SOLUTIONS TO HOMEWORK 4.

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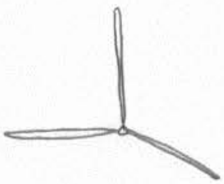
(1) $l = 1.8\,m$, $m = 20\,kg$ $\quad \nu = 2500 \frac{rev}{min} = \frac{2500}{60} \frac{rev}{s} \Rightarrow \omega = 2\pi\nu$

$\omega = 2\pi\left(\frac{2500}{60}\right) \frac{rad}{sec}$



$$I_R = \frac{1}{12} m l^2 + m\left(\frac{l}{2}\right)^2$$

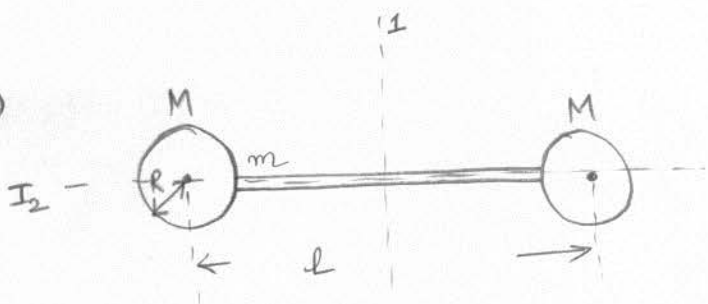
$$I_T = 3 I_R = 3\left[\frac{1}{12} m l^2 + \frac{m l^2}{4}\right]$$

$$K = \frac{1}{2} I_T \omega^2 = \frac{1}{2} \cancel{3} \left[m l^2 \cancel{3}\left(\frac{\cancel{4}}{\cancel{12}}\right)\right] \left(2\pi \frac{2500}{60}\right)^2 \ J$$

$$= \frac{\overset{10}{\cancel{20}}}{\cancel{2}} (1.8)^2 \left(2\pi \frac{2500}{60}\right)^2 \ J \Rightarrow \boxed{K = 2.2 \times 10^6 \ J}$$

Answer

(2) a)

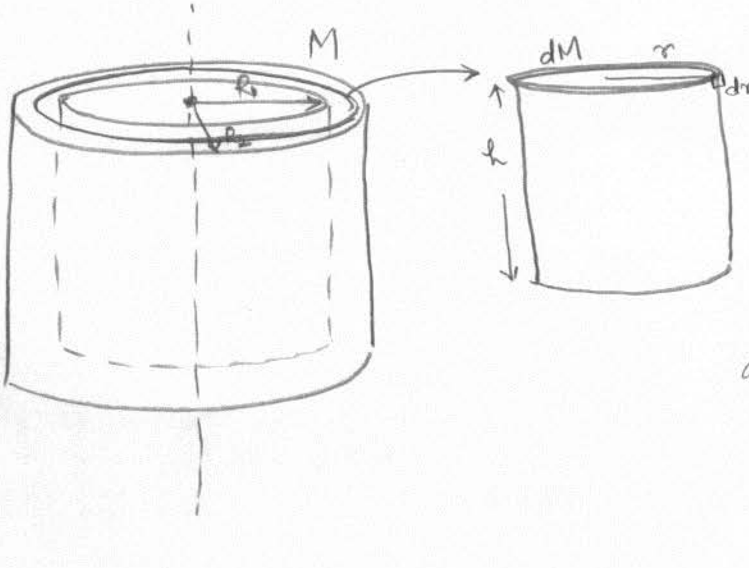


$$\boxed{I_1 = \frac{1}{12} m l^2 + 2\left[\frac{2}{5} M R^2 + M\left(\frac{l}{2}\right)^2\right]}$$

$$I_2 = 2\left[\frac{2}{5} M R^2\right]$$

(neglect radius of rod, ∵ thin rod).

