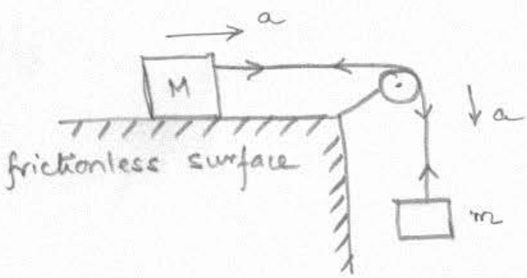
Applying Newton's Laws

Problem solving in Mechanics

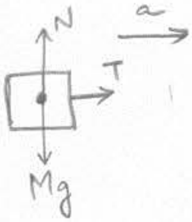
Problem 1

find (i) acceleration of sliding block
(ii) acc. of hanging block
(iii) Tension in the cord.



Free body diagram :

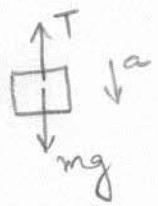
(I) M



$Mg = N$ (vertical component balance)

$T = Ma$ —(a) (T causes acceleration of M ∴ $T = Ma$)

(II) m



(T is the same because same string)

$mg - T = ma$ —(b) (net unbalalance force causing acceleration) $\Rightarrow$ $T = mg - ma$

from (a) & (b) solve for a [T is unknown, thus, we eliminate T]

$$Ma = mg - ma \Rightarrow a(M+m) = mg \Rightarrow \boxed{a = \frac{mg}{M+m}}$$

[check dimensions correct!]

Acceleration of sliding block is the same as the magnitude of acceleration of hanging block : [Since, connected to same string]

if block M moves by Δx_M and block m moves down by Δx_m

$$\Delta X_M = \Delta x_m$$

time derivative, $\frac{\Delta X_M}{\Delta t} = \frac{\Delta x_m}{\Delta t} \Rightarrow v_M = v_m$ (same velocity)

$$a_M = \frac{d^2 x_M}{d^2 t} = \frac{d^2 x_m}{dt} = a_m (= a) \quad \text{(same acceleration)}$$

$T = Ma$ from (a)

$$\boxed{T = \frac{Mmg}{M+m}}$$

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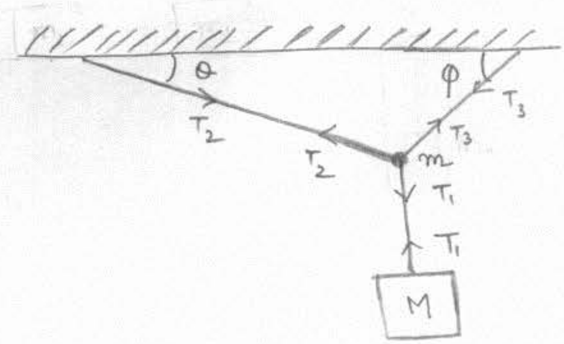
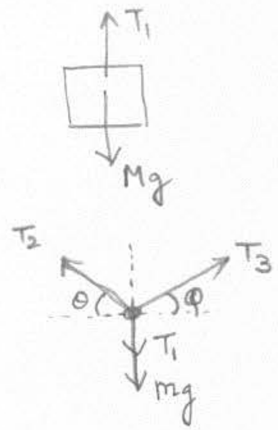
problem 2

Find the tension in the three cords





$$Mg = T_1 \quad \text{—(i)}$$

Vertical force balance

$$T_2 \sin\theta + T_3 \sin\varphi = T_1 + mg \quad \text{—(ii)}$$

Horizontal

$$T_2 \cos\theta = T_3 \cos\varphi \quad \text{—(iii)}$$

We have 3 equations and 3 unknowns (T_1, T_2 & T_3)
Thus, these can be solved for in terms of known quantities.

$$T_2 \sin\theta + T_3 \sin\varphi = (M+m)\,g \quad \rightarrow \text{from (i) \& (ii)}$$

$$T_2 \cos\theta - T_3 \cos\varphi = 0 \quad \rightarrow \text{(iii)}$$

Matrix method of solving eqn :

$$\begin{pmatrix} \sin\theta & \sin\varphi \\ \cos\theta & -\cos\varphi \end{pmatrix} \begin{pmatrix} T_2 \\ T_3 \end{pmatrix} = \begin{pmatrix} (M+m)\,g \\ 0 \end{pmatrix}$$

$$AT = B \Rightarrow T = A^{-1}B \qquad A^{-1} = \begin{pmatrix} -\cos\varphi & -\sin\varphi \\ -\cos\theta & \sin\theta \end{pmatrix} \frac{1}{|A|}$$

