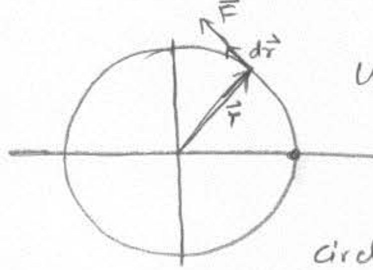
SOLUTIONS TO HOMEWORK 3.

INSTRUCTOR : ARIJIT BOSE.

(1) $\vec{F} = \left(\frac{F_0}{r}\right)(y\hat{i} - x\hat{j})$ $\quad F_0 = \text{const.} \quad r = \sqrt{x^2+y^2}$

a) $|\vec{F}| = \frac{F_0}{r}\sqrt{y^2+x^2} = F_0 \frac{r}{r} = F_0$ $\quad \left[\text{Use} : |\vec{F}|^2 = \vec{F}.\vec{F} \right]$

$\vec{F}.\vec{r} = \left(\frac{F_0}{r}\right)\left[xy - yx\right] = 0 \quad \therefore \vec{F} \perp \vec{r}$



b) $dW = \vec{F}.d\vec{r} \quad (r = 5m)$

Use: $d\vec{r} = -dx\hat{i} + dy\hat{j}$

$dW = \frac{F_0}{r}\left(-y\,dx - x\,dy\right)$

the body moves in a circle so $d\vec{r} \perp \vec{r}$

$\therefore W = \frac{F_0}{r}\left[-\int_{x_i}^{x_i}\sqrt{r^2-x^2}\,dx - \int_{y_i}^{y_i}\sqrt{r^2-y^2}\,dy\right]$

Check: $\vec{r}.d\vec{r} = -x\,dx + y\,dy$

and, $x^2+y^2 = 5^2 (= r^2)$

$\cancel{2}x\,dx + \cancel{2}y\,dy = 0$

$\Rightarrow x\,dx = -y\,dy$

it comes back to the same point.

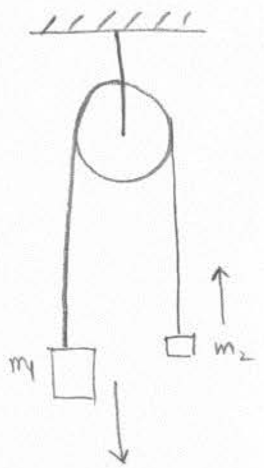
$\therefore \vec{r}.d\vec{r} = 0$

$W = \frac{F_0}{r}\left[0 + 0\right] = 0.$

$\therefore \boxed{W = 0}$

Since, The work done to take the particle around a closed path is zero. The force is conservative.

(2)

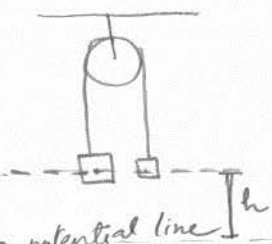


$t = 3.0$ sec $\quad u = 0 \quad v = 4$ m/s

$K_f = 80$ J $\quad h = 6.0$ m

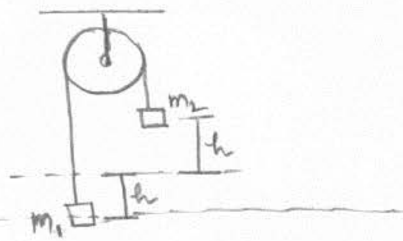
Suppose initially,

Initial,



$V = 0$, zero potential line

final:



$E_i = E_f$

(Energy conservation)

$E_i = m_2 g h + m_1 g h \qquad E_f = m_2 g (2h) + \frac{1}{2} m_2 v^2 + \frac{1}{2} m_1 v^2$

$\therefore (m_1 + m_2) g h = m_2 g (2h) + \frac{1}{2}(m_1 + m_2) v^2$

$m_1 g h = m_2 g h + \frac{1}{2}(m_1 + m_2) v^2 \Rightarrow \frac{1}{2}(m_1 + m_2) v^2 = (m_1 - m_2) g h$

Also, $\frac{1}{2}(m_1 + m_2) v^2 = 80\text{J}$ (given)

$\therefore (m_1 - m_2) g h = 80\text{ J} \Rightarrow m_1 - m_2 = \frac{80}{(9.8) \times 6} = 1.36$

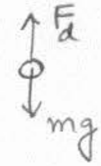
$\frac{1}{2}(m_1 + m_2) v^2 = 80\text{ J} \Rightarrow m_1 + m_2 = \frac{2 \times 80}{16} = 10$

$\therefore \quad 2m_1 = 11.36 \quad$ and $\quad 2m_2 = 10 - 1.36$

$\therefore \quad m_1 = 5.68$ kg $\quad$ and $\quad m_2 = 4.32$ kg

To 2 sig fig, $\quad m_1 = 5.7$ kg $\quad$ and $\quad m_2 = 4.3$ kg $\quad$ (Answer)

