POTENTIAL ENERGY.

The energy stored in a system due to the state of the system (eg: The deformed spring) is its potential energy.

Generally denoted by U.

Total Energy of a body:

$$E_T = \underbrace{K}_{\text{kinetic energy}} + \underbrace{U}_{\text{potential energy}}$$

[In conservative forces a potential energy can be defined such that the total energy of the body doesnot change by shifting positions, but the K & U change among themselves such that $E = K_1 + U_1 = K_2 + U_2$]

$$\Delta E = 0 = \underbrace{\Delta K}_{\text{change in Kinetic energy}} + \underbrace{\Delta U}_{\text{change in potential energy}} \Rightarrow \Delta K = -\Delta U$$

And, Work Energy theorem : $\Delta K = W$

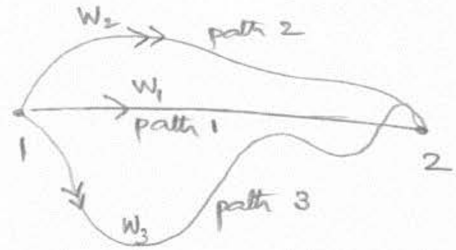
$$\Rightarrow W = -\Delta U$$

use, $W = \int \vec{F} . d\vec{r} \Rightarrow \boxed{\Delta U = -\int \vec{F} . d\vec{r}}$

Conservative and Nonconservative forces

<u>Conservative force</u> : The work done by the force on an object moving from one point to another depends only on the initial and final positions of the object, and is independent of the particular path taken

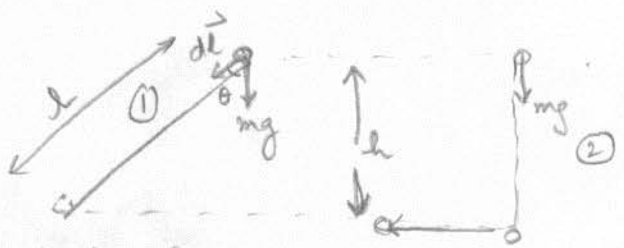
$$W = \int_1^2 \vec{F} \cdot d\vec{r}$$



*[Imagine friction;
work done to overcome friction
is path dependant, and hence friction
is Non-conservative force]

if $W_1 = W_2 = W_3$
then $\vec{F}$ is conservative.

eg:



$h = l\cos\theta$

path ①:

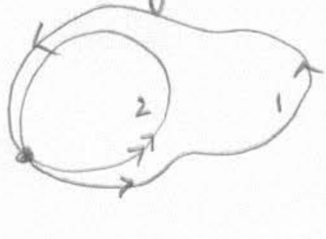
$$W_1 = \int \vec{F} \cdot d\vec{l}$$
$$= \int mg\, dl \cos\theta$$
$$= mg\cos\theta \int dl$$
$$= mg(l\cos\theta) = mgh$$

for path ②: $W_2 = \int \vec{F} \cdot d\vec{l}$
$$= mg \int dl = mgh$$

Thus, $\boxed{W_1 = W_2}$ $\therefore$ the Gravitational field is a conservative force field.

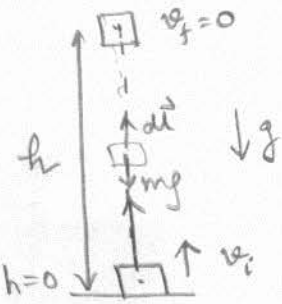
<u>Alternative defn:</u>

A force is conservative if the net work done by the force on an object moving around any closed path is zero.



$W_1 = W_2 = 0$

Gravitational potential energy $[\Delta U = mgh]$



Project a body upward with v_i velo.

$$W = \int \vec{F}\cdot d\vec{l} = \int_0^h mg\, dl \cos(\pi)$$

$$= -mg \int_0^h dl = -mgh$$

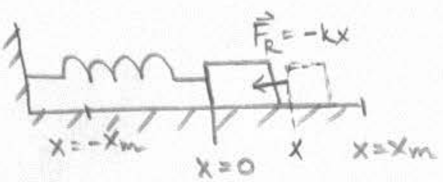
$$\Delta U = -W = mgh$$ (Gains potential by increasing height)

$$\Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = -\frac{1}{2} m v_i^2$$

Check, $\Delta E = \Delta K + \Delta U = -\frac{1}{2} m v_i^2 + mgh = 0$

$$\therefore \frac{1}{2} \cancel{m} v_i^2 = \cancel{m} gh \Rightarrow v_i = \sqrt{2gh}$$ (Same as kinetics)

Spring Potential energy.



