# PHY 121: MECHANICS

## SOLUTIONS TO PROBLEM SET 1

1) $\rho = 1.29 \text{ kg/m}^3$ $\quad V = 22.4 \, l \quad (1 \, l = 10^{-3} \text{ m}^3)$

$N = 6.0 \times 10^{23}$ molecules

mass of $V = 22.4 \, l = 22.4 \times 10^{-3} \text{ m}^3$ of air $M = \rho V$

$$M = 1.29 \frac{\text{kg}}{\text{m}^3} \times 22.4 \times 10^{-3} \text{ m}^3 = (1.29 \times 22.4 \times 10^{-3}) \text{ kg}$$

mass of an air molecule, $m = \frac{M}{N} = \frac{1.29 \times 22.4 \times 10^{-3}}{6.0 \times 10^{23}}$ kg/molecule.

$\boxed{m = 4.8 \times 10^{-26} \text{ kg}}$ Answer

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(2) (a) $[v] = LT^{-1}$

(b) $[p] = MLT^{-1}$

(c) $[a] = LT^{-2}$

(d) $[F] = MLT^{-2}$

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(3) LHS $= [E] = [\text{Force}][\text{displacement}] = MLT^{-2}L = ML^2T^{-2}$

RHS $= [mc^2] = M(LT^{-1})^2 = ML^2T^{-2}$

$\because$ LHS = RHS (dimensionally correct)

Given $E = 5 \times 10^{18}$ J $\quad c = 3 \times 10^{8}$ m/s. $\rightarrow$ both have 1 sig fig only

$$m = \frac{E}{c^2} = \frac{5 \times 10^{18} \text{ J}}{9 \times 10^{16} \text{ m}^2/\text{s}^2} = 6 \times 10^{1} \text{ kg} \quad (1 \text{ sig fig})$$

(4) $h_1 = 1.5\, m$ , $h_2 = 1.1\, m$ $\quad \Delta t = 6.2 \times 10^{-4}\, s$

$a_{avg} = ?$

$v_i$ = velocity just before hitting floor

$$\Rightarrow \quad v_i^2 = 2gh_1 \quad \Rightarrow \quad v_i = \sqrt{2gh_1} \quad \text{(directed downward)}$$

$v_f$ = velocity just after hitting floor

$$\Rightarrow \quad 0 = v_f^2 - 2gh_2 \quad \Rightarrow \quad v_f = \sqrt{2gh_2} \quad \text{(directed upward)}$$

$\Delta v = \sqrt{2gh_2} + \sqrt{2gh_1}$ = total change

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{\Delta t} = \frac{\sqrt{2 \times 9.8}\left(\sqrt{1.1} + \sqrt{1.5}\right)}{6.2 \times 10^{-4}}\ m\, s^{-2}$$

$$\boxed{a_{avg} = 1.6 \times 10^4\ m\, s^{-2}}$$

directed in the upward direction
(it is acting opposite to the direction of initial velocity $v_i$
i.e. in the direction opposite to gravity)

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(5) $u = 15.0$ m/s

(a) $\Delta t = 1$ sec. , $h = 11.0$ m

2 cases

1st stone going up

case (i)

case (ii)

1st stone coming down when 2nd hits it

1st stone $\quad h_1 = ut_1 - \frac{1}{2}gt_1^2 \quad -(i) \quad \downarrow g \ \uparrow u$

2nd stone $\quad h_2 = vt_2 - \frac{1}{2}gt_2^2 \quad -(ii)$

$$t_1 - t_2 = \Delta t$$
$$h_1 = h_2 = 11.0\ m$$

eqn (i) - (ii) we get

$$h_1 - h_2 = ut_1 - vt_2 - \frac{1}{2}g(t_1^2 - t_2^2)$$

$$0 = ut_1 - v(t_1 - \Delta t) - \frac{1}{2}g(t_1^2 - (t_1 - \Delta t)^2)$$

$$vt_1 = ut_1 + v\Delta t - \frac{1}{2}g(+2t_1(\Delta t) - (\Delta t)^2) \quad -(a)$$

$$h_1 = ut_1 - \frac{1}{2}gt_1^2 \Rightarrow \frac{1}{2}gt_1^2 - ut_1 + h_1 = 0$$

$$\Rightarrow gt_1^2 - 2ut_1 + 2h_1 = 0 \Rightarrow t_1 = \frac{2u \pm \sqrt{4u^2 - 4g(2h_1)}}{2g}$$

$$t_1 = \frac{u}{g} \pm \frac{\sqrt{u^2 - 2gh_1}}{g} = \frac{15}{10} \pm \frac{\sqrt{15^2 - 2.10.11}}{10}$$ let $g = 10\ ms^{-2}$

$$t_1 = 1.5 \pm 0.224 \quad \left.\begin{aligned} &= 1.723\ s \\ &= 1.276\ s \end{aligned}\right] \text{two possible } t_1$$

eqn (a) $\quad v(t_1 - \Delta t) = ut_1 - \frac{1}{2}g(+2t_1(\Delta t) - (\Delta t)^2)$

