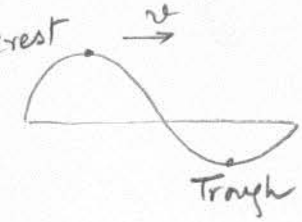
Longitudinal and Transverse waves.

a) * Transverse wave :



The particles of the medium oscillate in the direction $\perp$ to the direction of propagation of the wave.

* example: i) Waves on surface of fluids ⟶ surface of water ripples on a lake

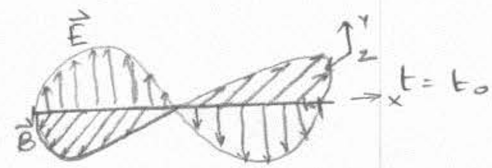
ii) Waves of disturbance in a string

(iii) Electromagnetic waves.

$$\vec{E}(t) = E_0 \sin(kx - \omega t)\,\hat{j}$$

$$\vec{B}(t) = B_0 \sin(kx - \omega t)\,\hat{k}$$

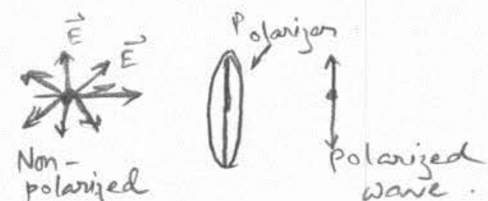
Waves of the Electric field and magnetic field



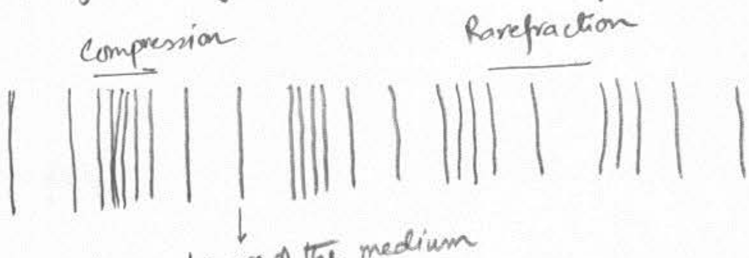
* Transverse waves can be polarized :

↓

what is polarization?

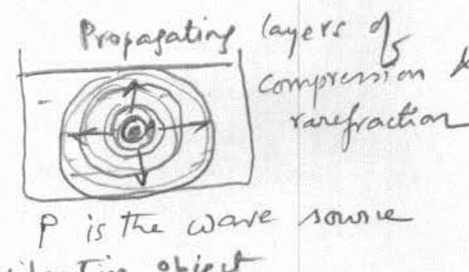


b) Longitudinal wave : The particles of the medium oscillate in the direction of propagation of the wave.



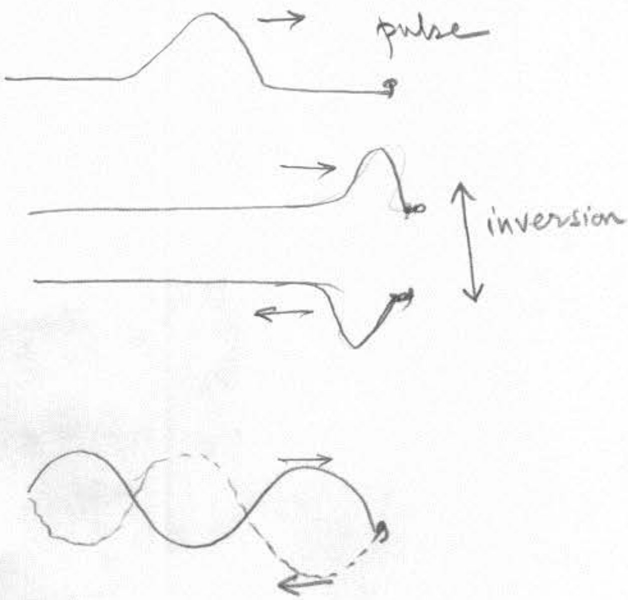
example: (i) Sound wave in air

(ii) disturbance within a fluid (water)

