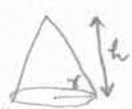
Useful Mathematical Formulas.

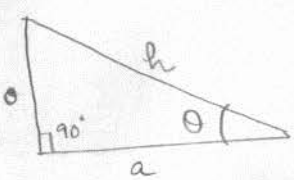
(1) Quadratic Formula.

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(2) AREAS AND Volumes

Shape	Volume	Surface area
Circle, radius r 	—	πr^2 (perimeter $= 2\pi r$)
Sphere, radius r 	$\frac{4}{3}\pi r^3$	$4\pi r^2$
Right circular cylinder radius r, height h 	$\pi r^2 h$	$2\pi r^2 + 2\pi r h$
Right circular cone, radius r, height h 	$\frac{1}{3}\pi r^2 h$	$\pi r^2 + \pi r\sqrt{r^2+h^2}$

(3) TRIGONOMETRIC FUNCTIONS & IDENTITIES.



$\sin\theta = \dfrac{o}{h}$ $\cos\theta = \dfrac{a}{h}$ $\tan\theta = \dfrac{o}{a}$

$\csc\theta = 1/\sin\theta$ $\sec\theta = 1/\cos\theta$ $\cot\theta = 1/\tan\theta$

$a^2 + o^2 = h^2$ (Pythagoras Theorem)

Quadrant picture and signs:

Notation:

r = radial distance (always +ive)

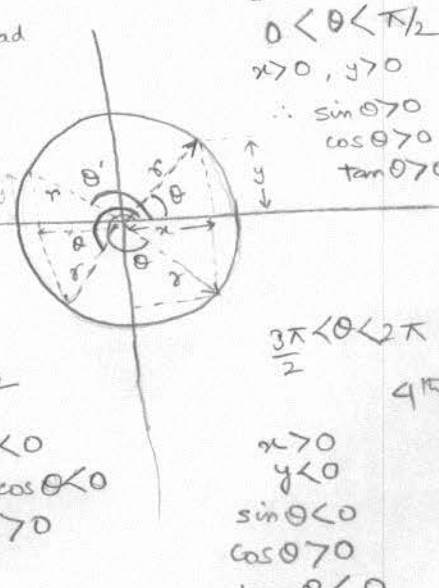
x = distance on x axis

y = distance on y axis

$\sin\theta = \dfrac{y}{r}$ $\cos\theta = \dfrac{x}{r}$ $\tan\theta = \dfrac{y}{x}$

1st Quad
$0 < \theta < \pi/2$
$x > 0, y > 0$
$\therefore \sin\theta > 0$, $\cos\theta > 0$, $\tan\theta > 0$

2nd Quad
$\frac{\pi}{2} < \theta < \pi$
$x < 0, y > 0$
$\sin\theta > 0$
$\cos\theta < 0$; $\tan\theta < 0$



$\pi < \theta < 3\pi/2$
3rd Quad
$x < 0, y < 0$
$\sin\theta < 0$, $\cos\theta < 0$
$\tan\theta > 0$

$\frac{3\pi}{2} < \theta < 2\pi$
4th Quad
$x > 0$
$y < 0$
$\sin\theta < 0$
$\cos\theta > 0$
$\tan\theta < 0$

$\sin^2\theta + \cos^2\theta = 1$; $\sec^2\theta - \tan^2\theta = 1$; $\csc^2\theta - \cot^2\theta = 1$

Multiple angle :

$\sin 2\theta = 2\sin\theta\cos\theta$; $\cos 2\theta = \cos^2\theta - \sin^2\theta$; $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

Sum / Diff. of angles :

$\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$2\cos A\cos B = \cos(A-B) + \cos(A+B)$

$2\sin A\cos B = \sin(A-B) + \sin(A+B)$

$\sin(\pi - \theta) = \sin\theta$

$\cos(\pi - \theta) = -\cos\theta$

$\tan(\pi - \theta) = -\tan\theta$

$\sin(\frac{\pi}{2} - \theta) = \cos\theta$

$\cos(\pi/2 - \theta) = \sin\theta$

$\tan(\frac{\pi}{2} - \theta) = \cot\theta$

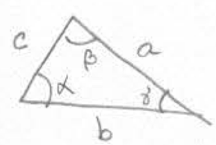
$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ $\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$

TRIANGLE LAWS



$\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$ [Law of Sines]

$c^2 = a^2 + b^2 - 2ab\cos\gamma$ [Law of Cosines]

(Will be useful for vector addition).