b)



$$dI = dM\, r^2$$

$$I = \int dI = \int_{R_1}^{R_2} dM\, r^2$$

$$dM = \rho\,(2\pi r\, dr)\, h$$

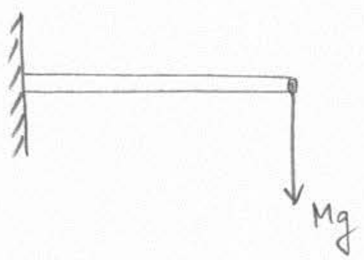
and $\rho = \frac{M}{(\pi R_2^2 - \pi R_1^2) h} = \frac{M}{\pi (R_2^2 - R_1^2) h}$

$$I = \int_{R_1}^{R_2} \rho\,(2\pi r\,dr)h\,r^2 = \rho\,2\pi h \int_{R_1}^{R_2} r^3\,dr = \rho\,2\pi h \left.\frac{r^4}{4}\right|_{R_1}^{R_2}$$

$$I = \rho\,2\pi h\,\frac{1}{4}\left(R_2^4 - R_1^4\right) = \frac{M}{\cancel{\pi}(R_2^2 - R_1^2)\cancel{h}}\,\cancel{2}\cancel{\pi}\cancel{h}\,\frac{1}{\cancel{4}\,2}\left(R_2^4 - R_1^4\right)$$

$$\boxed{I = \frac{M}{2}\left(R_2^2 + R_1^2\right)}$$ Answer.

(3)



$d = 4.8$ cm $\Rightarrow r = 2.4$ cm

$L = 5.3$ cm

$M = 1200$ kg

$\sigma = 3.0 \times 10^{10}\ \frac{N}{m^2}$

(a) stress, $\frac{F}{A} = \frac{Mg}{\pi r^2} = \frac{1200 \times 9.8}{\pi (2.4)^2 \times 10^{-4}} = 649.9 \times 10^4\ \frac{N}{m^2} \approx 6.5 \times 10^6\ \frac{N}{m^2}$ (2 sig fig)

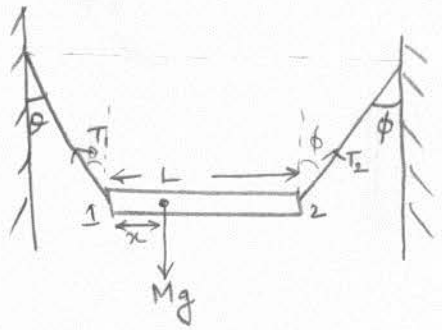
(b) strain, $\frac{\Delta l}{l}$

$$\sigma = \frac{F/A}{\Delta l / l} \Rightarrow \Delta l = \left(\frac{F}{A}\right)\frac{l}{\sigma} \Rightarrow \Delta l = 649.9 \times 10^6\ \frac{5.3 \times 10^{-2}}{3.0 \times 10^{10}}\ m$$

$\Delta l = 1.15 \times 10^{-3}$ m $\quad\therefore\quad \boxed{\Delta l = 1.2 \times 10^{-3}\ m\ (2\ \text{sig fig})}$

Answer.

(4)



vertical force balance

$T_1 \cos\theta + T_2 \cos\phi = Mg$ —(i)

$T_1 \sin\theta = T_2 \sin\phi$ —(ii) (horiz force bal.)

Balancing torque about pt 1.

$$Mg\, x = (T_2 \cos\phi)\, L \quad \text{—(iii)}$$

$$x = \frac{T_2 \cos\phi \; L}{Mg}$$

$$T_1 \cos\theta + T_2 \cos\phi = Mg$$

$$T_2 = T_1 \frac{\sin\theta}{\sin\phi} \Rightarrow T_1 = T_2 \frac{\sin\phi}{\sin\theta}$$

$$T_2 \frac{\sin\phi}{\sin\theta} \cos\theta + T_2 \cos\phi = Mg$$

$$\Rightarrow T_2 (\sin\phi \cos\theta + \cos\phi \sin\theta) = Mg \sin\theta$$

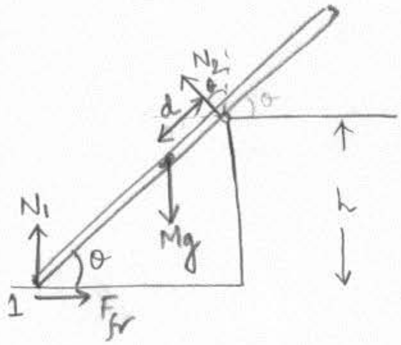
$$\Rightarrow T_2 \sin(\theta+\phi) = Mg \sin\theta \Rightarrow T_2 = \frac{Mg \sin\theta}{\sin(\theta+\phi)}$$

