Problem 3.1
A bat can sense its distance from a wall by emitting a sharp ultrasonic pulse \((f > 20 \text{ kHz})\) that reflects off the wall. Because a bat can detect frequencies as high as 100 kHz, it can gauge the distance from the wall through the time the echo takes to return to the bat. (Assume that the bat moves with speed much smaller than the speed of sound.)

a) If the bat is to determine the distance to a wall 8 m away with an error of less than \(\pm 0.2\) m, how accurately must it sense the time interval between emission and return of the pulse? (At \(20^\circ \text{C}\) and 1 atm, the speed of sound in air is \(v_a = 344 \text{ m/s}\).)

b) Suppose that the bat flies into a space filled with methane (swamp gas). By what factor will this gas distort the bat’s perception of distance? (At \(20^\circ \text{C}\) and 1 atm, the speed of sound in methane is \(v_m = 432 \text{ m/s}\).)

Problem 3.1 - Solution

a) The time \(t\) between the emission and the return of the bat’s pulse to a wall \(d\) meters away is \(t = \frac{2d}{v_a}\). Thus a distance uncertainty \(\Delta d\) corresponds to a time uncertainty

\[
\Delta t = \frac{2}{v_a} \Delta d.
\]

For \(\Delta d = \pm 0.2\) m and \(v_a = 344 \text{ m/s}\), we have

\[\Delta t = \pm 1.2 \times 10^{-3} \text{ s}.\]

b) Suppose the bat is now sending a pulse that covers a distance \(d_m\). The bat will receive the reflected pulse after a time \(t = \frac{2d_m}{v_m}\). If the bat perceives that it is in air, it will interpret this time delay as being due to an apparent distance

\[
d_{\text{app}} = \frac{v_a t}{2} = \frac{v_a}{v_m} d_m.
\]

This gives

\[d_{\text{app}} = 0.8d_m.

Problem 3.2
The equation of a transverse wave on a string is given by

\[
D(x,t) = 0.3 \sin(3x)\cos(1200t)
\]

where \(D\) and \(x\) are in cm and \(t\) in seconds.
a) Which type of wave is it?
b) What is the wavelength, wave number, frequency, and period, of this wave?
c) Carefully draw the wave ($D$ versus $x$) at $t = 0$, at $t = 1.31 \times 10^{-3}$ s, and at $t = 2.62 \times 10^{-3}$ s.
d) What is the maximum transverse speed?
e) Can we define a speed of propagation along the string of such a wave front?

**Problem 3.2 - Solution**

a) It is a standing wave.

b) 

\[ \lambda = 0.021 \text{ m} \]
\[ \omega = 1200 \text{ rad/s} \]
\[ f = 191 \text{ Hz} \]
\[ T = 5.24 \times 10^{-3} \text{ s} \]
\[ k = 0.030 \text{ m}^{-1} \]

c) 

\[ D(x,0) = 0.3 \sin(3x) \]
\[ D(x,1.31 \times 10^{-3}) = 0.3 \sin(3x) \cos(\pi/2) = 0 \]
\[ D(x,2.62 \times 10^{-3}) = 0.3 \sin(3x) \cos(\pi) = -0.3 \sin(3x) \]
d) The maximum transverse speed is

\[ \text{Max}\left[ \frac{\partial}{\partial t} D(x,t) \right] = 3.6 \, \text{m/s} \]

e) The phase speed is

\[ v = \frac{\omega}{k} = 4 \, \text{m/s} \]

but the speed with which this particular wave-front (standing wave) travels along the string is zero.

**Problem 3.3**

High-frequency sound can be used to produce standing-wave vibrations in a goblet. A standing-wave vibration in a goblet is observed to have four nodes and four antinodes equally spaced around the 20 cm circumference of the rim of the glass. If transverse waves move around the glass at 900 m/s, an opera singer would have to produce a high harmonic with what frequency to shatter the glass with a resonant vibration?

**Problem 3.3 - Solution**

The distance between adjacent nodes \((d_{\text{NN}})\) is one-quarter of the circumference, i.e., 5 cm.

\[ \lambda = 2d_{\text{NN}}; \quad \lambda = 0.10 \, \text{m} \]

Thus the frequency is \(f = v/\lambda = 9000 \, \text{Hz} \).

The singer must match this frequency quite precisely for some interval of time to force enough energy into the glass to crack it.

**Problem 3.4**

High-power lasers in factories are used to cut through cloth and metal. One such laser has a beam diameter of 0.1 cm and generates an electric field having an amplitude of 0.7 MV/m at the target. Find the amplitude of the magnetic field produced, the intensity of the laser, and the power delivered by the laser.