$$F_R = -kx$$

$$W = \int_{x=x_i}^{x=x_f} \vec{F}_R \cdot d\vec{x} = \int_{x_i}^{x_f} -kx\, dx$$

$$W = -\frac{1}{2} k (x_f^2 - x_i^2)$$

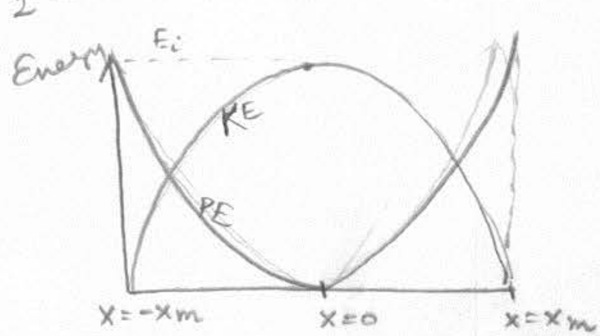
[Note $\vec{F}_R$ is a vector of magnitude kx, always pointing opposite direction as x $\therefore F_R = -kx$]

if $x_i = 0$ & $x_f = x$ then $W_s = -\frac{1}{2} k x^2$

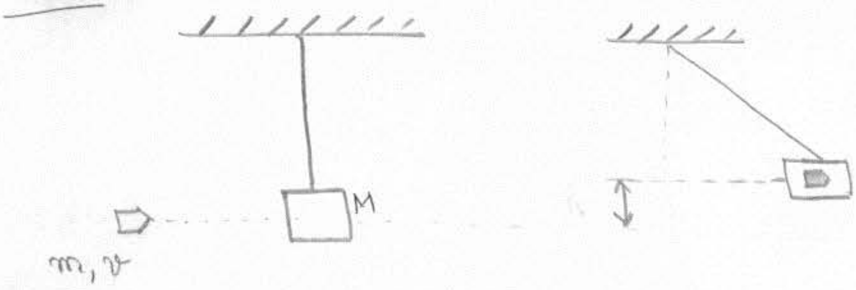
Now,
$$\Delta K = W = -\frac{1}{2} k x^2$$

$$\Delta U = -\Delta K = \frac{1}{2} k x^2$$

$$E(x) = E_i + \Delta K + \Delta U = E_i \text{ (constant)}$$



Problem



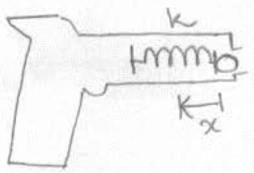
find h

Ballastic pendulum

Energy conservation

$$\frac{1}{2}mv^2 = (m+M)gh \Rightarrow h = \frac{1}{2}\frac{mv^2}{(m+M)g} = \frac{mv^2}{2(m+M)g}$$

Problem



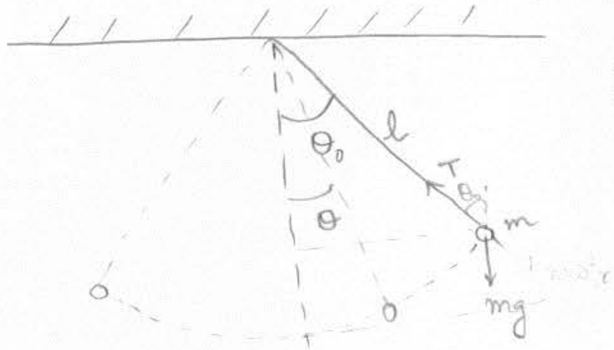
find the velocity of the projected dart ?

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

PE of spring = K.E of dart

[dart detaches from the spring when the spring is $(x=0)$ not extended]

Swinging Pendulum



Initial angle θ_0

mass m

length l

$$T = m\omega^2 l + mg\cos\theta$$

$$\cancel{m}g\sin\theta = \cancel{m}a$$

$$a = g\sin\theta$$

$$a \approx g\theta$$

$$F = g\theta$$

(force opposing increasing θ)

$$E_i = mgh$$

$$= mg(l - l\cos\theta_0)$$

$$E_i = mgl(1-\cos\theta_0) \quad \text{(at height } h\text{)}$$

$$E_f = \frac{1}{2}mv^2 \quad \text{(at bottom)}$$

$$E_f = E_i \Rightarrow \quad \frac{1}{2}\cancel{m}v^2 = \cancel{m}gl(1-\cos\theta_0)$$

$$\boxed{v = \sqrt{2gl(1-\cos\theta_0)}}$$ velocity at bottom

Power

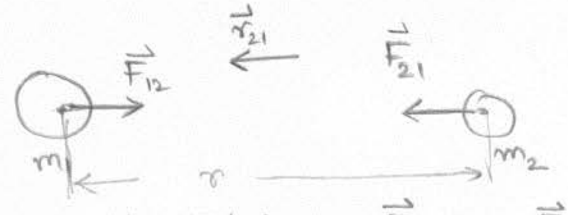
$$P_{avg} = W/\Delta t \quad \text{(average power)}$$

$$P = \frac{dW}{dt} = F\cos\theta\left(\frac{dx}{dt}\right) = F v \cos\theta$$

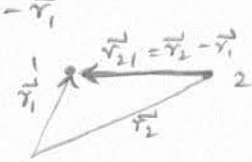
$$P = \vec{F}\cdot\vec{v} \quad \text{(instantaneous power)}$$

Newtons Law of Gravitation.