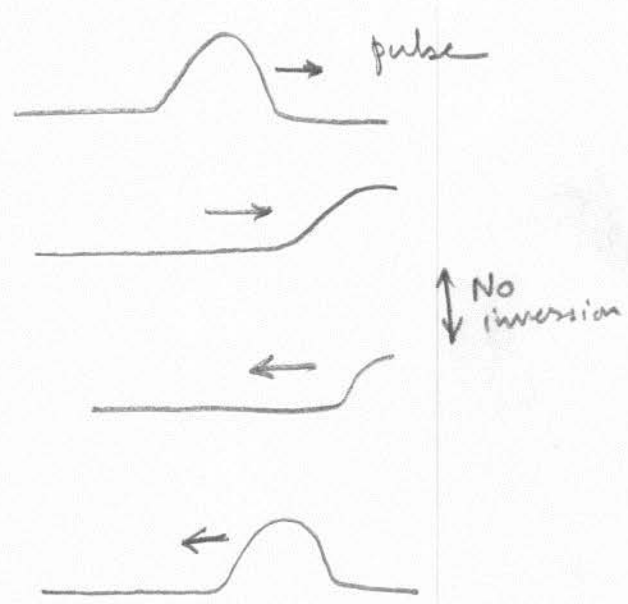


Reflection and Transmission

End is fixed to a support.

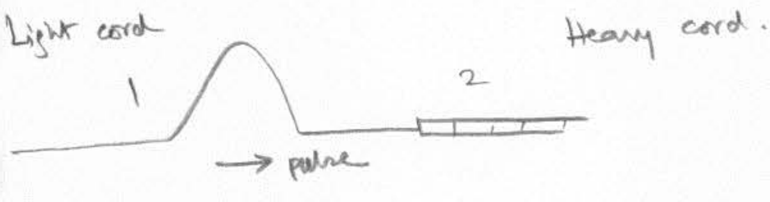


End is movable.

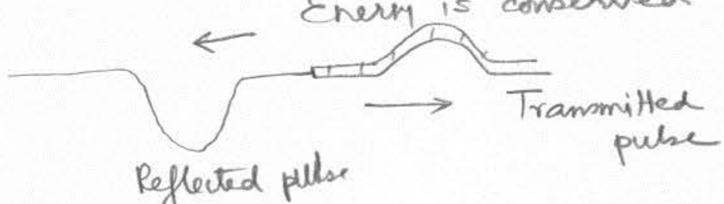


eg: sea waves.

Mixed case.



Energy is conserved



leads to

frequency of reflected & transmitted pulse is same

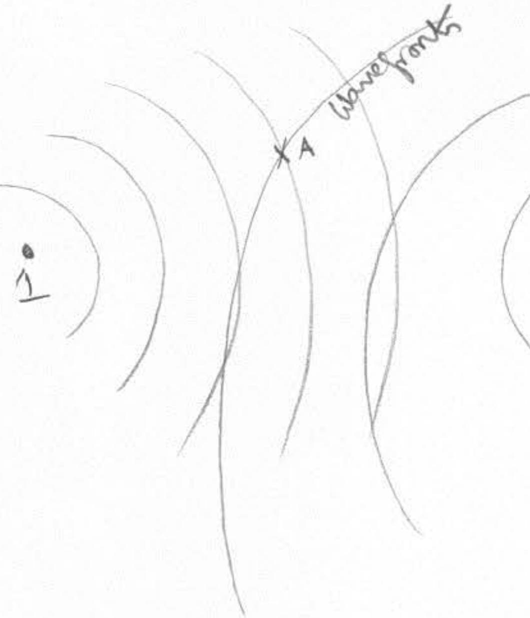
$\nu_I = \nu_R = \nu_T$

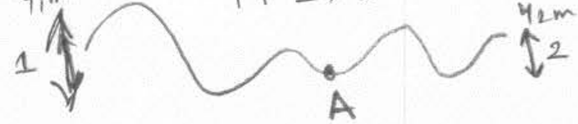
incident freq. $\downarrow$ ref. $\downarrow$ transmitted

$$\frac{v_1}{\lambda_1} = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} \quad v_2 \neq v_1, \quad \lambda_2 \neq \lambda_1$$

but $\frac{v_2}{\lambda_2} = \frac{v_1}{\lambda_1} = (\nu_T = \nu_R)$

Principle of Superposition





Two rocks striking a water surface.
They form ripples which propagate like waves.

Consider the point A.

