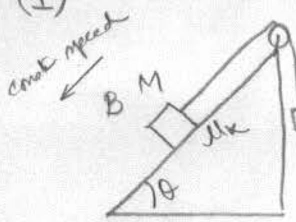
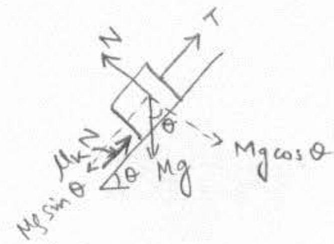
Homework 2.

SOLUTION SET.

(1)





Friction acts upward because block moves down the slope

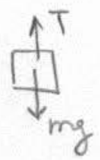
<u>Along plane</u>:

$$Mg\sin\theta = T + \mu_K N$$

<u>$\perp$ to plane</u>:

$$N = Mg\cos\theta$$

$$\therefore \quad Mg\sin\theta = T + \mu_K Mg\cos\theta \quad -(i)$$



$$T = mg \quad -(ii)$$

$$\therefore \quad M\cancel{g}\sin\theta = m\cancel{g} + \mu_k M\cancel{g}\cos\theta \qquad \text{from (i) \& (ii)}$$

$$\Rightarrow \quad m = M\sin\theta - \mu_K M\cos\theta$$

(i) $\boxed{m = M\left[\sin\theta - \mu_K\cos\theta\right]}$ <u>Answer.</u>

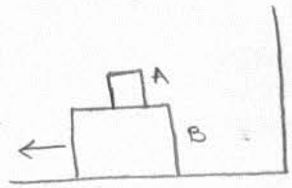
(ii) $M = 20$ kg, $\mu_K = 0.30$ $\quad \theta = 30°$

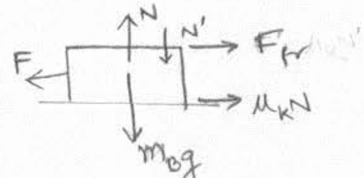
$$m = 20\left[\frac{1}{2} - 0.30\frac{\sqrt{3}}{2}\right] \text{kg} = \underline{4.8 \text{ kg}}$$ <u>Answer.</u>

(2) $m_A g = 1.20\ N$ $\quad m_B g = 3.60\ N$ $\quad \mu_K = 0.300$

(a)

Const speed



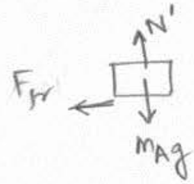


Horizontal eqlb:

$$F = F_{fr} + \mu_K N \quad \text{— (i)}$$

Vert eqlb: $N = m_B g + N'$ — (ii)

Block A:

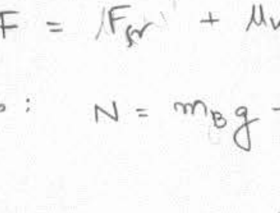


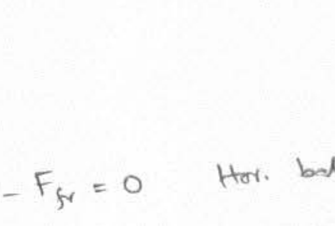
(iii) — $F_{fr} = 0$ Hor. balance

(iv) — $N' = m_A g$ vertical balance.

$$F = \mu_K (m_B g + m_A g) = 0.300\,(3.60 + 1.20)\,N = \underline{\underline{1.44\ N}}.$$ Answer

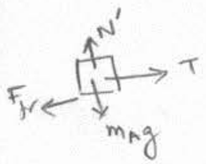
(b)





$F = F_{fr} + \mu_K N$ — (i)

$N = N' + m_B g$ — (ii)



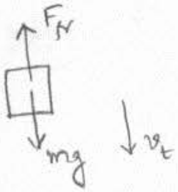
$T = F_{fr}$ (iii) $(F_{fr} = \mu_K N')$

