body.

$$F_b = F_g$$

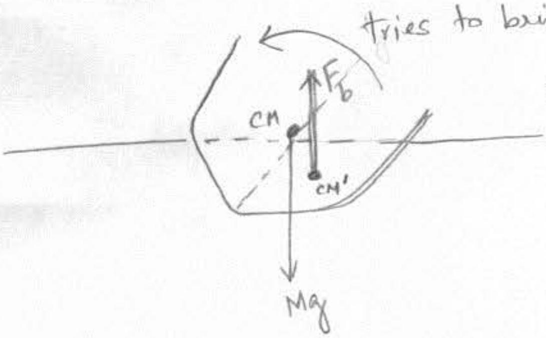


$$F_g = F_b$$

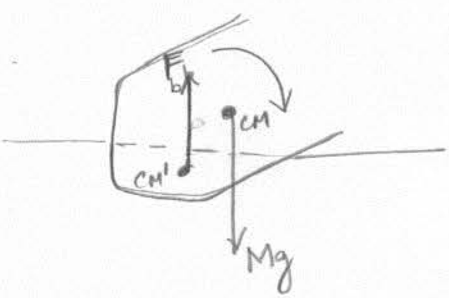
$$F_g = V \rho_b g = Mg$$

$$F_b = V' \rho g$$

tries to bring back to original position (torque)



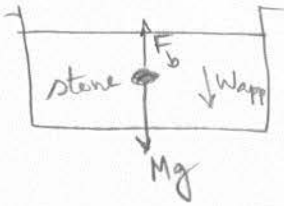
slightly tilted



body topples.
heavy tilt.

Apparent Weight in a Fluid.

$$W_{app} = \text{weight} - F_b$$



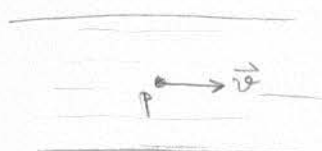
$$W_{app} = Mg - F_b$$

Ideal fluids in Motion

dynamics of fluid.

ideal fluid.

(i) Steady flow :

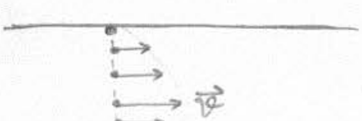


Velocity at any pt P is always constant (Laminar flow)

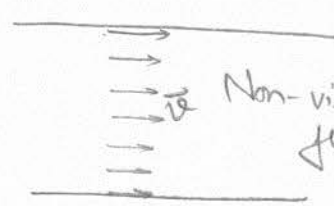
(ii) Incompressible flow

(iii) Non-viscous flow :

Viscocity → how risistive the fluid is to flow
↓
(is The reason behind the drag force its like friction.)



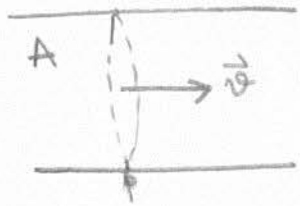
← Viscous fluid



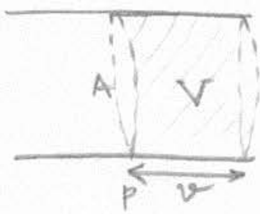
Non-viscous fluid

(iv) Irrotational fluid (no terbulance)

The Equation of Continuity.

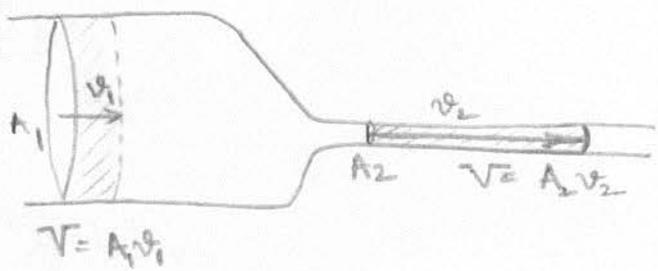


in unit time
Volume that
crosses A
is vA



$V = vA$

Volume that crosses any area of cross section per unit time (vA) is a constant for a fluid flow without any source or sink.



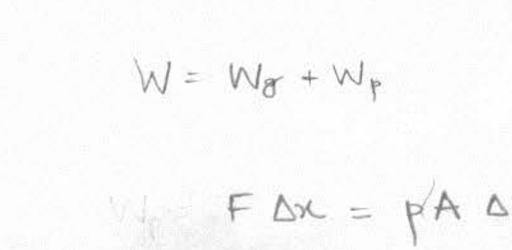
$$V = \boxed{A_1 v_1 = A_2 v_2} = \text{constant}$$

Equation of Continuity.