(3) $\quad v_t = 120$ km/hr $\qquad m = 55$ kg



$\downarrow v_t \quad \therefore F_d = mg$

$$P = -F_d v_t = -(mg)\, v_t$$

$$P = -(55 \times 9.8)\, 120 \times \frac{5}{18} \ \ J s^{-1}$$

(a) $\quad P = 17,966.67 \ J s^{-1} \approx 18000$ W $\quad$ (2 sig fig)

(b)
$$P = -F_d' v_t'$$
$$= -mg\, v_t'$$
$$P = -(55 \times 9.8)\, 15 \times \frac{5}{18} \ \text{W}$$
$$P = 2245.8 \ \text{W}$$
$$P \approx 2200 \ \text{W} \quad (2 \text{ sig fig})$$

Note:

$\downarrow v_t' \qquad F_d' = mg$

What happens here is

earlier $\quad F_d = \underbrace{c}_{\text{depends on area of c.s.}} v_t^2$

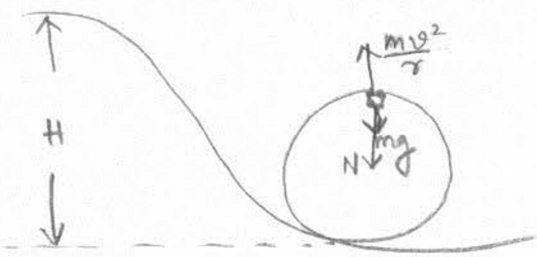
Now, $\quad F_d' = c' v_t'^2$ $\;\to$ this $v_t' < v_t$

and $c' > c$

So, c' & v_t' are so adjusted

such that, $\quad F_d' = F_d = mg$

(4) $m = 1500\ kg$ $H = 23\ m$ $r = \frac{15}{2}\ m$



We need to find N.

$N + mg = \frac{mv^2}{r}$

Need v?

Energy conservation.

$E_i = mgH$

$E_f = \frac{1}{2}mv^2 + mg(2r)$

$E_i = E_f$

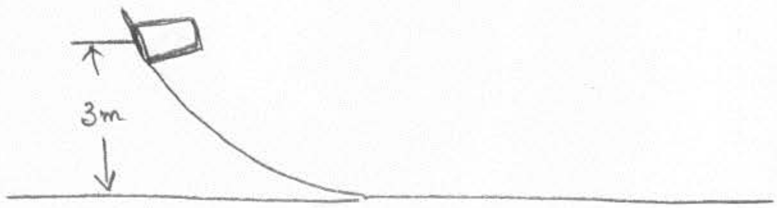
$\cancel{m}gH = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g(2r) \Rightarrow$ $v^2 = 2(gH - 2gr)$

$\therefore N = \frac{m}{r}2g(H-2r) - mg = mg\left[\frac{2(H-2r)}{r} - 1\right] = mg\left(\frac{2H-5r}{r}\right)$

$N = \overset{100}{\cancel{1500}} \times 9.8 \left(\frac{2(23) - 5(15/2)}{\cancel{15}/2}\right) = 16,660\ N$

$\therefore$ $\boxed{N \approx 17,000\ N \quad (2\ \text{sig fig})}$

(5)



Energy conservation,

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 \Rightarrow v^2 = 2gh \Rightarrow v = \sqrt{2gh}$$

a) $v = \sqrt{2(9.8)3} = 7.67\ ms^{-1} \approx \underline{7.7\ ms^{-1}\ (2\ \text{sig fig})}$

b) Energy dissipated, $E = \frac{1}{2}mv^2 = \frac{1}{2}(2)\,2(9.8)3 = \underline{58.8\ J}$

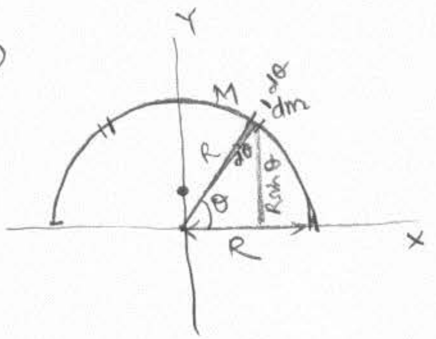
(c) $Fd = E \Rightarrow F = \frac{E}{d} \Rightarrow \mu_K = \frac{E}{Nd}$

Work done against friction = KE lost

$N = mg$ $\quad \therefore \mu_K = \frac{E}{mgd} = \frac{58.8}{2\times 9.8\times 9} = 0.33$

Thus, $\boxed{\mu_K = 0.33}$ Answer.