$N' = m_A g$ (iv)

$$F = \mu_K (m_A g) + \mu_K (m_A g + m_B g) = (2 m_A g + m_B g)\,\mu_K$$

$$F = (2 \times 1.20 + 3.60)\ 0.300 = \underline{\underline{1.80\ N}}$$ Answer

(3) $v_{t1} = 100$ km/hr $\quad v_{t2} = 300$ km/hr $\quad C$ (drag coefficient)



$$F_{fr} = mg \Rightarrow \frac{1}{2} C \rho A v_t^2 = mg \Rightarrow A = \left(\frac{2mg}{C\rho}\right)\frac{1}{v_t^2}$$

Ratio : $\dfrac{A_1}{A_2} = \dfrac{v_{t2}^2}{v_{t1}^2} = \dfrac{9}{1}$

Ratio of cross section ; slower position : faster position

$\boxed{A_1 : A_2 = 9 : 1}$ Answer

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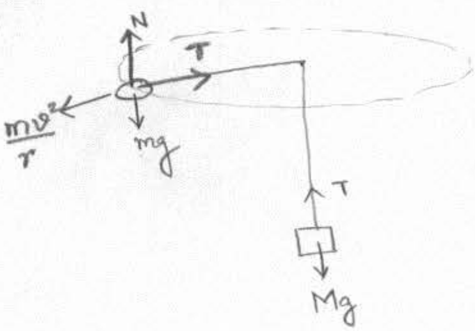
(4)



For dice,

$\perp$ to table equilibrium :

$N = mg \quad -(i)$

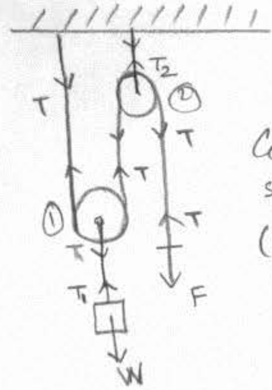
radial eqlb :

$\dfrac{mv^2}{r} = T \quad -(ii)$

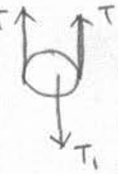
For hanging mass, $\quad T = Mg \quad -(iii)$

$\dfrac{mv^2}{r} = Mg \Rightarrow \boxed{v = \sqrt{\dfrac{Mgr}{m}}}$ Answer

(5)

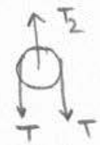


Constant speed lift (No acceleration)

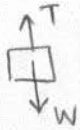


①

$2T = T_1$ —(i)



$2T = T_2$ —(ii)



$W = T_1$ (iii)

$F = T$ —(iv)

∴ Tension in chains, $\boxed{\begin{aligned} T_1 &= W \\ T_2 &= 2T = T_1 = W \end{aligned}}$ Answer

$F = T = \frac{T_1}{2} = \frac{W}{2}$ ∴ $\boxed{F = \frac{W}{2}}$ Answer

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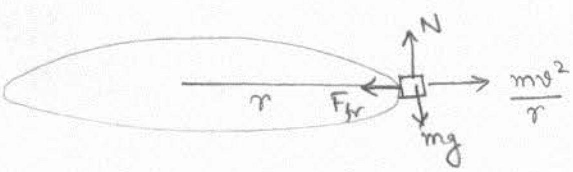
(6)

$r = 220$ m

$v = 25.0$ m/s.



Non-inertial frame (centrifugal force outward)

$F_{fr} = \frac{mv^2}{r}$

for radial equilibrium

$F_{fr} = \mu_{s_{min}} N$

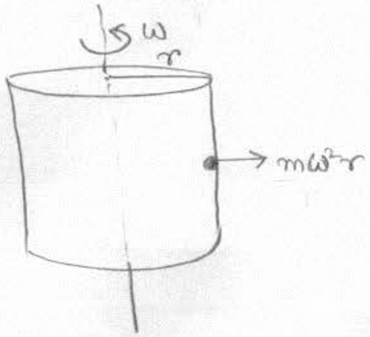
$N = mg$

∴ $\mu_{s_{min}} (\cancel{m} g) = \frac{\cancel{m} v^2}{r}$ $\Rightarrow$ $\boxed{\mu_{s_{min}} = \frac{v^2}{r g}}$

Thus, $\mu_{s_{min}} = \frac{25^2}{220 (9.8)} = \underline{0.290}$ is the minimum coefficient of friction required to prevent sliding.