(4) LOGARITHMS :

* if $y = A^x$ then $x = \log_A y$

Natural log → $y = e^x \Rightarrow \ln y = x$

Common log → $y = 10^x \Rightarrow \log_{10} y = x$

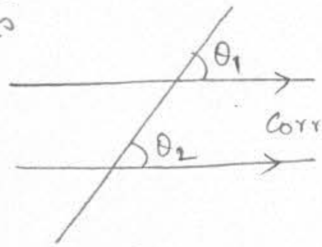
* $\log(AB) = \log A + \log B$

$\log(A-B) = \log A - \log B$

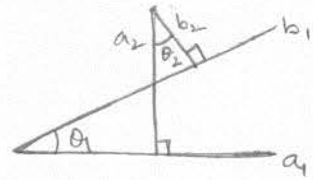
(5) PLANE GEOMETRY :

[Note: Will be very useful in breaking force into normal & Tangential components when dealing with free body diagrams]

* Equal angles

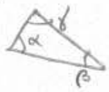


$\theta_1 = \theta_2$
Corresponding angles for parallell lines



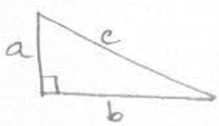
If $a_1 \perp a_2$ and $b_1 \perp b_2$
then $\theta_1 = \theta_2$

* Sum of interior angles of a plane Δ is 180°



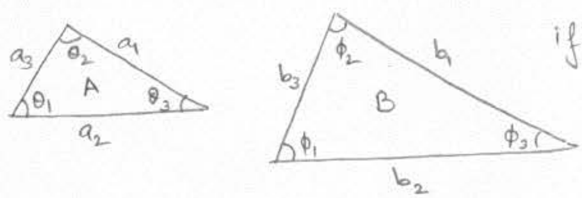
$\alpha + \beta + \gamma = \pi$

* Pythagoras theorem



$a^2 + b^2 = c^2$
for right angled triangle

* Similar Triangles



Triangle A is similar to B
if $\theta_1 = \phi_1$; $\theta_2 = \phi_2$; $\theta_3 = \phi_3$

(i) if ($\theta_1 = \phi_1$ and $\theta_2 = \phi_2$) then ($\theta_3 = \phi_3$) is true
and the Δs are similar

because
$\theta_1 + \theta_2 + \theta_3 = \pi$
$\phi_1 + \phi_2 + \phi_3 = \pi$

(ii) ratio of corresponding sides of similar Δs are equal

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

* Congruent triangles : Two Δs are congruent only if they are similar and of the same size.

Two Δs are congruent if any of these is known to be true

(a) three corresponding sides are equal (SSS)

(b) Two sides and enclosed angle (A) are equal (SAS)

(c) Two angles and enclosed side are equal (ASA)

(6) VECTORS.