Attractive forces always



$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$



By Newton's third law, $\vec{F}_{12} = -\vec{F}_{21}$

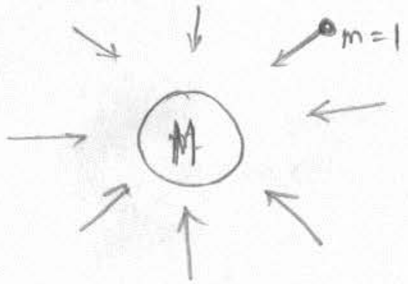
Newtons law of gravitation: $F_{21} \propto \frac{1}{r^2}$ $F_{21} \propto m_2$ $F_{21} \propto m_1$

$$\boxed{F_{21} = G\frac{m_1 m_2}{r^2}} \quad \text{and} \quad \boxed{\vec{F}_{21} = -G\frac{m_1 m_2}{r^2}\hat{r}_{21}}$$

$G = 6.67 \times 10^{-11}\ N\,m^2/kg^2$ (Gravitation const)

[It can be shown that the gravitational force exerted on a particle outside a sphere, with a spherically symmetric mass distribution, is the same as if the entire mass of the sphere was concentrated at its center.] → Comes from Gauss' law applied to Gravitational field.

Gravitational field: Gravitational force acting on a unit mass placed at a distance r away.



Unit mass at r would feel a force with field.

$$\vec{E} = G\frac{M}{r^2} \quad \left[\because\ F = \frac{GM}{r^2}(m) \atop {=1}\right]$$

Note: $F = \left(G\frac{M}{r^2}\right)m = gm$

Think about Earth's gravitational field

$$E = \frac{GM}{R^2} = g$$

and $F = mg$

Gravitational potential energy

$$U = -G\frac{m_1 m_2}{r}$$

$$F = -\frac{dU}{dr} = -\frac{G m_1 m_2 (-1)}{r^2}$$

$$F = G\frac{m_1 m_2}{r^2}$$

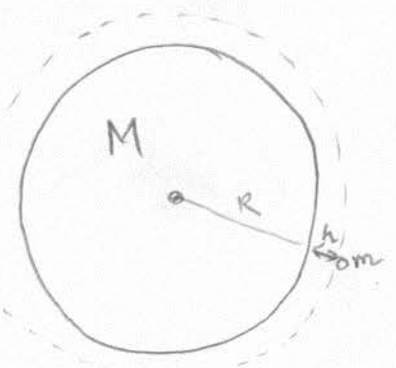
$$F = -\frac{dU}{dr} \Rightarrow -F\,dr = dU$$

$$-\underbrace{\int F\,dr}_{} = \Delta U$$

$$-W = \Delta U$$

Gravitational potential near Earth's surface

Choose, $U = 0$ (on the surface of Earth)



$$U = 0 = -\frac{GM}{R}m \quad \text{[We choose it to be the scale]}$$

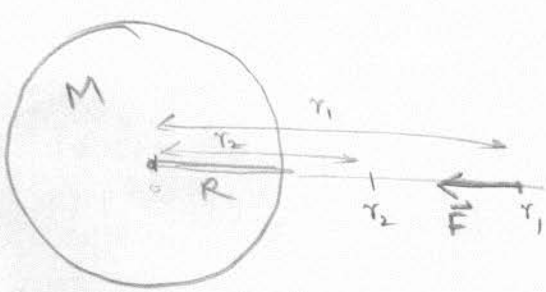
$$U(h) = -\frac{GM}{(R+h)}m = -\frac{GM}{R(1+\frac{h}{R})}m = -\frac{GM}{R}\left(1-\frac{h}{R}\right)m$$

$$U(h) = \underset{\overset{\shortparallel}{0}}{-\frac{GMm}{R}} + \underset{\overset{\shortparallel}{g \text{ (as earlier)}}}{\frac{GM}{R^2}}\,hm$$

$\boxed{U(h) = mgh}$ as seen earlier.

Consider bringing an object towards the earth from distance r_1 to r_2

$$W = \int \vec{F}\cdot d\vec{r} = -GMm\int_{r_1}^{r_2}\frac{1}{r^2}\,dr$$



$$W = -GMm\left.\frac{(-1)}{r}\right|_{r_1}^{r_2} = -GMm\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

if $r_1 \Rightarrow \infty$ & $r_2 = r$ (+h)

$$W = GMm\left(+\frac{1}{r}\right) = \frac{GMm}{r}$$

$$U = -\frac{GMm}{r}$$

at $r \to \infty$ $\quad U(\infty) = 0$

at r, $\quad U(r) = -\frac{GMm}{r}$

Escape Velocity:

At what speed must a body of mass m be thrown upward so that it escapes from the Earth's gravitational field.

$$\Delta U = U(\infty) - U(R) = \frac{GMm}{R}$$

$$\Delta K = \frac{1}{2} m v_{esc}^2$$

$$\therefore \frac{1}{2} \cancel{m} v_{esc}^2 = \frac{GM\cancel{m}}{R} \Rightarrow \boxed{v_{esc} = \sqrt{\frac{2GM}{R}}}$$

$$v_{esc} = 1.12 \times 10^4 \text{ m/s}$$
$$= 11.2 \text{ km/s} \quad (\text{for Earth})$$