Thus, $$x = \frac{\cancel{Mg} \sin\theta}{\sin(\theta+\phi)} \frac{\cos\phi \; L}{\cancel{Mg}} = L \frac{\sin\theta \cos\phi}{\sin(\theta+\phi)}$$ Answer.

$$x = (6.10\text{ m}) \frac{\sin(36.9)\cos(53.1)}{\sin(36.9+53.1)} = 2.199\text{ m}$$

$$\boxed{x = 2.20\text{ m}}$$ (3 sig fig's) Answer.

(5)



L , M

Force balance, vertical

$$N_2 \cos\theta + N_1 = Mg$$

$$N_2 \sin\theta = F_{fr} \quad -(i)$$

Balance torque about 1,

$$Mg\left(\frac{L}{2}\cos\theta\right) = N_2\left(\frac{L}{2}+d\right) \qquad \text{where} \quad h = \left(\frac{L}{2}+d\right)\sin\theta$$

$$\Rightarrow \quad \frac{L}{2}+d = \frac{h}{\sin\theta}$$

$$Mg\,\frac{L}{2}\cos\theta = N_2\,\frac{h}{\sin\theta} \quad -(ii)$$

Maximum force of friction is when $\theta = \theta_{min}$

$$F_{fr} = \mu_s N_1$$

$\therefore$ from (i) $\quad N_2 \sin\theta_{min} = \mu_s N_1 \quad$ & $\quad N_2 \cos\theta_{min} + N_1 = Mg$

$$Mg\,\frac{L}{2}\cos\theta_{min} = N_2\,\frac{h}{\sin\theta_{min}}$$

$$\longrightarrow \quad N_2 = Mg\,\frac{L}{2}\cos\theta_{min}\,\frac{\sin\theta_{min}}{h}$$

$$\therefore \quad \mu_s = \frac{N_2 \sin\theta_{min}}{N_1}$$

Use N_2 in other eqn.

$$Mg\,\frac{L}{2}\cos^2\theta_{min}\,\frac{\sin\theta_{min}}{h} + N_1 = Mg$$

$$\therefore \quad N_1 = Mg\left[1 - \frac{L}{2h}\cos^2\theta_{min}\sin\theta_{min}\right]$$

Thus,
$$\mu_s = \frac{\cancel{Mg}\,\frac{L}{2h}\cos\theta_{min}\sin^2\theta_{min}}{\cancel{Mg}\left[1 - \frac{L}{2h}\cos^2\theta_{min}\sin\theta_{min}\right]}$$

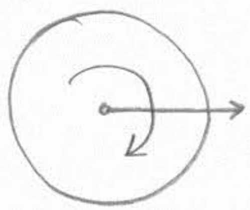
$$\boxed{\mu_s = \frac{L}{2h}\,\frac{\cos\theta_{min}\sin^2\theta_{min}}{\left[1 - \frac{L}{2h}\cos^2\theta_{min}\sin\theta_{min}\right]}}$$

Answer.

$$\boxed{\mu_s = 0.339}$$ plugging in the values

Answer.

(6)



$\hat{j}$ $r = 33\ cm$ $v = 80\ km/hr$

$\hat{i}$

a) $\vec{v} = 0$

b) $\vec{a} = 0$

c) $\vec{v} = 80\ \frac{km}{hr}\ \hat{i}$

d) $\vec{a} = -\frac{v^2}{r}\ \hat{j} = -\frac{\left(80 \times \frac{5}{18}\right)^2}{33 \times 10^{-2}}\ \frac{m^{\cancel{2}} s^{-2}}{\cancel{m}}\ \hat{j} = -\left(1.5 \times 10^3\ ms^{-2}\right)\hat{j}$