Answer

(7)



$2r = 800m$

$a = 9.8\ m/s^2$

centrifugal acceleration

$$a = \omega^2 r = 9.8\ m s^{-2}$$

$$\therefore \omega = \sqrt{9.8/r} = \sqrt{(9.8)/400}\ s^{-1} = \frac{2\pi}{T}$$

No. of rotations per second $= \frac{1}{T} = \frac{\sqrt{(9.8)/400}}{2\pi}\ s^{-1}$

No. of rotations per minuite $\Rightarrow \frac{1}{T} = \frac{\sqrt{(9.8)/400}}{2\pi} \times 60\ min^{-1}$

(i) $\nu = \frac{1}{T} = 1.49$ rotations per minuite. Answer

(ii) $g = 3.70\ ms^{-2}$

$$g = \omega^2 r \Rightarrow \omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{3.70}{400}}\ s^{-1} = \frac{2\pi}{T}$$

No. of rotations per min.; $\nu = \frac{1}{T} = \frac{\omega}{2\pi} = \sqrt{\frac{3.70}{400}}\ \frac{1}{2\pi} \times 60\ min^{-1}$

$\nu = \frac{1}{T} = 0.92$ rev/min. Answer

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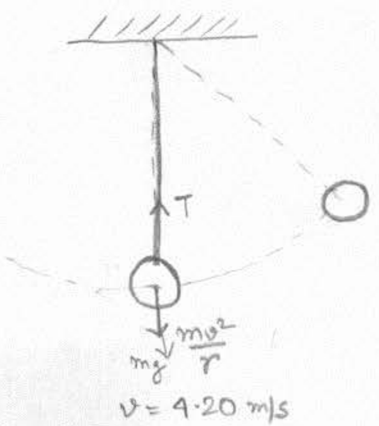
(8)



$W = 71.2\ N$ $\quad r = 3.80\ m$

$mg + \frac{mv^2}{r} = T$ (Radial force balance)

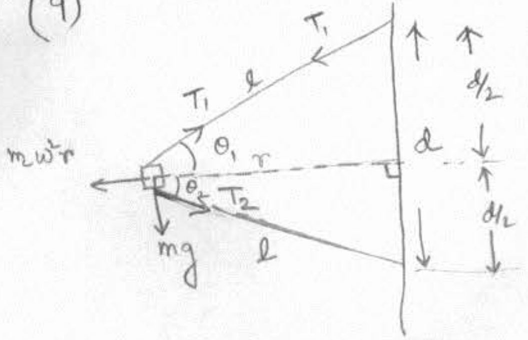
(i) $a_R = \frac{v^2}{r} = \frac{(4.20)^2\ m^2s^{-2}}{3.80\ m} = 4.64\ ms^{-2}$ Answer

Radial acceleration

$a_t = 0$

(ii) $T = W + \left(\frac{W}{g}\right) a_R = \left[71.2 + \frac{71.2}{9.8}\ 4.64\right] N = 105\ N$ Answer

(9)



$T_1 = 80.0$ N $\quad m = 4.00$ kg

$l = 1.25$ m

$d = 2.00$ m

Radial :

$$m\omega^2 r = T_1 \cos\theta_1 + T_2 \cos\theta_2 \quad -(i)$$

Vertical :

$$mg + T_2 \sin\theta_2 = T_1 \sin\theta_1 \quad -(ii)$$

$$\left.\begin{aligned} \cos\theta_1 &= \frac{r}{l} \\ \cos\theta_2 &= \frac{r}{l} \end{aligned}\right] \therefore \cos\theta_1 = \cos\theta_2 \Rightarrow \theta_1 = \theta_2 \quad -(iii)$$

and $\sin\theta_1 = d/2 / l = \dfrac{d}{2l} \Rightarrow \theta_1 = \sin^{-1}\left(\dfrac{d}{2l}\right) \quad -(iv)$

from (i) & (ii)

$$m\omega^2 r = \cos\theta\,(T_1 + T_2)$$

$$mg = (T_1 - T_2)\sin\theta \Rightarrow \boxed{T_2 = T_1 - \frac{mg}{\sin\theta}}$$