(6) (a)



$x_{cm} = 0 \qquad Y_{cm} = ?$

$$y_{cM} = \frac{\int_{\theta=0}^{\pi} dm\ R\sin\theta}{\int_{\theta=0}^{\pi} dm}$$

$dm = \lambda(R\,d\theta)$

λ = linear density

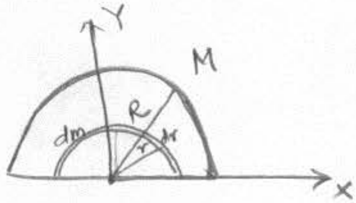
$\lambda = \frac{M}{\pi R}$

$$y_{cM} = \frac{\int_0^{\pi} \lambda R^2 d\theta \sin\theta}{\int_0^{\pi} \lambda R\, d\theta} = \frac{\cancel{\lambda} R^{\cancel{2}}(-\cos\theta)_0^{\pi}}{\cancel{\lambda}\cancel{R}\pi} = \frac{R}{\pi}(1+1)$$

$y_{cM} = \frac{2R}{\pi}$ $\quad \therefore$ position of CM; $(x_{cm}, y_{cm}) = \left(0, \frac{2R}{\pi}\right)$

Answer.

(b)



$$x_{cm} = 0 \qquad y_{cm} = ?$$

$$dm = \pi r \, dr$$

$$y_{cm} = \frac{\int dm \; 2r/\pi}{\int dm}$$

$$y_{cm} = \frac{\int_0^R \pi r \, dr \; \frac{2r}{\pi}}{\int_0^R \pi r \, dr} = \frac{2 \int_0^R r^2 \, dr}{\pi \int_0^R r \, dr} = \frac{2}{\pi} \frac{R^3}{3} \frac{1}{R^2/2} = \frac{4}{3} \frac{R}{\pi}$$

$$\therefore \left(x_{cm}, y_{cm} \right) = \left(0, \frac{4R}{3\pi} \right)$$ Answer.

(7)
a) Conservation of momentum,

throwing 1st weight

$$m_T \cdot 0 = \underbrace{(m_s + m_w)}_{\text{mass of skater + skate + second load}} v_{s1} - \underbrace{m_w}_{\text{first load}} \underbrace{(v_{W1} - v_{s1})}_{\text{velocity of weight w.r.t. stationary observer}}$$

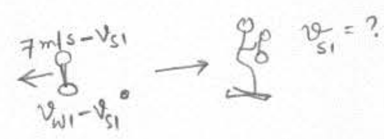
$$(m_W + m_s) v_{s1} = m_W (v_{W1} - v_{s1})$$

m_T = Total mass initially $= (40 + 3 + 2 \times 5)$ kg

m_s = mass of skater + skate $= (40 + 3)$ kg

$m_W = 5$ kg

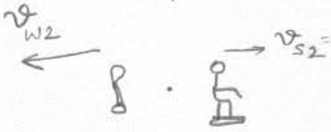
$v_{W1} = 7$ m/s wrt skater



$$\therefore (m_W + m_s) v_{s1} + m_W v_{s1} = m_W v_{W1}$$

$$\therefore v_{s1} = \frac{m_W v_{W1}}{(m_s + 2m_W)} = \frac{5(7)}{53} = \underline{\underline{0.66 \text{ m/s}}}$$ Answer.

(b) Momentum conservation,



$$(m_s + m_W)\, v_{s1} = -m_W v_{W2} + m_s v_{s2}$$

initial momentum

v_{W2} = velocity of the weight 2 w.r.t a stationary observer

$v_{W2} = 7 - v_{s2}$

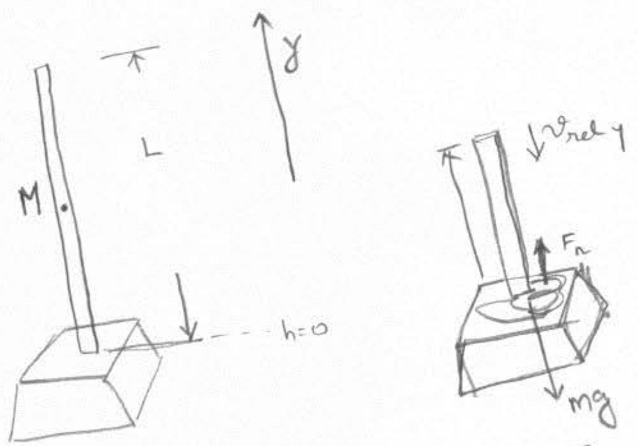
$$v_{s2} = \frac{(m_s + m_W)\, v_{s1} + m_W v_{W2}}{m_s}$$

v_{s2} = velocity of skater after throwing second wt. w.r.t stationary observer

$$43\, v_{s2} = 48\,(0.66) + 5\,(7 - v_{s2})$$

$$\therefore \quad 48\, v_{s2} = 48\,(0.66) + 35 \quad \Rightarrow \quad \boxed{v_{s2} = 1.39 \text{ m/s}}$$ **Answer.**

(8)



We consider the system as the mass m that has fallen on the weight (after some time t)

Let (m) denote the mass of the system (that part of the rope on scale).