e) $\vec{v} = -80\ \frac{km}{hr}\ \hat{i}$

f) $\vec{a} = \left(1.5 \times 10^3\ ms^{-2}\right)\hat{j}$

g) $\vec{v} = \left(80\ \frac{km}{hr}\right)\hat{i}$

h) $\vec{a} = 0$

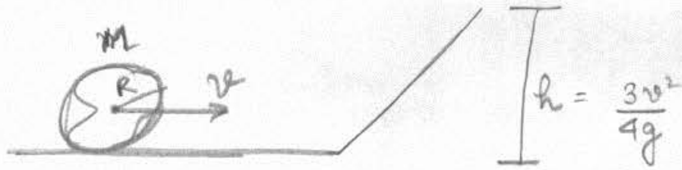
i) $\vec{v} = 160\ \frac{km}{hr}\ \hat{i}$

j) $\vec{a} = -1.5 \times 10^3\ ms^{-2}\ \hat{j}$

k) $\vec{v} = 0$

l) $\vec{a} = \left(1.5 \times 10^3\ ms^{-2}\right)\hat{j}$

(7)



Energy conservation :

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh \qquad \text{where} \quad v = \omega R \ \text{(for rolling)}$$

$$\therefore \frac{1}{2}I\frac{v^2}{R^2} = mgh - \frac{1}{2}mv^2$$

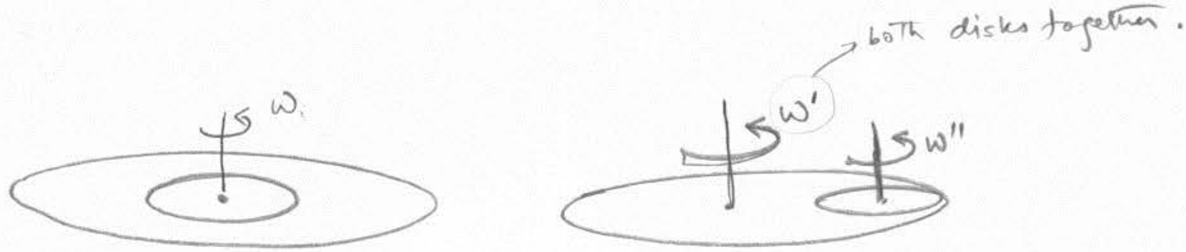
$$= \frac{3v^2}{4\cancel{g}}m\cancel{g} - \frac{1}{2}mv^2 = \frac{1}{4}mv^2$$

$$I = \left(\frac{1}{\cancel{4}_2}m\cancel{v^2}\right)\frac{\cancel{2}R^2}{\cancel{v^2}} = \frac{1}{2}mR^2$$

a) $\boxed{I = \frac{1}{2}mR^2}$ Answer.

b) The body may be a cylinder of radius R .

(8) $M = 10m$ $\qquad R = 3.0\, r$ $\qquad \omega = 20$ rad/sec.



Conservation of angular momentum:

$$I_S \omega + I_L \omega = I_L \omega' + \left[I_S + m(R-r)^2 \right] \omega' + I_S \omega''$$

$$\cancel{\tfrac{1}{2}} \cancel{m} \cancel{r^2} \omega + \cancel{\tfrac{1}{2}} (10\cancel{m})(3\cancel{r})^2 \omega = \cancel{\tfrac{1}{2}} (10\cancel{m})(3\cancel{r})^2 \omega' + \left[\cancel{\tfrac{1}{2}} \cancel{m} \cancel{r^2} + \overset{\times 2}{\cancel{m}} 4\cancel{r^2} \right] \omega' + \cancel{\tfrac{1}{2}} \cancel{m} \cancel{r^2} \omega''$$

$$\omega + 90\,\omega = 90\,\omega' + 9\,\omega' + \omega''$$

$$\therefore \quad \boxed{\omega'' + 99\,\omega' - 91\,\omega = 0} \quad \text{— (i)}$$

Conservation of Energy.

$$\cancel{\tfrac{1}{2}} I_S \omega^2 + \cancel{\tfrac{1}{2}} I_L \omega^2 = \cancel{\tfrac{1}{2}} I_L \omega'^2 + \cancel{\tfrac{1}{2}} \left[I_S + m(R-r)^2 \right] \omega'^2 + \cancel{\tfrac{1}{2}} I_S \omega''^2$$

$$\omega^2 + 90\,\omega^2 = 90\,\omega'^2 + 9\,\omega'^2 + \omega''^2$$

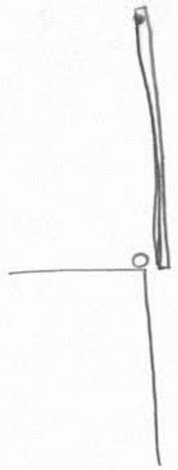
$$\boxed{\omega''^2 + 99\,\omega'^2 - 91\,\omega^2 = 0} \quad \text{— (ii)}$$