× displacement of a particle at pt A due to disturbance from wave 1 is $y_1(x,t) = y_{1m} \cos(k_1 x - \omega_1 t + \phi_1)$

× displacement of a particle at pt A due to disturbance from wave 2 is $y_2(x,t) = y_{2m} \cos(k_2 x - \omega_2 t + \phi_2)$

Total disp:

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$y(x,t) = y_{1m} \cos(k_1 x - \omega_1 t + \phi_1) + y_{2m} \cos(k_2 x - \omega_2 t + \phi_2)$$

Interference →

- $y_1(x,t)$ & $y_2(x,t)$ may produce effects that have similar effect and sum up i.e. in phase and the superposition is bigger.
  * Constructive interference.

- $y_1(x,t)$ & $y_2(x,t)$ may nullify the effects i.e. out of phase
  * destructive interference.

Coherent waves cause perfect constructive & destructive interference

$$y_1(x,t) = y_0 \cos(kx + \omega t)$$
$$y_2(x,t) = y_0 \cos(kx + \omega t + \phi)$$

same $\boxed{\omega \,\&\, k.}$ for coherent waves.

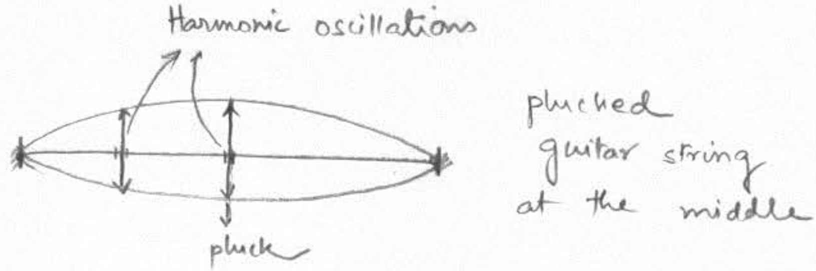
$$y = y_1 + y_2 = 2y_0 \cos(kx + \omega t)$$ Constructive interference if $\phi = 0$ or 2π $(\underbrace{2n}_{\text{even}}\pi)$

$$y = y_1 + y_2 = y_0 \cos(kx + \omega t) + y_0 \cos(kx + \omega t + \pi)$$
$$= y_0 \cos(kx + \omega t) - y_0 \cos(kx + \omega t)$$
$$y(x,t) = 0$$

[if $\phi = \pi$ or 3π . . .
$(\underbrace{2n+1}_{\text{odd}})\pi$

Standing Waves.

a) Simplest example:



The ends are fixed

b) Superposition of a travelling wave & its reflected wave

$y_i = y_m \sin(kx - \omega t) \rightarrow$ incident wave

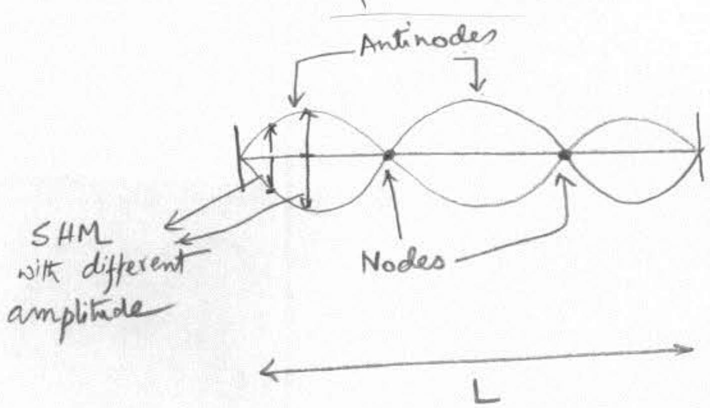
$y_r = y_m \sin(kx + \omega t) \rightarrow$ reflected wave

If two sinosoidal waves of the same amplitude and wavelength travel in opposite directions along a stretched string, their interference with each other produces a <u>standing wave</u>.

$$y = y_i + y_r = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t)$$

$$= y_m\left[\sin(kx - \omega t) + \sin(kx + \omega t)\right] = 2y_m \sin(kx)\cos(\omega t)$$

$$\therefore \overbrace{y(x,t)}^{\text{Displacement}} = \underbrace{\left[2y_m \sin kx\right]}_{\substack{\text{Amplitude at} \\ \text{position } x}} \underbrace{\cos\omega t}_{\text{oscillating term}}.$$



The position of nodes and Antinodes are fixed (stationary)

To obtain the position of the nodes:

$\sin kx = 0$ at nodes, $\because$ Amplitude of SHM at nodes is zero.