The velocity of the system remains zero, so $\left(\frac{dv_y}{dt}\right) = 0$

$$F_{net\,ext\,y} + \frac{dm}{dt} v_{rel\,y} = m \frac{dv_y}{dt}$$

↓

Net external force in y direction

$$(F_n - mg) + \frac{dm}{dt} v_{rel\,y} = 0 \quad -(i)$$

Let dm denote the mass of the short rope segment of length dl that falls on the scale during time dt. Since, the rope is uniform the relation between dm and dl is

$$\frac{dm}{dl} = \frac{M}{L} \Rightarrow \frac{dm}{dt} = \frac{M}{L}\frac{dl}{dt} \text{ (ii) (divide by } dt \text{ on both sides)}$$

$\frac{dl}{dt}$ is the impact speed of the segment, so $v_{rel\,y} = -\frac{dl}{dt}$

($v_{rel\,y}$ is negative because up is the +ive y direction and the rope is falling).

Substituting this in the (ii) gives

$$\frac{dm}{dt} = -\frac{M}{L} v_{rel\,y} \quad \text{— (iii)}$$

Now, from (i) & (iii)

$$(F_n - mg) - \frac{M}{L} v_{rel\,y}^2 = 0$$

$$\therefore \quad F_n = mg + \frac{M}{L} v_{rel\,y}^2$$

Until the rope touches scale, each point along the rope falls with constant acceleration g; $\Delta y = -\frac{L}{2}$

$$v_{rel\,y}^2 = v_{rel\,y0}^2 + 2a_y(\Delta y) = 0 + 2(-g)(-L/2) = gL$$

$$(m = \frac{M}{2} \text{ finally})$$

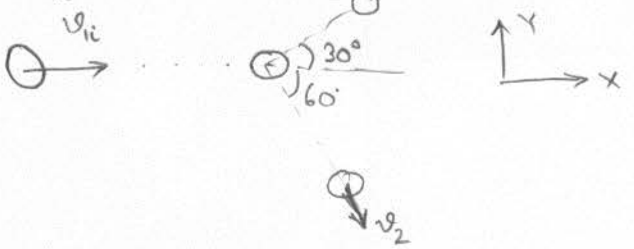
$$F_n = mg + \frac{M}{L} gL$$

$$F_n = \frac{M}{2} g + Mg = \frac{3}{2} Mg$$

$$\boxed{F_n = \frac{3}{2} Mg} \text{ Answer}$$

(9)

(a) $m_1 = 5\text{ kg}$ $\quad v_{1i} = 2\text{ m/s}$ $\quad v_{2i} = 0$ $\quad m_2 = 5\text{ kg}$



Conservation of Linear momentum (x direction)

$$m_1 v_{1i} = m_1 v_1 \cos 30^\circ + m_2 v_2 \cos 60^\circ$$

$$\cancel{5} \times 2 = \cancel{5}_1 v_1 \frac{\sqrt{3}}{2} + \cancel{5}\, v_2 \frac{1}{2}$$

$$\therefore\ 2 = \frac{\sqrt{3}}{2} v_1 + \frac{v_2}{2} \Rightarrow \boxed{4 = \sqrt{3}\, v_1 + v_2}\ \text{(i)}$$

(y direction):

$$v_1 \sin 30^\circ = v_2 \sin 60^\circ$$

$$v_1 \frac{1}{\cancel{2}} = v_2 \frac{\sqrt{3}}{\cancel{2}} \Rightarrow v_1 = \sqrt{3}\, v_2 \quad \text{-(ii)}$$

$$4 = \sqrt{3}\left(\sqrt{3}\, v_2\right) + v_2 \Rightarrow 4 v_2 = 4 \Rightarrow \boxed{v_2 = 1\text{ m/s}}$$

$$\boxed{v_1 = \sqrt{3}\text{ m/s}}$$ <u>Answer</u>.

(b) Check KE conservation:

$$\cancel{\tfrac{1}{2}} \cancel{m_1} v_{1i}^2 = \cancel{\tfrac{1}{2}} \cancel{m_2} v_2^2 + \cancel{\tfrac{1}{2}} \cancel{m_1} v_1^2$$

$$v_{1i}^2 = v_2^2 + v_1^2 \quad \text{(must be true if Elastic collision)}$$

$$4 = 3 + 1 \quad \text{true}$$

Thus, Collision is <u>Elastic</u>.