$$T = A^{-1}B = \frac{1}{|A|}\begin{pmatrix} -\cos\varphi & -\sin\varphi \\ -\cos\theta & \sin\theta \end{pmatrix}\begin{pmatrix} (M+m)\,g \\ 0 \end{pmatrix}$$

$$; \ |A| = \sin\theta\cos\varphi - \sin\varphi\cos\theta = -\sin(\theta+\varphi)$$

$$T = \begin{pmatrix} \dfrac{+\cos\varphi\,(M+m)g}{\sin(\theta+\varphi)} \\ \\ \dfrac{+\cos\theta\,(M+m)g}{\sin(\theta+\varphi)} \end{pmatrix} \Rightarrow \boxed{\begin{aligned} T_2 &= \frac{(\cos\varphi)(M+m)g}{\sin(\theta+\varphi)} \\ T_3 &= \frac{(\cos\theta)(M+m)g}{\sin(\theta+\varphi)} \end{aligned}} \quad \& \ T_1 = Mg$$

Another solving method

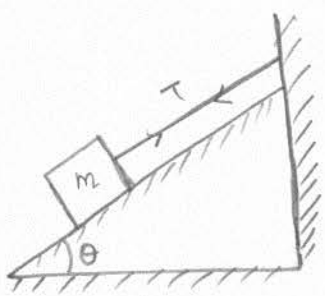
$$T_2 \sin\theta\cos\theta + T_3 \sin\varphi\cos\theta = (M+m)g\cos\theta$$
$$T_2 \cos\theta\sin\theta - T_3 \cos\varphi\sin\theta = 0$$

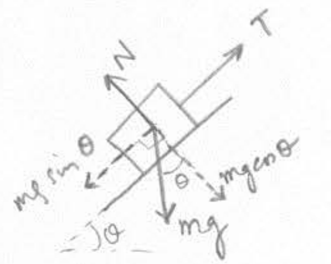
$$T_3(\sin\varphi\cos\theta + \cos\varphi\sin\theta) = (M+m)g\cos\theta$$

$$T_3 = \frac{(M+m)g\cos\theta}{\sin(\theta+\varphi)} \quad \text{and} \quad T_2 = \frac{\cos\varphi\,(M+m)g}{\sin(\theta+\varphi)} \quad \& \ T_1 = Mg$$

Problem 3.

Find T and N.





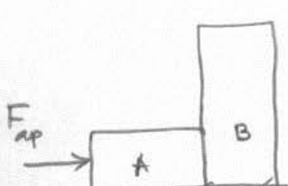
Forces ⊥ to plane

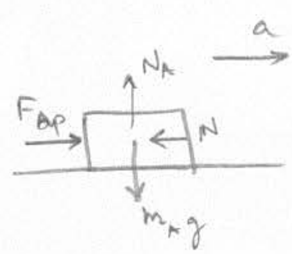
$$N = mg\cos\theta$$
$$T = mg\sin\theta$$

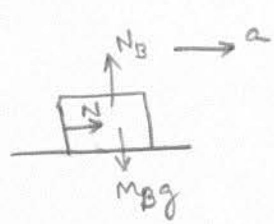
frictionless slide

Problem 4.

m_A, m_B, F_{ab} is the applied force on A.





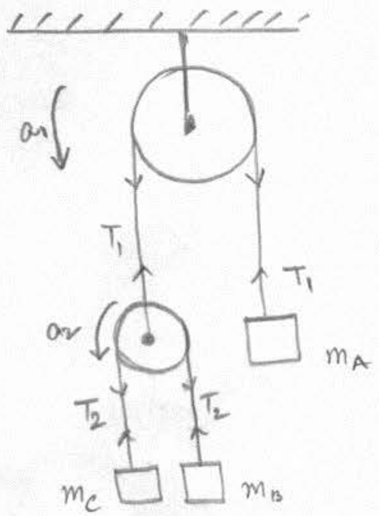


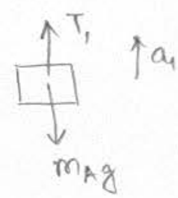
$$F_{ap} - N = m_A a \quad -(i) \qquad N = m_B a \quad -(ii)$$

$$F_{ap} - m_B a = m_A a \quad \Rightarrow \quad \boxed{a = \frac{F_{ap}}{(m_A + m_B)}}$$

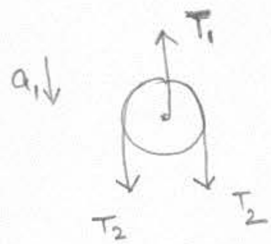
Problem 5.

A double Atwood machine has frictionless, massless pulleys and cords. Determine
(a) acceleration of masses
(b) Tension in cords.





