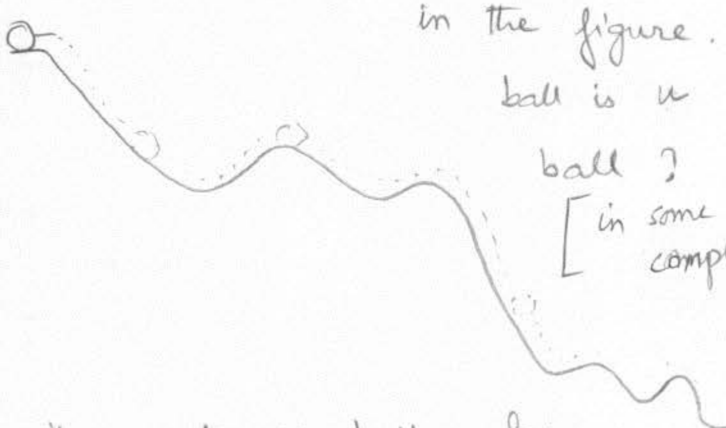
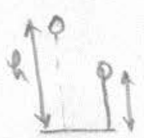
Kinetic Energy and Work.

Energy.:



A ball slides down a nonuniform slope as in the figure. The initial speed of the ball is u find the final speed of the ball?
[in some cases it is complicated to work the kinetics.]

eg: if you drop a ball from a hight h, it bounces with a retardation 'a' find the hight h' it reaches after bouncing?



Another alternative to solve these problems is by using Energy.

Kinetic energy: Energy associated with the state of motion of an object.

if mass m, velocity v

$$\boxed{K_1 = \frac{1}{2} m v^2}$$

$$1\ J = 1\ kg\ \frac{m^2}{s^2}$$

* unit J (joule) in S.I.

$$[J] = M L^2 T^{-2}$$

* Aolther unit of energy is calorie (cal) or Kcal (kilo calorie)

$$1\ cal = 4.18\ J \quad \& \quad 1\ Kcal = 10^3\ cal$$

[1 cal is the heat required to rise the temp. of 1g of water from 3.5°C to 4.5°C at 1atm pressure]

Work.

A force acting on a body slows it down or makes it move fast. (changes kinetic energy, $\frac{1}{2}m\underline{v}^2$)

* Work W is energy transferred to or from an object by means of a force acting on the object.
Energy transferred to the object is positive work, and energy transferred from the object is negative work.

Work done : $W = \vec{F}.\vec{d}$ (work done by a constant force)

$$W = \int_{x_0}^{x_1} F(x)\,dx \quad \text{(if F is changing with x as F(x))}$$

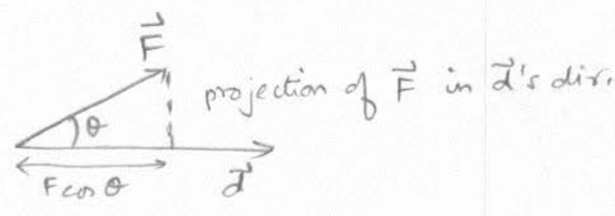
$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} \quad \left(\text{where } \vec{F} = F_x(x,y,z)\,\hat{i} + F_y(x,y,z)\,\hat{j} + F_z(x,y,z)\,\hat{k},\ d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right)$$

$$\boxed{W = \int_{\vec{r}_1}^{\vec{r}_2} F_x\,dx + \int_{\vec{r}_1}^{\vec{r}_2} F_y\,dy + \int_{\vec{r}_1}^{\vec{r}_2} F_z\,dz}$$

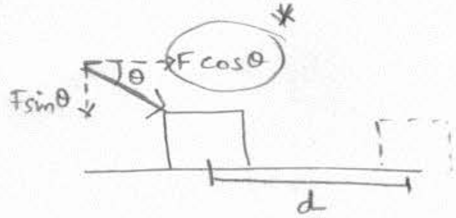
$$\begin{cases} \vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \\ \vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \end{cases}$$

* Constant force, case

$$W = \vec{F}.\vec{d} = F d \cos\theta$$



example:



$$W = (F\cos\theta).d$$

A force $\vec{F}$ does +ive work if it has a component in the same direction as the displacement

$$W = \vec{F}.\vec{d} = \underbrace{F d \cos\theta}_{-\text{ive if } \theta = \pi} \longrightarrow W = \vec{F}.\vec{d} = F d \cos\theta$$

$-$ive if $\theta = \pi$
$W = -$ ive

if $\theta = 0$, $W = +$ive

For many forces acting on a body

$$W = \vec{F}_{net} \cdot \vec{d} \qquad (\text{if } \vec{F}_{net} = \text{const})$$

else, $W = \int \vec{F}_{total} \cdot d\vec{r}$

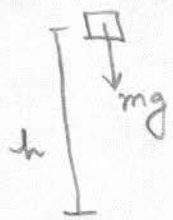
Work Energy theorem

$$\begin{pmatrix}\text{change in} \\ \text{Kinetic energy} \\ \text{of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done} \\ \text{on particle}\end{pmatrix}$$

$$\Delta K = K_f - K_i = W$$

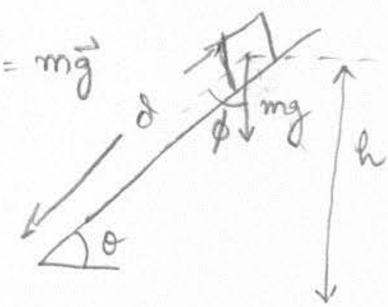
$$\Rightarrow K_f = W + K_i$$

Work done by a Gravitational Force



$$W = \vec{F} \cdot \vec{h} = mgh$$

$\vec{F} = m\vec{g}$



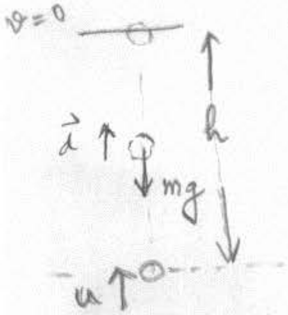
$$W = \vec{F} \cdot \vec{d} = mg(d \cos\phi)$$

$$W = mgh$$

$$(\because h = d\cos\phi)$$

What is the velocity with which we should throw a ball of mass 'm' for it to reach a height 'h'?