Solve for ω'

from (i) $\quad \omega'' = 91\omega - 99\omega'$

in (ii) $\quad (91\omega - 99\omega')^2 + 99\,\omega'^2 - 91\omega^2 = 0 \Rightarrow \omega^2(91^2 - 91) + \omega'^2(+99^2 + 99) - 2(91\,\omega)(99\,\omega') = 0$

(9)



$L = 0.60\ m \qquad M = 1\ kg$

$m = 0.20\ kg \qquad \omega_i = 2.4\ \frac{rad}{sec}$

$\omega_f = ?$

Conservation of angular momentum:

$$I\omega_i = I_T\,\omega_f$$

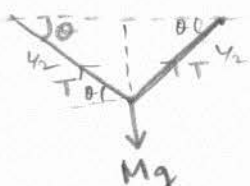
$$I = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = 0.12$$

$$I_T = \left[\frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2\right] + mL^2 = 0.192$$

$$\omega_f = \frac{I\omega_i}{I_T} = \frac{0.12 \times 2.4}{0.192}\ \frac{rad}{sec} = 1.5\ rad\ s^{-1}$$

(10)

a)



$L = 5m$

$$2T\sin\theta = Mg$$

$$T\sin\theta = \frac{Mg}{2}$$

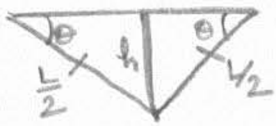
on boat,

$T\cos\theta = F.$

$$\therefore \quad T^2 = \left(\frac{Mg}{2}\right)^2 + F^2 \Rightarrow T = \sqrt{\left(\frac{Mg}{2}\right)^2 + F^2}$$

$$\boxed{T = 70.7\ N}$$

(b)



$$h = \frac{L}{2}\sin\theta$$

and, $\tan\theta = \frac{Mg/2}{F} \Rightarrow \theta = \tan^{-1}\left(\frac{Mg/2}{F}\right)$

$$\theta = \tan^{-1}(1) = 45°.$$

$$h = \frac{L}{2}\frac{1}{\sqrt{2}} = \frac{L}{2\sqrt{2}} = \underline{\underline{1.77\ m}}$$

(c) $d = \cancel{2}\frac{L}{\cancel{2}}\cos\theta = 5\frac{1}{\sqrt{2}} = \underline{3.54\ m}$

(d) $T_{max} = 500\ N$

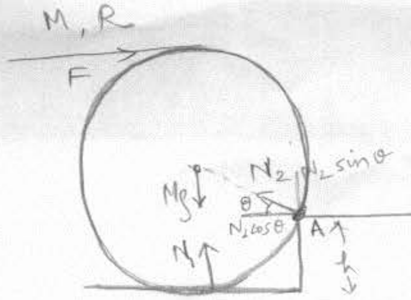
$$T_{max}\cos\theta_m = F_{max}$$

$$T_{max}\sin\theta_m = \frac{Mg}{2}$$

$$T_{max}^2 = F_{max}^2 + \left(\frac{Mg}{2}\right)^2 \Rightarrow F_{max}^2 = T_{max}^2 - \left(\frac{Mg}{2}\right)^2$$

$\underline{F_{max} = 497.5\ N}$. $\approx 500\ N$

(11)



$F = N_2 \cos\theta$

$Mg = N_1 + N_2 \sin\theta$

Torque balance about A

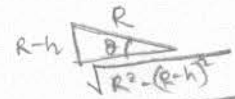
$$F(2R-h) + N_1(R\cos\theta) = Mg\,R\cos\theta.$$

