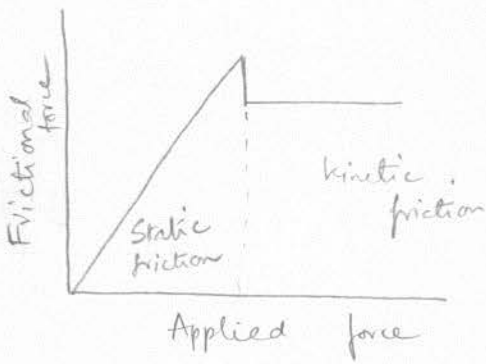
FRICTION.

Frictional force opposes any $\underbrace{\text{motion}}_{\downarrow}$ / $\underbrace{\text{tendency of motion}}_{\text{static friction.}}$

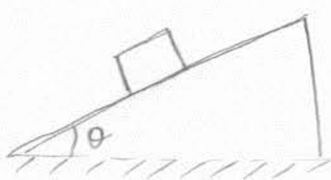
kinetic friction



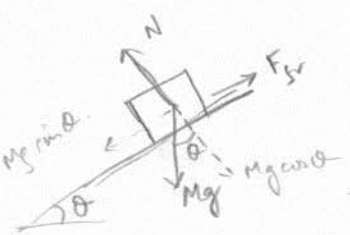
- Idea about origin of friction
- $F_{fr} = \mu_K N$ (kinetic friction)
- $F_{fr} = F_{app}$. (before motion starts)
- $F_{fr} = F_{app} < \mu_S N$ (When motion is about to start) $\mu_K < \mu_S$

$F_{fr,\,max} = \mu_S N$ (motion starts)

Problem



Block of mass M rests on a inclined plane. What should be the angle θ of the inclined plane if the block just starts falling.



(i) When the mass M is <u>at rest</u>

forces along the incline: $Mg \sin\theta = F_{fr}$

(applied force = force of friction)

$N = Mg\cos\theta$

(ii) When mass M just starts moving (at $\theta = \theta_m$)

$$\left.\begin{aligned} Mg \sin\theta_m &= \mu_S N \\ N &= Mg\cos\theta_M \end{aligned}\right] \Rightarrow \cancel{Mg} \sin\theta_m = \mu_S \cancel{Mg} \cos\theta_M$$

$$\boxed{\tan\theta_m = \mu_s}$$

$$\theta_m = \tan^{-1}\mu_s$$

Note: θ_M is independant of the mass of the body. Its just dependant on μ_S (which is the nature of the roughness of the surface of contact)

(iii) after body starts moving.

$$Mg \sin\theta - F_{fr} = Ma \qquad F_{fr} = \mu_k N$$

$$N = Mg\cos\theta \qquad \theta > \theta_M$$

$$\therefore \cancel{M}g \sin\theta - \mu_k \cancel{M}g\cos\theta = \cancel{M}a$$

$$\Rightarrow \boxed{a = g(\sin\theta - \mu_k \cos\theta)}$$

Drag forces, velocity dependant friction and Terminal velocity

Imagine a ball falling through a liquid.

D = drag force

$$D = \frac{1}{2} C \rho A v^2$$

v → rel. velocity of the ball

A → area of cross section taken ⊥ to velocity.

ρ → density of the liquid

C → drag coefficient (material dependant)

$$Mg - D = Ma \quad \text{(for non eqlb.)}$$

but D increases gradually (D↑ if v↑) and equals Mg

$$Mg - D = 0$$

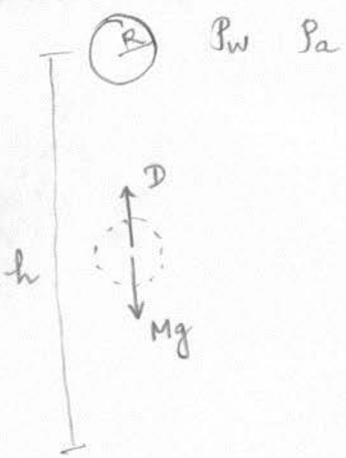
$$\frac{1}{2} C\rho A v^2 - Mg = 0 \quad \Rightarrow \quad v_t = \sqrt{\frac{2(Mg)}{C\rho A}}$$

v_t → terminal velocity

A raindrop of radius 'R' falls from a cloud at a hight 'h' above the ground. The drag coefficient for the drop is C. Assume that the drop is spherical throughout its fall. The density of water is ρ_w and density of air is ρ_a

(a) what is the terminal speed of the drop?

(b) What would be the drop's speed just before impact if there were no drag force?