Let us throw the ball with a initial velocity u.



Work picture,

$$W = \vec{F} \cdot \vec{d} = mgh(\cos\pi)$$

$$W = -mgh$$

Change in K.E, $\Delta K = \frac{1}{2}m0^2 - \frac{1}{2}mu^2 = -\frac{1}{2}mu^2$.

Now, $\Delta K = W$ [Work energy theorem]

$$\frac{1}{2}\cancel{m}u^2 = \cancel{m}gh \quad \Rightarrow \quad \boxed{u = \sqrt{2gh}}$$

Check with what we get using kinetics:

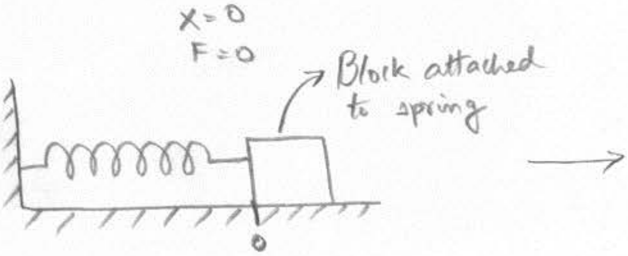
$$0 = u^2 - 2gh \quad \Rightarrow \quad u = \sqrt{2gh} \qquad \text{Same result}$$

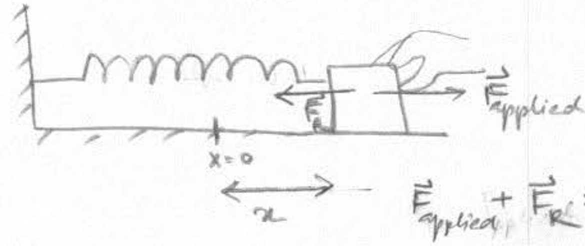
Law of Conservation of Energy.

* Energy can neither be created not destroyed, it can just be transferred from one body to another.
* Total energy of an isolated system is constant. (cannot change) $E_i = E_f \Rightarrow \Delta E = 0$
* The total energy E of a system can change only by amounts of energy that are transferred to or from the system. (eg. in the form of work done or heat)

$$W = \Delta E$$

Spring (Simple harmonic Oscillator)

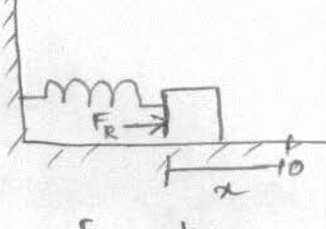




$$\vec{F}_{applied} + \vec{F}_R = 0$$

$$\boxed{F_R = -kx}$$ Hook's Law.

The restoring force by spring changes with x (displacement)



$$F_R = -kx$$

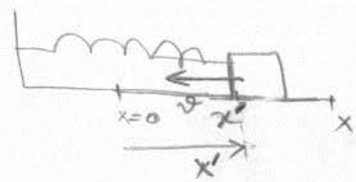
Work done by spring

$$W = \int_0^x F(x')\,dx' = \int_0^x -kx'\,dx' = -k\,\frac{x'^2}{2}\Big|_0^x = -\frac{1}{2}kx^2$$

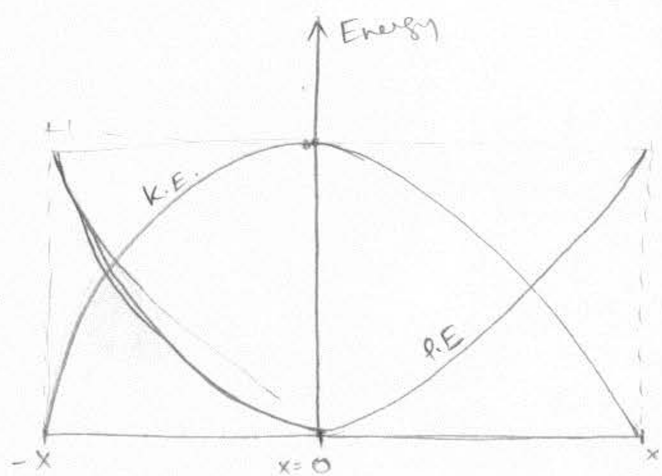
$$\boxed{W = -\frac{1}{2}kx^2}$$

Now, $W = \Delta K$

$$\Delta K = \underset{(x=0)}{K_f} - \underset{\substack{\text{zero} \\ x'=X \\ v=0}}{K_i} = \frac{1}{2}mv^2 = -\frac{1}{2}kx^2 \Rightarrow v^2 = -\frac{kx^2}{m}$$



Kinetic energy potential energy
from energy conservation



* Gravitational potential energy

$$\Delta U = mgh.$$
$$\Delta U = -W = -(-mgh) = mgh.$$

$E_T = \frac{1}{2}mv_i^2$ (ball projected upward)
$= mgh =$ constant