(a) $T_2 = 80.0\,\text{N} - \dfrac{4.00 \times 9.8}{2.00/2 \times 1.25}\,\text{N} = \underline{\underline{31\,\text{N}}}$ Answer

(b) $\omega^2 = \dfrac{(T_1 + T_2)\cos\theta}{mr} = \dfrac{(T_1 + T_2)\,r}{m r l} = \dfrac{80 + 31}{4 \times 1.25} = 22.2$

$\omega = 4.71\ s^{-1} \quad \therefore \nu = \dfrac{1}{T} = \dfrac{\omega}{2\pi} = \dfrac{4.71}{2\pi}\ s^{-1} = 0.750\ s^{-1}$

$\therefore$ Number of revolutions per minute, $\nu = 45.0$ rev/min.

(c) For lower cord just going slack $T_2 = 0$.

eqn (ii) gives $\quad \omega^2 = \cos\theta \left(\dfrac{T_1}{mr}\right) = \dfrac{T_1 \cancel{\cos\theta}}{m\, l\, \cancel{\cos\theta}} \Rightarrow \omega = \sqrt{\dfrac{T_1}{ml}}$

$\omega = \sqrt{\dfrac{80}{4 \times 1.25}}\ s^{-1} = 4\ s^{-1}$

$\nu = \dfrac{1}{T} = \dfrac{\omega}{2\pi} \times 60\ \text{min}^{-1} = 38.2\ \dfrac{\text{rev}}{\text{min}}$

Answer

(d) If the number of revolutions/min is less than part (c) then the block can not be in equilibrium anymore and it will drop down with an acceleration.

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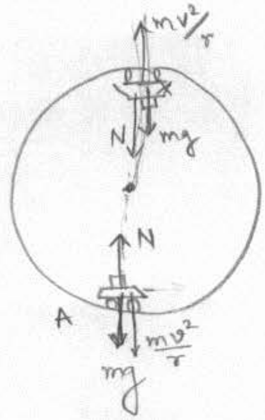
(10) $m = 1.60$ kg $\quad v = 12.0$ m/s $\quad r = 5.00$ m



(a) Radial equilibrium at A:

$$N = \frac{mv^2}{r} + mg = \frac{1.60\,(12)^2}{5} + (1.6)(9.8)$$

$\boxed{N = 61.8 \text{ N}}$ at pos. A. (upward)

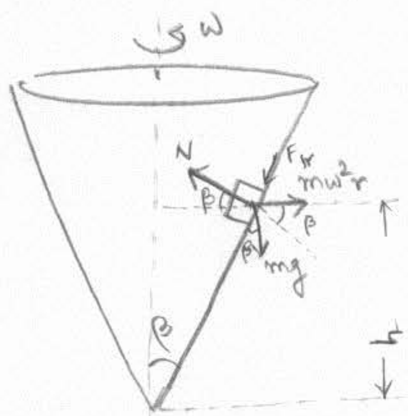
(b) Radial equilibrium at B:

$$N + mg = \frac{mv^2}{r} \Rightarrow N = \frac{mv^2}{r} - mg = 30.4 \text{ N}$$

$\boxed{N = 30.4 \text{ N}}$ at position B (downward)

$$\omega = \frac{2\pi}{T} \; ; \; \mu_s$$

(11)



(a) When the cone is rotated at a higher ω, it has a tendency to fly up in this case F_{fr} will be acting downward along the slope