Case (ii) $\quad v(1.723 - 1) = 15(1.723) - \frac{1}{2}10(+2(1.723)(1) - 1)$

$v = 18.8$ m/s $\quad$ [stone 1 is coming down when stone 2 is also coming down and they both strike while falling]

Case (i) $\quad v(1.276 - 1) = 15(1.276) - 5(+2(1.276)(1) - 1)$

$v = 41.2$ m/s $\quad$ [Stone 1 is going up and stone 2 comes and hits it]

The two possible velocities are $\boxed{v = 18.8\ m/s \text{ or } v = 41.2\ m/s.}$

Answer.

(b) If second stone is thrown after 1.30 sec, in this time the 1st stone will have already reached its maximum height and would be coming down ($\because$ $\Delta t > 1.276\,s$)

Case (ii) is only possible :

$$v'(1.723 - 1.3) = 15(1.723) - \frac{1}{2}\cdot 10\left(+2(1.723)(1.3) - (1.3)^2\right)$$

$$\Rightarrow \boxed{v' = 28.1 \ \ m/s} \quad \text{Answer}$$

I have used $g = 10\ m/s^2$ ; in case you use $g = 9.8\ m\,s^{-2}$, you will get slightly different answers.

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(6) $d = 190\ m$ $\quad v_{max} = 305\ m/min$ $\quad a = 1.22\ m/s^2$

$$v_{max}^2 = 0 + 2as \Rightarrow s = \frac{v_{max}^2}{2a} = \frac{\left(\frac{305}{60}\frac{m}{sec}\right)^2}{2 \ \ 1.22\ m/s^2} = 10.6\ m$$

a) distance, $\underline{s = 10.6\ m}$

b) to come to rest from $v_{max}$, it travells the same distance $s$ again

Thus, $h = d - 2s = 190\,m - 2(10.6)\,m = 168.8\,m$ is the distance travelled without acceleration with velocity $v_{max}$

$$\therefore \text{ time, } t = \frac{h}{v_{max}} = \frac{168.8\ m}{\frac{305}{60}\frac{m}{sec}} = 33.2 \ sec.$$

acceleration time, $v_{max} = a\,t_a \Rightarrow$ $\quad t_a = \frac{v_{max}}{a} = \frac{305}{60} \times \frac{1}{1.22}\ sec$

$$t_a = 4.17 \ sec.$$

Total time, $t_{TOT} = 2(4.17) + 33.2 = \underline{41.5 \ sec.}$

(7) $d_x = 40\,m$ , $d_y = 50\,m$ $t = 2$ sec.

a) $v_{x0} = \frac{d_x}{t} = \frac{40\,m}{2\,sec} = \underline{20\ m/s}$

b) $d_y = v_{y0}t - \frac{1}{2}gt^2 \Rightarrow 50 = 2v_{y0} - \frac{1}{\cancel{2}}\cdot(9.8)\cdot\cancel{4}^2$

$\underline{v_{y0} = 34.8\ \frac{m}{s}}$

(c) $0 = v_{y0} - gt \Rightarrow t = \frac{v_{y0}}{g} = \frac{34.8\ m/s}{9.8\ m/s^2} = 3.55\ s$

$d'_x = v_{ox}t = 20\ \frac{m}{s}\ 3.55\ s = \underline{71\ m}$ (horizontal distance)

(d) $R = 2\times d'_x = 2\times 71\ m = \underline{142\ m}$

(e) If there was no acceleration due to gravity, the ball would continue to move in a straight line with the constant initial velocity of projection

$\vec{v} = v_{x0}\hat{i} + v_{y0}\hat{j} = (20\hat{i} + 34.8\hat{j})\ m/s$

$\vec{r}_1 = 3\hat{i} + 5\hat{j} + 2\hat{k}$ (initial position vector)

$\vec{r}(t) = (3 + v_{x0}t)\hat{i} + (5 + v_{y0}t)\hat{j} + 2\hat{k}$

$$\boxed{\vec{r}(t) = (3 + 20t)\hat{i} + (5 + 34.8t)\hat{j} + 2\hat{k}}$$

(8) $\vec{r} = (2.00\,t^3 - 5.00\,t)\,\hat{i} + (6.00 - 7.00\,t^4)\,\hat{j}$

$\vec{r}$ in meters & $t$ in sec.

at $t = 2.00$ sec.