If $\vec{a}$ and $\vec{b}$ are two vectors :

(i) Commutativity : $\vec{a}+\vec{b} = \vec{b}+\vec{a}$

(ii) Associativity : $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$

(iii) Existence of negetive vector : for every $\vec{a}$ there is $(-\vec{a})$ in opposite direction

$$\vec{a}+(-\vec{a})=0$$

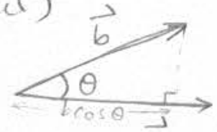
(iv) If m and n are scalars,

$$m(\vec{a}+\vec{b}) = m\vec{a}+m\vec{b} \quad \text{and} \quad (m+n)\vec{a} = m\vec{a}+n\vec{a}$$

$$|m\vec{a}| = m|\vec{a}| \quad ; \quad (mn)\vec{a} = m(n\vec{a}) \quad ; \quad 0\vec{a} = 0$$

Scalar product : (Dot product)

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$



Projection of $\vec{b}$ on $\vec{a}$

* $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ Commutative

if $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

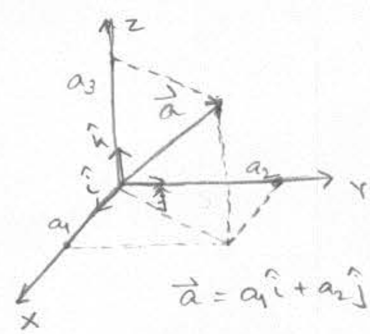
$\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

Comes from dot product of basis vectors (cartesian cord) $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$; $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

* $\vec{a}\cdot\vec{b} = a_1b_1+a_2b_2+a_3b_3$

* $|\vec{a}| = a = \sqrt{a_1^2+a_2^2+a_3^2}$; $|\vec{b}| = b = \sqrt{b_1^2+b_2^2+b_3^2}$

* $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{ab} = \dfrac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$

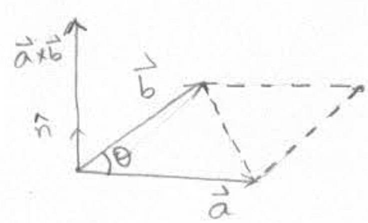


$\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

* Unit vector in the direction of $\vec{a}$ is $\hat{a}$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{a}$$

Vector product : (Cross product)



$|\vec{a}\times\vec{b}|$ = Area of parallellogram

$|\vec{a}\times\vec{b}| = 2$ Area of $\Delta = (ab\sin\theta)$

Direction of $\vec{a}\times\vec{b}$ is ⊥ to both $\vec{a}$ and $\vec{b}$ (i.e. ⊥ to plane containing $\vec{a}$ & $\vec{b}$)

* $\vec{a}\times\vec{b} = ab\sin\theta\,\hat{n}$

* $\vec{a}\times\vec{b}$ is anti-commutative ie $\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \hat{i}(a_2 b_3 - a_3 b_2) + \hat{j}(a_3 b_1 - a_1 b_3) + \hat{k}(a_1 b_2 - a_2 b_1)$$

* $\sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}|\,|\vec{b}|} = \dfrac{|\vec{a} \times \vec{b}|}{a\, b}$

(7) Binomial expansion:

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{2!}x^2 \pm \frac{n(n-1)(n-2)}{3!}x^3 + \dots = \sum_{k=0}^{\infty} (\pm 1)^k \, {}^nC_k \, x^k$$

$$(x+y)^n = x^n\left(1 + \frac{y}{x}\right)^n = x^n\left(1 + n\frac{y}{x} + \frac{n(n-1)}{2!}\frac{y^2}{x^2} + \dots\right)$$

(8) Permutation and Combination

* Choosing r things out of n distinct possibilities without repition, and order doesnot matter

no. of ways $= {}^nC_r = \dfrac{n!}{r!\,(n-r)!}$

* Choosing r things out of n distinct possibilities without repition

no. of ways $= {}^nP_r = \dfrac{n!}{(n-r)!}$

(9) Other Expansions:

$$e^x = 1 + x + \frac{x^2}{2!} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$$

$$\tan\theta = \theta - \frac{\theta^3}{3} + \frac{2}{15}\theta^5 + \dots \qquad |\theta| < \pi/2$$

In general: Taylor series $f(x) = f(0) + \left(\dfrac{df}{dx}\right)_{x=0} x + \left(\dfrac{d^2f}{dx^2}\right)_{x=0} \dfrac{x^2}{2!} + \dots$