To reach terminal speed the drag force must completely balance the weight of the drop.

$$D = Mg$$

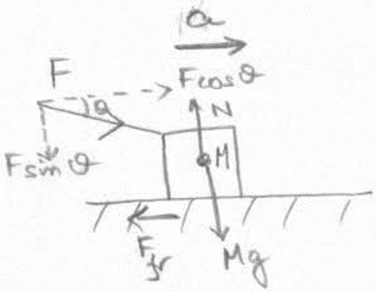
$$\Rightarrow \frac{1}{2} C \rho_a A v_t^2 = V \rho_w g$$

$$\Rightarrow v_t^2 = \frac{2\left(\frac{4}{3}\pi R^3\right)\rho_w g}{C\rho_a (\pi R^2)} = \frac{8\rho_w g R}{3C\rho_a}$$

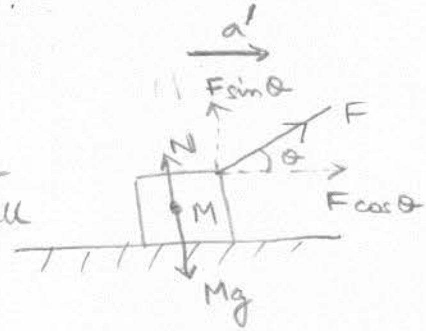
(a) $$\therefore \boxed{v_t = \sqrt{\frac{8\rho_w g R}{3C\rho_a}}}$$

(b) $$v^2 = 2gh \Rightarrow \boxed{v = \sqrt{2gh}}$$

Is it easier to push or pull?



Same mass, same magnitude of F to push/pull



$$\left.\begin{aligned} F\cos\theta - \mu N &= Ma \\ Mg + F\sin\theta &= N \end{aligned}\right\}$$

$$Ma = F\cos\theta - \mu\left(Mg + F\sin\theta\right)$$

$$\Rightarrow \quad a = \frac{F\cos\theta - \mu Mg - \mu F\sin\theta}{M}$$

$$\left.\begin{aligned} F\cos\theta - \mu N &= Ma' \\ Mg - F\sin\theta &= N \end{aligned}\right\}$$

$$Ma' = F\cos\theta - \mu(Mg - F\sin\theta)$$

$$\Rightarrow a' = \frac{F\cos\theta - \mu Mg + \mu F\sin\theta}{M}$$

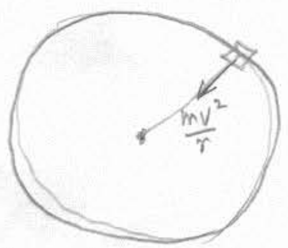
for same M, F and θ

$\boxed{a' > a}$ $\Rightarrow$ It is easier to pull than to push.

Uniform circular motion

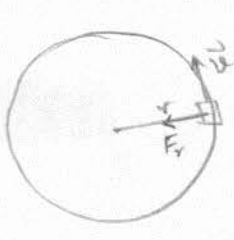
Friction as the centripetal force for uniform circular motion, changing the direction of velocity, keeping its magnitude constant.

$$a = v^2/R \qquad \boxed{F = \frac{mv^2}{R} = F_{fr}}$$



* Idea of reference frame & pseudo forces
* centrifugal force.

eg: A car of mass m is moving at a very high speed v. It is moving in a circle of radius r. If the car is just not skidding at this velocity, find the coefficient of friction?



$$F_{fr} = \frac{mv^2}{r} = F_{applied} \qquad N = mg$$

$$\mu_s N = \frac{mv^2}{r} \quad \text{(for just about to skid)}$$

$$\boxed{\mu_s = \frac{v^2}{rg}}$$

What should the car do so that it has no chance of skidding?

(i) reduce v (maintain same r)

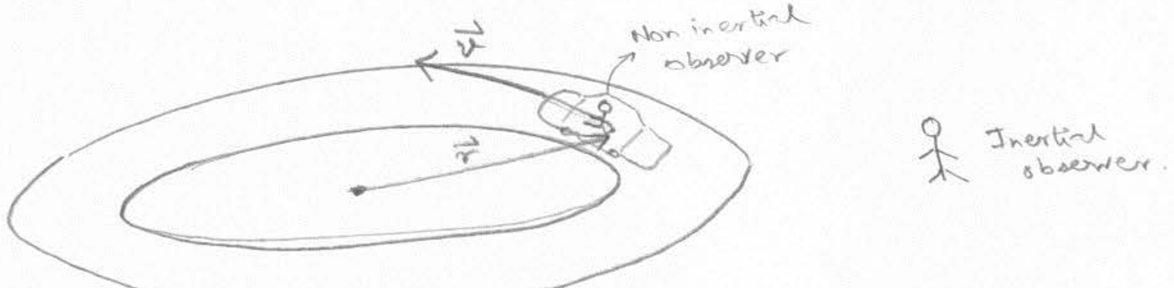
$$F_{fr} = \frac{mv^2}{r} = F_{applied}$$

$$F_{fr} = \frac{mv^2}{r} < F_{max,fr} = \frac{mv_{max}^2}{r}$$

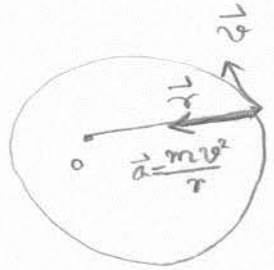
(ii) or increase r

Centripetal force and centrifugal force (a pseudo-force)

Consider a car moving in a circle of radius r, with a constant velocity v. There is a man sitting in the car. (the non-inertial / accelerating observer) The total mass of the car and man is m. There is another person who stands on the street and observes the car moving in circle.