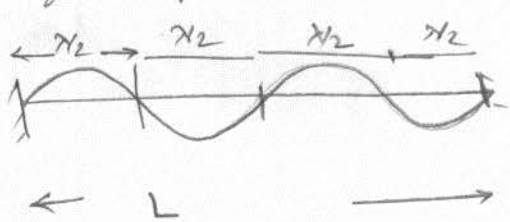
$kx = n\pi$ for $n = 0, 1, 2, \dots$

Now, $k = \frac{2\pi}{\lambda}$

$\therefore \quad x = \frac{n\pi}{k} = \frac{n\cancel{\pi}\lambda}{2\cancel{\pi}} = \frac{n\lambda}{2}$

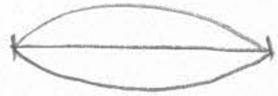
$\therefore \boxed{x = \frac{n\lambda}{2}}$, $n = 0, 1, 2 \dots$

The zero amplitudes, i.e. the nodes occur at x values separated by $\frac{\lambda}{2}$



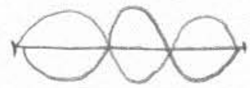
$L = n\left(\frac{\lambda}{2}\right)$ must be true because only integer number of half waves can fit in a string length

$L = \lambda/2$; $n=1$ fundamental mode,



$L = 2\left(\frac{\lambda}{2}\right) = \lambda$; $n=2$ first harmonic,

$L = 3\left(\frac{\lambda}{2}\right)$; $n=3$, second harmonic,



SOUND WAVES.

Longitudinal waves.

* Speed of sound? Yesterday we saw for a string : $v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$

for sound, $\boxed{v_{sound} = \sqrt{\frac{B}{\rho}}}$

B = Bulk modulus $= -\frac{\Delta p}{\Delta V / V}$

ρ = density of medium

$v_{air} = 331$ m/s

Wave eqn for sound:

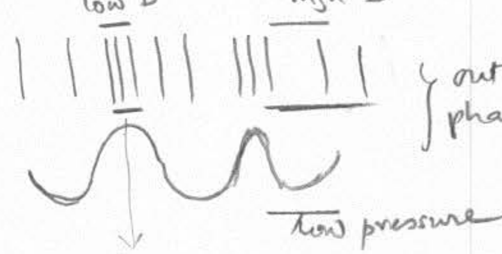
$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 D}{\partial t^2}$$

D is displacement from mean position.

The displacement causes pressure fluctuations and is equivalent to a pressure wave

$$D(x,t) = D_m \cos(kx - \omega t)$$

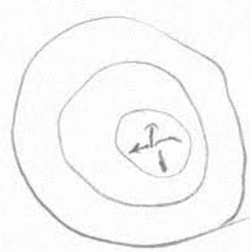
$$\Delta p(x,t) = \Delta p_m \sin(kx - \omega t)$$



out of phase by $\frac{\pi}{2}$

Intensity of sound,

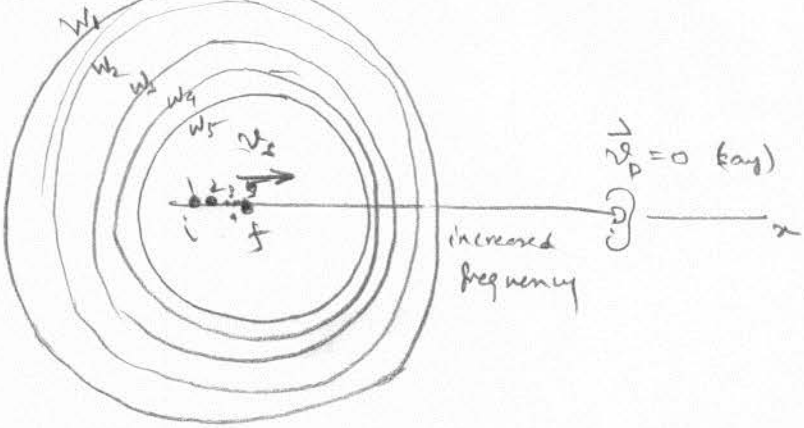
$$I = \frac{P_s}{4\pi r^2} = \frac{1}{2} \rho v \omega^2 s_m^2$$



decibel scale: $\beta = (10\ dB) \log\left(\frac{I}{I_0}\right)$

$I_0 = 10^{-12}\ W/m^2$ standard

The Doppler Effect



$$f' = f \, \frac{v \pm v_D}{v \pm v_S}$$

f' → detected freq

f → emitted freq

v = speed of sound through air

v_S = source speed

v_D = detector speed.