$N_1 = Mg - N_2 \sin\theta$

$N_1 = Mg - \frac{F}{\cos\theta}\sin\theta = Mg - F\tan\theta$

a) $\boxed{N_1 = Mg - \frac{F(R-h)}{\sqrt{h(2R-h)}}}$ Answer

$\tan\theta = ?$



$= \sqrt{(R-R+h)(R+R-h)}$

$= \sqrt{h(2R-h)}$

$\tan\theta = \frac{R-h}{\sqrt{h(2R-h)}}$

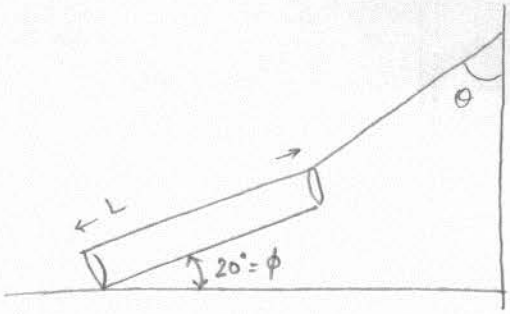
b) $N_2\cos\theta = F$

Horizontal force $= F$ Answer

c) vertical component, $N_2 \sin\theta = Mg - N_1$

$\boxed{N_2 \sin\theta = \frac{F(R-h)}{\sqrt{h(2R-h)}}}$ Answer

(12)



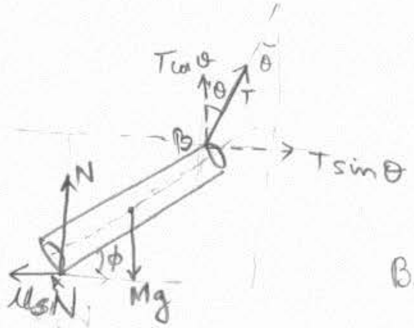
$M = 100$ kg

$L = 4$ m

$r = 12$ cm

$\mu_s = 0.6$

T & θ are required.



$$T\sin\theta = \mu_s N \qquad \text{— (i)}$$

$$T\cos\theta + N = Mg \qquad \text{— (ii)}$$

Balance Torque about A.

$$Mg\left(\frac{L}{2}\cos\phi\right) + (T\sin\theta)\,L\sin\phi = (T\cos\theta)\,L\cos\phi \qquad \text{— (iii)}$$

Eqn (i) and (iii) are equations with 2 unknowns T & θ, we can solve for both.

$$\text{(i)} \rightarrow \quad T\sin\theta = \mu_s\left(Mg - T\cos\theta\right)$$

$$T\left(\sin\theta + \mu_s\cos\theta\right) = \mu_s Mg$$

$$\text{(iii)} \rightarrow \quad TL\left(\cos\theta\cos\phi - \sin\theta\sin\phi\right) = Mg\,\frac{L}{2}\cos\phi$$

$$TL\cos(\theta+\phi) = Mg\,\frac{L}{2}\cos\phi$$

$$\frac{\cancel{L}\cos(\theta+\phi)}{\sin\theta + \mu_s\cos\theta} = \frac{\cancel{Mg}\,\frac{\cancel{L}}{2}\cos\phi}{\mu_s\cancel{Mg}} \Rightarrow \frac{\cos(\theta+\phi)}{\sin\theta+\mu_s\cos\theta} = \frac{\cos\phi}{\mu_s}$$

Torque balance about B,

$$N \cancel{L} \cos\phi + \mu_s N \cancel{L} \sin\phi = Mg \frac{\cancel{L}}{2} \cos\phi$$

$$N (\cos\phi + \mu_s \sin\phi) = \frac{Mg}{2} \cos\phi$$

$$N = \frac{Mg \cos\phi}{2} \left(\frac{1}{\cos\phi + \mu_s \sin\phi} \right) = 402.17 \text{ N}$$

Now from (i) $\quad T \sin\theta = (0.6)(402.17) = 241.3 \text{ N}$

from (ii) $\quad T \cos\theta = Mg - N = 577.83 \text{ N}.$

$\therefore \quad T = 626.19 \text{ N} \approx 626 \text{ N}.$ (3 sig fig)

and (i) divide by (ii) $\quad \tan\theta = \frac{241.3}{577.83} \Rightarrow \theta = 22.67°.$

$\therefore$ $\boxed{\begin{array}{ll} T = 626 \text{ N} & \\ \theta = 22.7° & \text{(3 sig fig)} \end{array}}$ Answer.