What does the inertial observer see?



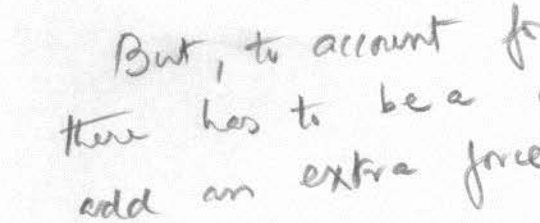
$\vec{a}$ = centripetal acceleration

$\vec{F} = m\vec{a} = \frac{mv^2}{r}$ = centripetal force.

The force required to change the direction of $\vec{v}$ of the car and make it move in a circle.

What does the non-inertial observer see?

The non-inertial observer sees no motion in this horizontal direction

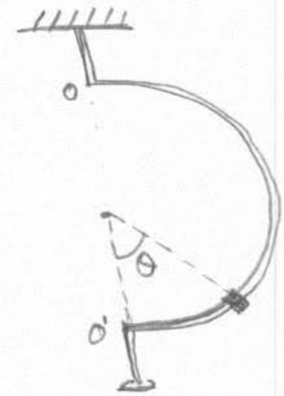


Hence if you look at the problem from his reference frame, there is no resultant horizontal (radial) force.

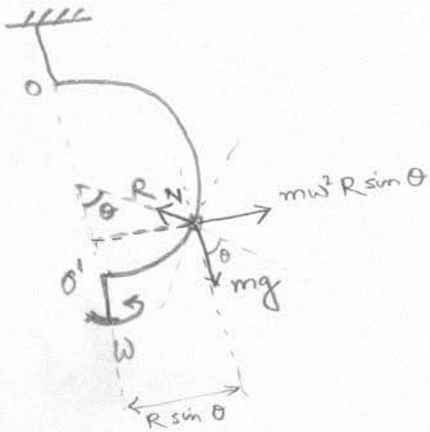
But, to account for the fact that he is moving in a circle there has to be a centripetal force; therefore we have to add an extra force (pseudo force) when looking at things from a noninertial frame of reference in the opposite direction of the centripetal force, so that the man in the car doesnot have any resultant force in the horizontal direction.

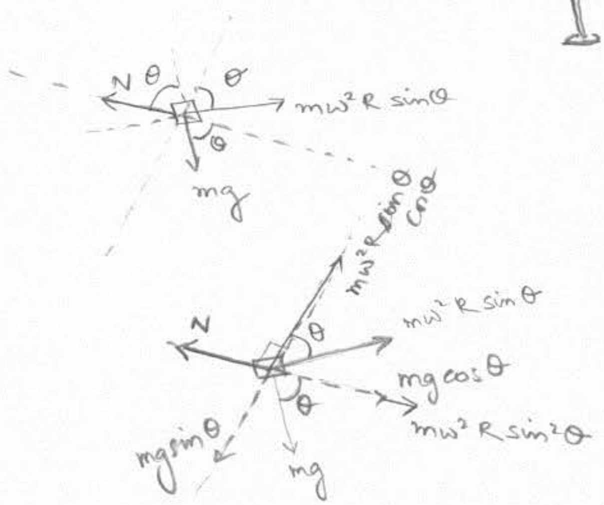
Problem

A sleeve A can slide freely along a smooth rod bent in the shape of a half-circle of radius R. The system is set in rotation with a constant angular velocity ω about a vertical axis OO'. Find the angle θ corresponding to the steady position of the sleeve.



Forces on the sleeve A.





Tangential force balance

$$mg\sin\theta = m\omega^2 R\sin\theta\cos\theta \quad -(i)$$

Radial balance

$$N = mg\cos\theta + m\omega^2 R\sin^2\theta \quad -(ii)$$

from (i) $\quad g = \omega^2 R\cos\theta \quad \Rightarrow \left(1 - \frac{\omega^2 R\cos\theta}{g}\right) = 0$ for balance.

Now, $\quad 1 - \frac{\omega^2 R\cos\theta}{g} = 0$ is true for $\theta = \theta_0 = \cos^{-1}\left(\frac{g}{\omega^2 R}\right)$,

which is real if $\omega^2 R > g$