**Solution 3.4 - Solution**

\[ B_{\text{max}} = B_0 = \frac{E_0}{c}; \quad B_{\text{max}} = \frac{7 \times 10^5 \, \text{N/C}}{3 \times 10^8 \, \text{m/s}} = 2.33 \, \text{mT} \]

\[ I = \frac{E_0^2}{2\mu_0 c}; \quad I = 6.50 \times 10^8 \, \text{W/m}^2 \]
\[ P = I A; \quad \text{with} \quad A = \frac{\pi}{4} (10^{-3} \text{ m})^2; \quad P = 511 \text{ W}. \]

**Problem 3.5**

A wave solution to Maxwell’s equations is given by

\[ \vec{E} = E_0 \cos \left( 2\sqrt{3} z \right) \cos \left( 7.0 \times 10^{10} t \right) \hat{x} \]

where \( z \) is measured in centimeters, \( t \) in seconds, and \( E_0 \) is assumed positive.

a) What is the wavelength and frequency of the wave (in SI units)?

b) What is the index of refraction of the medium? (The index of refraction \( n \) is defined as the ratio of the speed of light in vacuum \( c \) to the speed \( v \) in a given medium.)

c) Give the expression for the associated magnetic field, \( \vec{B} \), in terms of \( E_0, z, \) and \( t \).

[Hints: Read Giancoli 31-4 and 31-5. Express the electric field wave as sum of two traveling waves.]

d) What is the time-averaged Poynting vector for \( x = y = 3 \text{ cm}, z = \sqrt{3} \text{ cm} \). Compare this result with question (e) in Problem 3.2 and briefly comment on it.

[Hints: Read Giancoli 31-8 and example 31-2.]

**Problem 3.5 - Solution**

a) Any standing wave of the form \( \cos(kz)\cos(\omega t) \) has a wavelength of \( 2\pi/k \) and a frequency of \( \omega/2\pi \). For our wave, \( k = 2\sqrt{3} \times 10^2 \text{ m}^{-1} \) and \( \omega = 7.0 \times 10^{10} \text{ rad s}^{-1} \), so

\[ \lambda = 1.814 \times 10^{-2} \text{ m}, \quad f = 1.114 \times 10^{10} \text{ Hz}. \]

b) The index of refraction of the medium is

\[ n = \frac{c}{v} = \frac{c}{\omega/k} = \frac{(3 \times 10^8 \text{ m s}^{-1})}{(7 \times 10^{10} \text{ s}^{-1})/(2\sqrt{3} \times 10^2 \text{ m}^{-1})} = 1.48. \]

c) To find \( \vec{B} \), we first picture our \( \vec{E} \)-field as the linear superposition of travelling waves, one travelling in the \( +\hat{z} \) direction and one travelling in the \( -\hat{z} \) direction. Using the trigonometry identity

\[ 2 \cos \alpha \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta), \]

we can rewrite our \( \vec{E} \)-field as

\[ \vec{E} = \frac{1}{2} E_0 \left[ \cos(2\sqrt{3} \times 10^2 z - 7 \times 10^8 t) \right] \hat{x} + \frac{1}{2} E_0 \left[ \cos(2\sqrt{3} \times 10^2 z + 7 \times 10^8 t) \right] \hat{x}. \]
The $\vec{B}$-field associated with the $\vec{E}$-field wave propagating in the $+\hat{z}$ direction must point in the $+\hat{y}$ direction (assuming $E_0$ is positive). Similarly, for the wave propagating in the $-\hat{z}$ direction, $\vec{B}$-field must point in the $-\hat{y}$ direction. Thus our total $\vec{B}$-field wave must be

$$\vec{B} = \frac{1}{2} B_0 \left[ \cos(2\sqrt{3} \times 10^2 z - 7 \times 10^8 t) \right] \hat{y} - \frac{1}{2} B_0 \left[ \cos(2\sqrt{3} \times 10^2 z + 7 \times 10^8 t) \right] \hat{y}.$$ 

Using the identity

$$\cos(\alpha - \beta) - \cos(\alpha + \beta) = 2 \sin \alpha \sin \beta,$$

we can write

$$\vec{B} = B_0 \sin(2\sqrt{3} \times 10^2 z) \sin(7 \times 10^8 t) \hat{y}.$$ 

We see that in a standing wave, $\vec{B}$ is $\pi/2$ out of phase relative to $\vec{E}$ both in space and in time. The values of $B_0$ is related to $E_0$ by

$$B_0 = \frac{k}{\omega} E_0 = \frac{n}{c} E_0.$$ 

d) The instantaneous Poynting vector (Giancoli, formula 31-18)

$$\vec{S} = \frac{1}{\mu_0} \left( \vec{E} \times \vec{B} \right)$$

for this wave at any point in space will have a time dependence of the form

$$\vec{S} \propto \sin(\omega t) \cos(\omega t).$$

The average over one full period in time is zero, so at all points

$$\vec{S} = 0.$$ 

This result tells us that standing electromagnetic waves do not transmit energy through space.