[* Incompressible fluid]

Bernoulli's Equation.

Conservation of Energy:



Changes at the input & output end

$$W = \Delta K$$

$$\Delta K = \frac{1}{2}\Delta m\, v_2^2 - \frac{1}{2}\Delta m\, v_1^2$$

$$= \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2)$$

$$W = W_g + W_p$$

$$W_g = -\Delta m\, g (y_2 - y_1)$$
$$= -\rho g\, \Delta V\, (y_2 - y_1)$$

$$F\Delta x = pA\,\Delta x) = p\,\Delta V \Rightarrow W_p = -p_2 \Delta V + p_1 \Delta V$$
$$= -(p_2 - p_1)\Delta V$$

$$W = W_g + W_p = \Delta K \Rightarrow -\rho g\, \Delta V (y_2 - y_1) - \Delta V (p_2 - p_1) = \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2)$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 = \text{constant}$$

Check: $v_1 = v_2 = 0$ fluid at rest.

$$P_1 + \rho g y_1 = P_2 + \rho g y_2 \quad \text{(True)}$$

if $y_1 = y_2 \Rightarrow P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

speed & pressure compensate each other.

Oscillations.

Motion of particles

(i) Translational (ii) Rotational (iii) Vibrational.

- Oscillations
  - ideal oscillations
  - damped oscillations
  - forced oscillations