Blance along slope:

$$mg\cos\beta + \mu_s N = m\omega^2 r \sin\beta \quad -(i)$$

Blance $\perp$ to slope:

$$N = m\omega^2 r\cos\beta + mg\sin\beta \quad -(ii)$$

$$mg\cos\beta + \mu_s\left(m\omega^2 r\cos\beta + mg\sin\beta\right) = m\omega^2 r\sin\beta$$

$$\Rightarrow \omega^2\left(\mu_s m r\cos\beta - m r\sin\beta\right) = -mg\cos\beta - \mu_s m g\sin\beta$$

$$\omega_{max}^2 = \frac{\cancel{m}g(\cos\beta + \mu_s \sin\beta)}{\cancel{m}r(\sin\beta - \mu_s\cos\beta)} \Rightarrow \omega_{max} = \sqrt{\frac{g}{r}\frac{(\cos\beta+\mu_s\sin\beta)}{(\sin\beta - \mu_s\cos\beta)}}$$

$$\therefore \quad T_{min} = \frac{2\pi}{\omega_{max}} = 2\pi\sqrt{\frac{r}{g}\frac{(\sin\beta - \mu_s\cos\beta)}{(\cos\beta+\mu_s\sin\beta)}} \qquad \frac{r}{h} = \tan\beta$$

$$T_{min} = 2\pi\sqrt{\frac{h}{g}\tan\beta\left(\frac{\sin\beta-\mu_s\cos\beta}{\cos\beta+\mu_s\sin\beta}\right)} = 2\pi\sqrt{\frac{h}{g}\left(\frac{\tan\beta-\mu_s}{\cot\beta+\mu_s}\right)}$$

(b)
Now, when the cone is rotated at a lower ω, the block has a tendency to fall down the slope due to its weight. In this case the F_{fr} acts upward and stops it from falling.

Balance along slope:

$$mg\cos\beta - \mu_s N = m\omega^2 r\sin\beta$$

Balance $\perp$ to slope:

$$N = m\omega^2 r\cos\beta + mg\sin\beta$$

$$\therefore \quad mg\cos\beta - \mu_s(m\omega^2 r\cos\beta + mg\sin\beta) = m\omega^2 r\sin\beta$$

$$\therefore \quad \omega^2(\cancel{m}r\sin\beta + \mu_s\cancel{m}r\cos\beta) = \cancel{m}g\cos\beta - \mu_s\cancel{m}g\sin\beta$$

$$\omega_{min}^2 = \frac{g(\cos\beta - \mu_s\sin\beta)}{r(\sin\beta+\mu_s\cos\beta)}$$

$$T_{max} = \frac{2\pi}{\omega_{min}} = 2\pi\sqrt{\frac{r(\sin\beta+\mu_s\cos\beta)}{g(\cos\beta-\mu_s\sin\beta)}} = 2\pi\sqrt{\frac{h}{g}\frac{(\tan\beta+\mu_s)}{(\cot\beta-\mu_s)}}$$

$$\therefore \quad \boxed{\underbrace{2\pi\sqrt{\frac{h}{g}\frac{(\tan\beta-\mu_s)}{(\cot\beta+\mu_s)}}}_{T_{min}} < T < \underbrace{2\pi\sqrt{\frac{h}{g}\frac{(\tan\beta+\mu_s)}{(\cot\beta-\mu_s)}}}_{T_{max}}}$$

Answer

(12) $m = 2.00\ kg$, $k = 400\ \frac{N}{m}$, $x = 0.220\ m$

$\theta = 37°$

(a) $\frac{1}{2} m v^2 = \frac{1}{2} k x^2$ (energy in spring provides the KE for horizontal motion)

$$v = \sqrt{\frac{k x^2}{m}} = \sqrt{\frac{400\,(0.22)^2}{2}} = \underline{\underline{3.11\ m s^{-1}}}$$ Answer

(b) $\frac{1}{2} m v^2 = m g h$ (KE initially, takes the block to height h with PE only)

$$h = \frac{v^2}{2g} = \frac{k x^2}{2 m g} = \frac{400\,(0.22)^2}{2 \times 2 \times 9.8} = 0.494\ m$$

$$d = \frac{h}{\sin\theta} = \frac{0.494}{\sin 37} = \underline{\underline{0.821\ m}}$$ Answer.