(a) $\vec{r}\,(t = 2.00\text{ sec}) = \left[2.00\,(2)^3 - 5.00\,(2)\right]\hat{i} + \left[6 - 7\,(2)^4\right]\hat{j}$

$$\boxed{\vec{r} = 6.00\,\hat{i} - 106.\,\hat{j}}$$

(b) $\vec{v} = \dfrac{d\vec{r}}{dt} = (2 \times 3t^2 - 5)\,\hat{i} + (-7 \times 4.\,t^3)\,\hat{j}$

$\vec{v} = (6.00\,t^2 - 5.00)\,\hat{i} - 28.0\,t^3\,\hat{j}$

$\vec{v}\,(t = 2.00\text{ sec}) = (6 \times 4 - 5)\,\hat{i} - 28\,(2)^3\,\hat{j} = 19.0\,\hat{i} - 224\,\hat{j}$

$$\boxed{\vec{v} = 19.0\,\hat{i} - 224\,\hat{j}}$$

(c) $\vec{a} = \dfrac{d\vec{v}}{dt} = 12\,t\,\hat{i} - 28 \times 3\,t^2\,\hat{j}$

$\vec{a}\,(t = 2.00\text{ sec}) = 12 \times 2\,\hat{i} - 28 \times 3 \times (2)^2\,\hat{j}$

$$\boxed{\vec{a} = 24.0\,\hat{i} - 336\,\hat{j}}$$

(d) The particle follows the path traced by $\vec{r}$

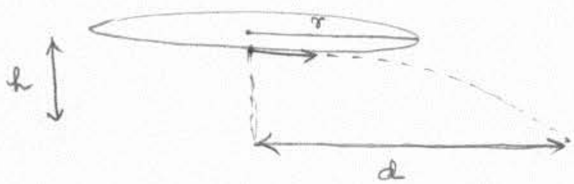
and $\dfrac{d\vec{r}}{dt} = \vec{v}$ is the tangent to the path at every instant.

at $t = 2.00$ sec $\dfrac{\vec{v}}{|\vec{v}|} = \hat{v} = \dfrac{19.0\,\hat{i} - 224\,\hat{j}}{\sqrt{19^2 + 224^2}} = \dfrac{19}{224.8}\,\hat{i} - \dfrac{224}{224.8}\,\hat{j}$

This is the direction (unit vector) of the tangent vector

to the trajectory at $t = 2.00$ sec.

(9) $r = 2.0\,m$ $h = 3.0\,m$ $d = 12\,m$




$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$ (time of flight)

$d = v t \Rightarrow v = \frac{d}{t} = d\sqrt{\frac{g}{2h}}$

$v = d\sqrt{\frac{g}{2h}}$ (velocity of swirling stone)

Magnitude of centripetal acceleration,

$$a = \frac{v^2}{r} = d^2 \frac{g}{2h} \frac{1}{r} = 144 \times \frac{9.8}{2\times 3} \times \frac{1}{2} \ ms^{-2}$$

$\boxed{a = 117.6 \ ms^{-2}}$ Answer. $\Rightarrow a = 120 \ ms^{-2}$ (2 sig fig)

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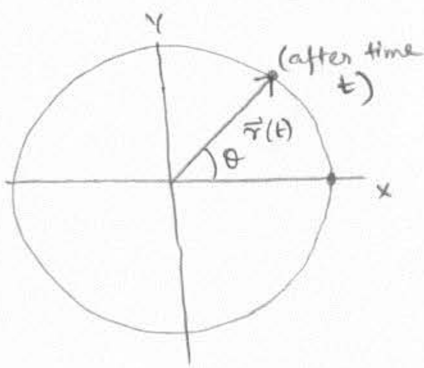
(10) $r$ $T$

at $t=0$




at $t=0$

$\vec{r}(t=0) = r\,\hat{i}$

$\vec{r}(t=t) = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$

$\theta = 2\pi$ when $t = T$. $\Rightarrow$ when $t=t$, $\theta = \frac{t}{T}2\pi = \left(\frac{2\pi}{T}\right)t$

$\therefore \boxed{\vec{r}(t) = r\cos\left(\frac{2\pi}{T}t\right)\hat{i} + r\sin\left(\frac{2\pi}{T}t\right)\hat{j}}$ $\omega = \frac{2\pi}{T}$

$\vec{v}(t) = \frac{d\vec{r}}{dt} = -r\omega\sin\omega t\,\hat{i} + r\omega\cos\omega t\,\hat{j}$