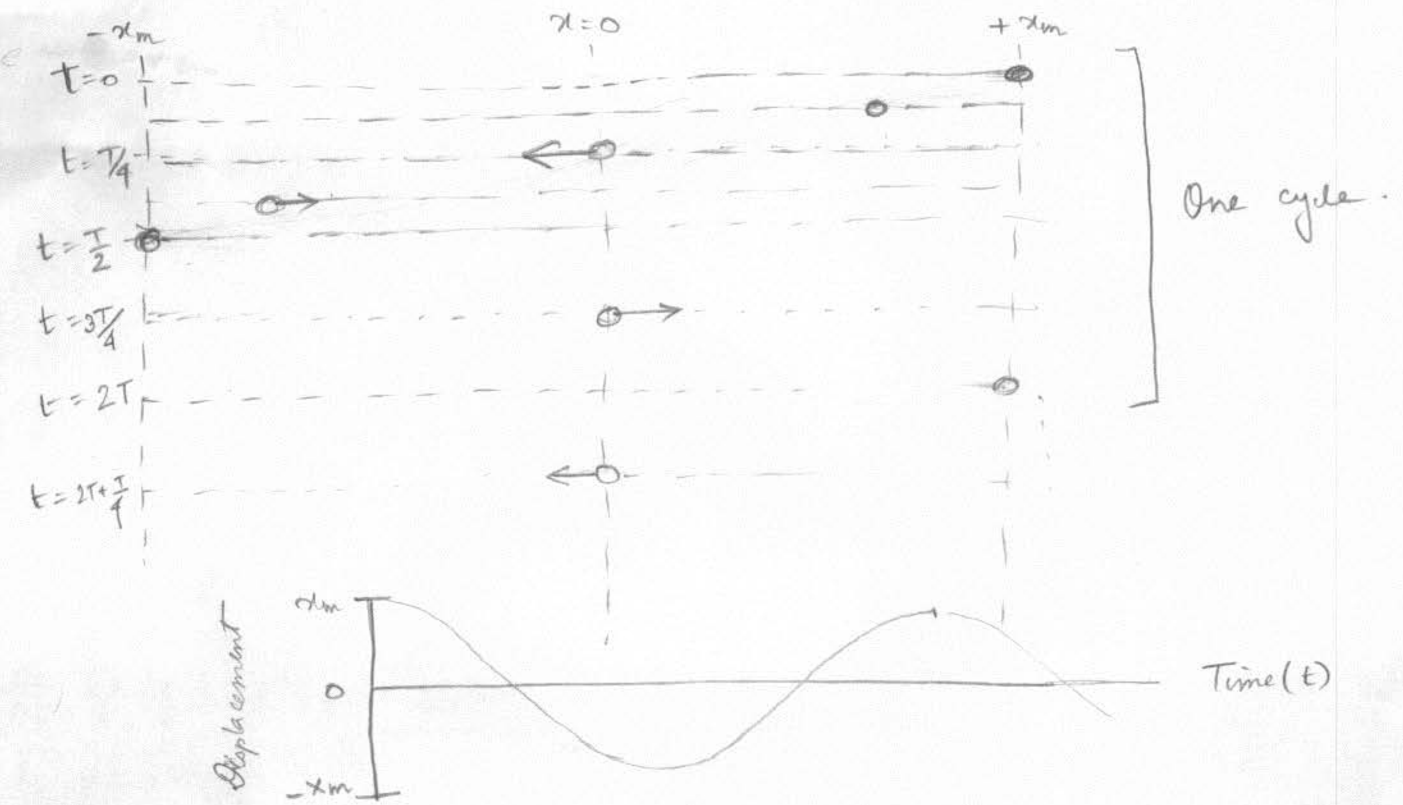
Simple harmonic motion

$T = \frac{1}{f}$ [units: second]

Time taken for one complete oscillation (cycle)

$f = \frac{1}{T}$ [units: $\frac{1}{s}$ = Hz (oscillation per second)]

frequency: number of oscillations per second.



$$\underbrace{x(t)}_{\text{displacement at time } t} = \underbrace{x_m}_{\text{Amplitude}} \cos \underbrace{(\omega t + \phi)}_{\text{phase}}$$

ω → Angular frequency

t → time

ϕ → phase constant. initial phase

or

$x = x_m \sin(\omega t + \phi')$

equivalent

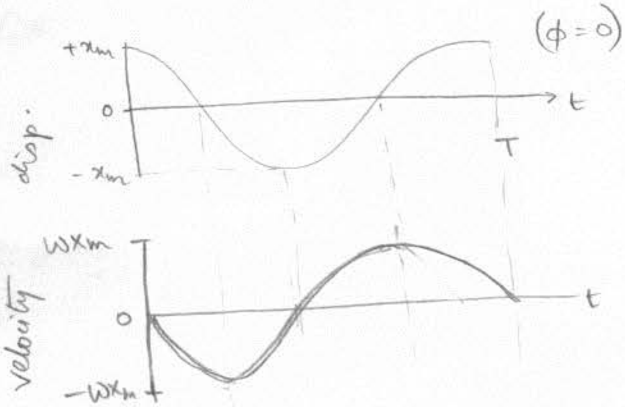
Angular frequency : $\omega = \frac{2\pi}{T} = 2\pi f$.

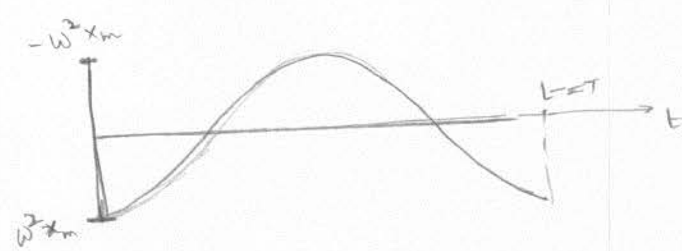
because,
$$x(t+T) = x(t)$$
$$\cos(\omega(t+T)+\phi) = \cos(\omega t + \phi)$$
$$\cos(\omega t + \phi + \omega T) = \cos(\omega t + \phi) \Rightarrow \omega T = 2\pi \Rightarrow \omega = \frac{2\pi}{T}$$

<u>Velocity</u>: $v(t) = \frac{dx}{dt} = \frac{d}{dt}\left[x_m \cos(\omega t + \phi)\right] = -(x_m \omega) \sin(\omega t + \phi)$

$$= -v_m \sin(\omega t + \phi)$$

Same angular freq of oscillation.





<u>Acceleration</u>: $a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left[-\omega x_m \sin(\omega t + \phi)\right]$

$$a(t) = -\omega^2 \underbrace{x_m \cos(\omega t + \phi)}$$

$$a(t) = -\omega^2 x(t)$$

acceleration is proportional to displacement opposite in sign.

$$\vec{F}(t) = m a(t) = -m\omega^2 x(t)$$
$$= -kx(t) \quad [\text{SHM spring}]$$

$k = m\omega^2$ ⇒ $\omega^2 = \left(\frac{k}{m}\right)$, same for the same spring and m (mass) attached.

↳ property of the spring

$\omega = \sqrt{\frac{k}{m}}$ (angular frequency)

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ (period)

Energy of SHM.