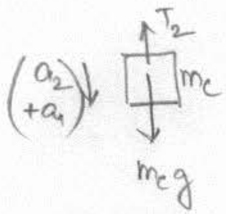
$$T_1 - m_A g = m_A a_1 \quad -\text{(i)}$$

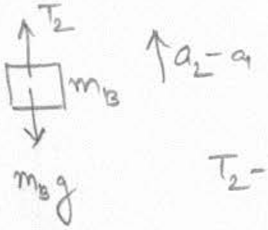


$$2T_2 - T_1 = \underbrace{m_{pulley}}_{\text{negligible}} a_2$$

$$2T_2 - T_1 = 0$$

$$2T_2 = T_1 \quad -\text{(ii)}$$





$$T_2 - m_B g = m_B(a_2 - a_1) \quad -\text{(iii)}$$

$$m_C g - T_2 = m_C(a_1 + a_2) \quad -\text{(iv)}$$

$$2T_2 = m_A(g + a_1) \quad -\text{(i) \& (ii)}$$

$$T_2 = m_B(a_2 - a_1 + g) \quad -\text{(iii)}$$

$$T_2 = m_C(g - a_1 - a_2) \quad -\text{(iv)}$$

$\therefore$

$$\frac{m_A}{2}(g + a_1) = m_B(a_2 - a_1 + g) \qquad \frac{m_A}{2}(g + a_1) = m_C(g - a_1 - a_2)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$a_1\left(\frac{m_A}{2} + m_B\right) - a_2 m_B = g\left(m_B - \frac{m_A}{2}\right) \qquad a_1\left(\frac{m_A}{2} + m_C\right) + a_2 m_C = g\left(m_C - \frac{m_A}{2}\right)$$

Solve for a_1 & a_2

$$a_1\left(\frac{m_A}{2}+m_B\right) - a_2 m_B = g\left(m_B - \frac{m_A}{2}\right)$$

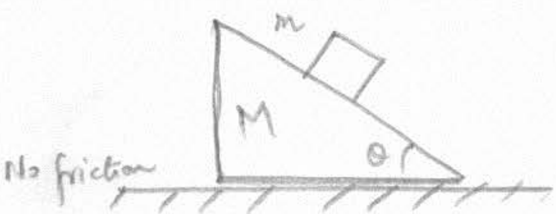
$$a_1\left(\frac{m_A}{2}+m_C\right) + a_2 m_C = g\left(m_C - \frac{m_A}{2}\right)$$

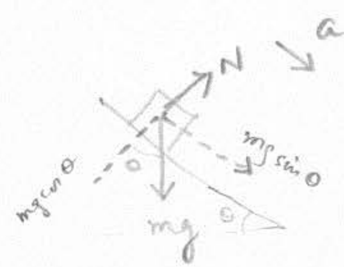
$$\underbrace{\begin{pmatrix} \frac{m_A}{2}+m_B & -m_B \\ \frac{m_A}{2}+m_C & m_C \end{pmatrix}}_{A} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} g\left(m_B - \frac{m_A}{2}\right) \\ g\left(m_C - \frac{m_A}{2}\right) \end{pmatrix}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \frac{\begin{pmatrix} m_C & m_B \\ -\frac{m_A}{2}-m_C & \frac{m_A}{2}+m_B \end{pmatrix}}{\det A} \begin{pmatrix} g\left(m_B - \frac{m_A}{2}\right) \\ g\left(m_C - \frac{m_A}{2}\right) \end{pmatrix} = \frac{1}{\det A}\begin{pmatrix} m_C\, g\left(m_B - \frac{m_A}{2}\right) + m_B\, g\left(m_C - \frac{m_A}{2}\right) \\ -g\left(m_B - \frac{m_A}{2}\right)\left(\frac{m_A}{2}+m_C\right) + g\left(\frac{m_A}{2}+m_B\right)\left(m_C - \frac{m_A}{2}\right) \end{pmatrix}$$

Use a's to find T's.

Problem 6

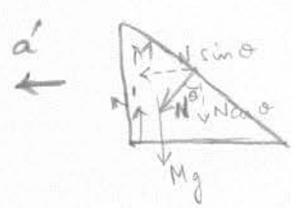




$$N = mg\cos\theta$$

$$\cancel{m}g\sin\theta = \cancel{m}a$$

$$\boxed{a = g\sin\theta}$$



$$N\sin\theta = Ma'$$

$$N\cos\theta + Mg = N'$$

$$a' = \frac{N\sin\theta}{M}$$

$$a' = \frac{mg\cos\theta\sin\theta}{M}$$

$$\boxed{a' = \frac{mg\sin 2\theta}{2M}}$$