$\boxed{\vec{v}(t) = -r\left(\frac{2\pi}{T}\right)\sin\left(\frac{2\pi}{T}t\right)\hat{i} + r\left(\frac{2\pi}{T}\right)\cos\left(\frac{2\pi}{T}t\right)\hat{j}}$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = -r\omega^2 \cos\omega t\ \hat{i} - r\omega^2 \sin\omega t\ \hat{j}$$

$$\boxed{\vec{a}(t) = -r\left(\frac{2\pi}{T}\right)^2 \cos\left(\frac{2\pi}{T}t\right)\hat{i} - r\left(\frac{2\pi}{T}\right)^2 \sin\left(\frac{2\pi}{T}t\right)\hat{j}}$$

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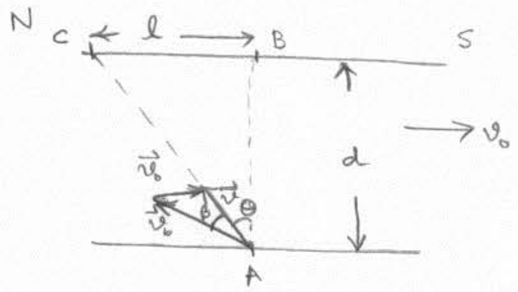
(11)




$v_b$

$$\vec{v} = \vec{v}_b + \vec{v}_0$$

$\vec{v}$ is the velocity with which the boat actually moves, and this $\vec{v}$ must take him to the destination C. To do that he rows with $\vec{v}_b$

let angle between $\vec{v}$ and AB be $\theta$

" " $\vec{v}_b$ and $\vec{v}$ be $\beta$

If it takes time $t$ to cross the river then,

$$v_b \cos(\theta+\beta)\, t = d = v\cos\theta . t \quad -(i)$$ (component $\perp$ to river flow direction)

$$\left[v_b \sin(\theta+\beta) - v_0\right] t = l = v\sin\theta . t \quad -(ii)$$

and $$\tan\theta = \frac{l}{d} \quad -(iii)$$

divide (ii) by (i) we get. $$\frac{v_b \sin(\theta+\beta) - v_0}{v_b\cos(\theta+\beta)} = \frac{l}{d} = \tan\theta.$$

$$\tan(\theta+\beta) - \frac{v_0}{v_b\cos(\theta+\beta)} = \frac{l}{d} \Rightarrow \tan(\theta+\beta) = \frac{l}{d} + \frac{v_0}{v_b\cos(\theta+\beta)}$$

$$\Rightarrow \sin(\theta+\beta) = \frac{l}{d}\cos(\theta+\beta) + \frac{v_0}{v_b} \Rightarrow \sin(\theta+\beta) - \frac{l}{d}\cos(\theta+\beta) = \frac{v_0}{v_b}$$

$$\cos\theta\sin(\theta+\beta) - \sin\theta\cos(\theta+\beta) = \frac{v_0}{v_b}\cos\theta$$ (mult by $\cos\theta$ & use $\tan\theta = \frac{l}{d}$)

$$\sin(\theta+\beta-\theta) = \frac{v_0}{v_b}\cos\theta \Rightarrow \sin\beta = \frac{v_0}{v_b}\cos\theta$$

$$\Rightarrow \beta = \sin^{-1}\left(\frac{v_0}{v_b}\frac{d}{\sqrt{l^2+d^2}}\right)$$

Thus, the direction with respect to the $\perp$ line to the stream is at an angle $\boxed{\theta+\beta = \tan^{-1}\left(\frac{d}{l}\right) + \sin^{-1}\left(\frac{v_0}{v_b}\frac{d}{\sqrt{l^2+d^2}}\right)}$ Answer

$$\boxed{t = \frac{d}{v_b\cos(\theta+\beta)}}$$ Answer

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(12) $m$ (mass)

$$\vec{r}(t) = a\cos(\omega t)\hat{i} + a\sin(\omega t)\hat{j} + c\hat{k}$$

$a, \omega, c$ are constants

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = -a\omega\sin(\omega t)\hat{i} + a\omega\cos(\omega t)\hat{j}$$

$$\vec{v}(t) = -a\omega\sin(\omega t)\hat{i} + a\omega\cos(\omega t)\hat{j}$$

$$\boxed{\vec{p}(t) = -am\omega\sin(\omega t)\hat{i} + am\omega\cos(\omega t)\hat{j}}$$ Answer

$$\vec{L} = \vec{r}\times\vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a\cos\omega t & a\sin\omega t & c \\ -am\omega\sin\omega t & am\omega\cos\omega t & 0 \end{vmatrix} = \hat{i}(-am\omega c\cos\omega t) - \hat{j}(+cam\omega\sin\omega t) + \hat{k}(a^2m\omega\cos^2\omega t + a^2m\omega\sin^2\omega t)$$

Answer

$$\boxed{\vec{L} = -am\omega c\cos(\omega t)\hat{i} - am\omega c\sin(\omega t)\hat{j} + a^2m\omega\hat{k}}$$