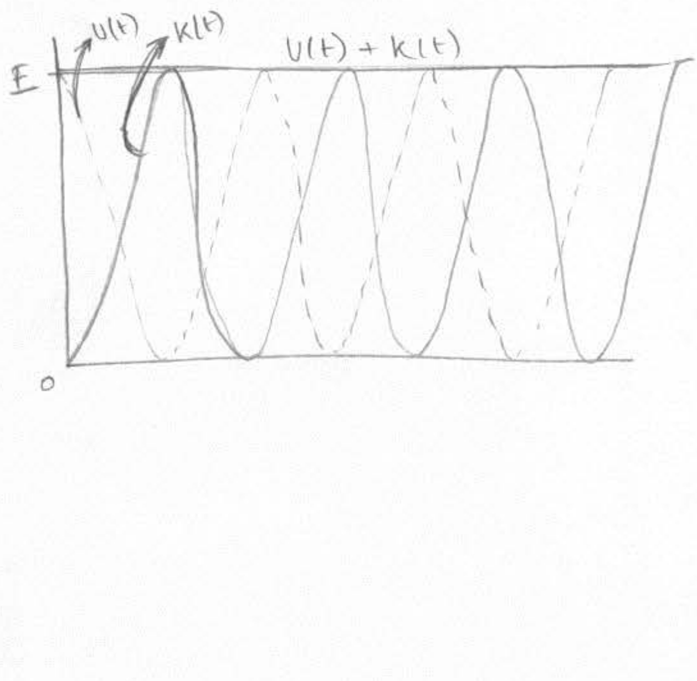
$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}k\left[x_m \cos(\omega t + \phi)\right]^2 = \frac{1}{2}kx_m^2 \cos^2(\omega t + \phi)$$

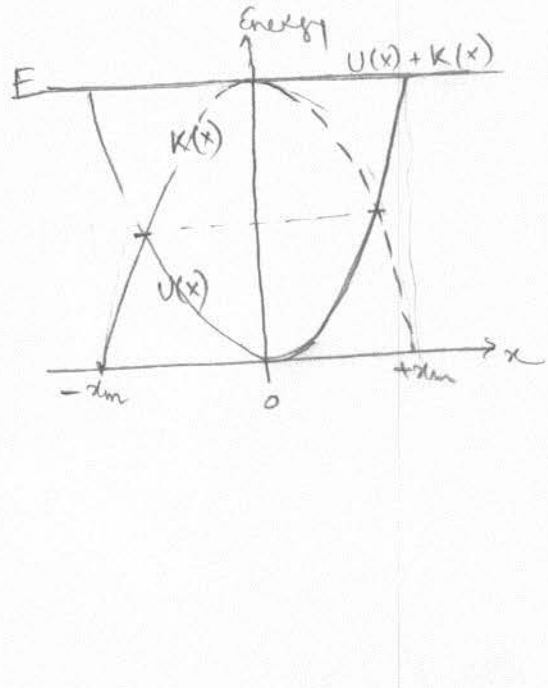
$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}m\left[-x_m\omega \sin(\omega t + \phi)\right]^2 = \frac{1}{2}m x_m^2 \omega^2 \sin^2(\omega t + \phi)$$

$$K(t) = \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

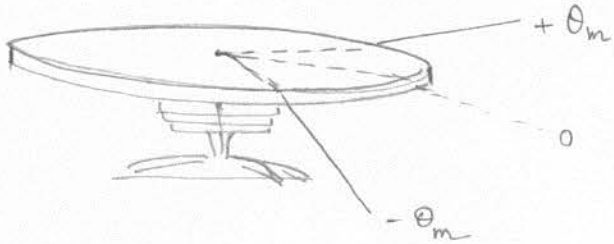
$$E = K(t) + U(t) = \frac{1}{2}kx_m^2\left[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)\right]$$

$$\boxed{E = \frac{1}{2}kx_m^2} \text{ or } E = \frac{1}{2}m\omega^2 x_m^2$$





Angular SHM (Torsional pendulum)



$$\theta(t) = \theta_m \cos(\omega t + \phi)$$

$$\omega(t) = \frac{d\theta}{dt} = -\theta_m \omega \sin(\omega t + \phi)$$

$$\alpha(t) = \frac{d\omega}{dt} = -\theta_m \omega^2 \cos(\omega t + \phi)$$

$$\alpha(t) = -\omega^2 \theta(t) \Rightarrow I\alpha(t) = -I\omega^2 \theta(t) \Rightarrow \boxed{\begin{array}{l} \tau(t) = -K\theta(t) \\ \quad \text{compare} \\ F(t) = -k\,x(t) \end{array}}$$

$$\boxed{K = I\omega^2}$$

↓ torsion constant

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{K}}$$