Waves and Modern Physics

PHY 123 - Spring 2012

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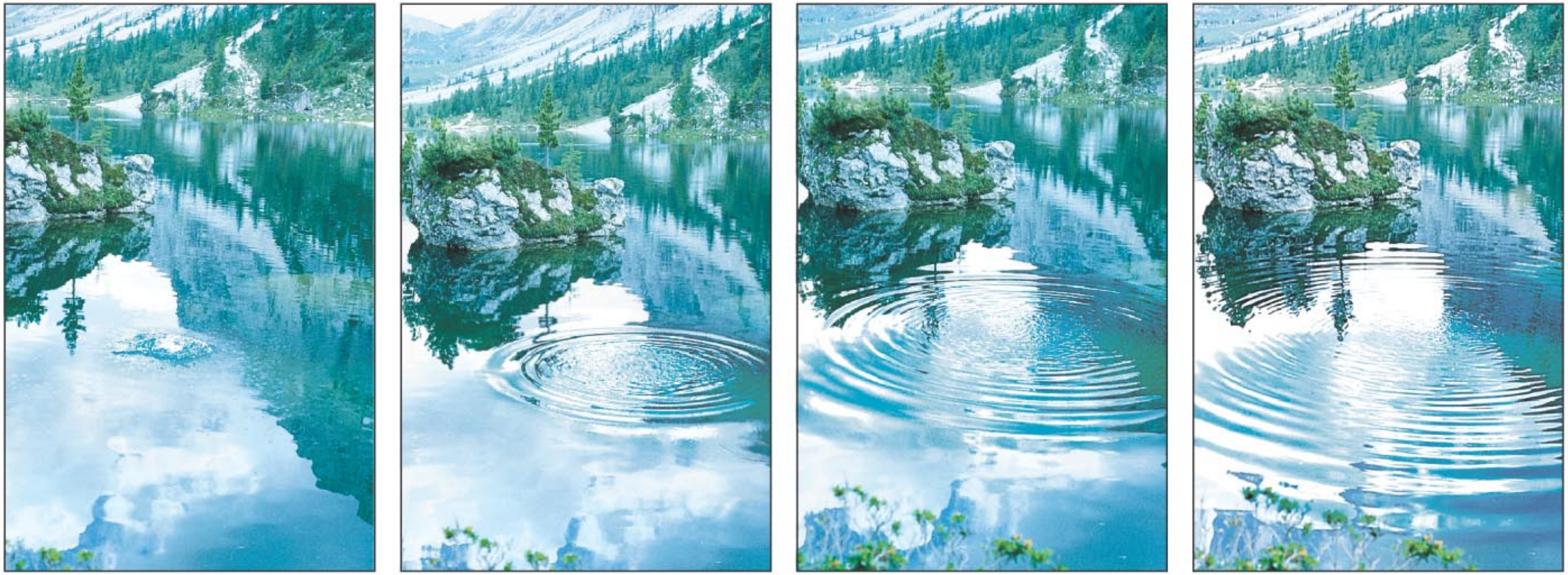
Course Web Site

http://www.pas.rochester.edu/~badolato/PHY_123/PHY_123.html

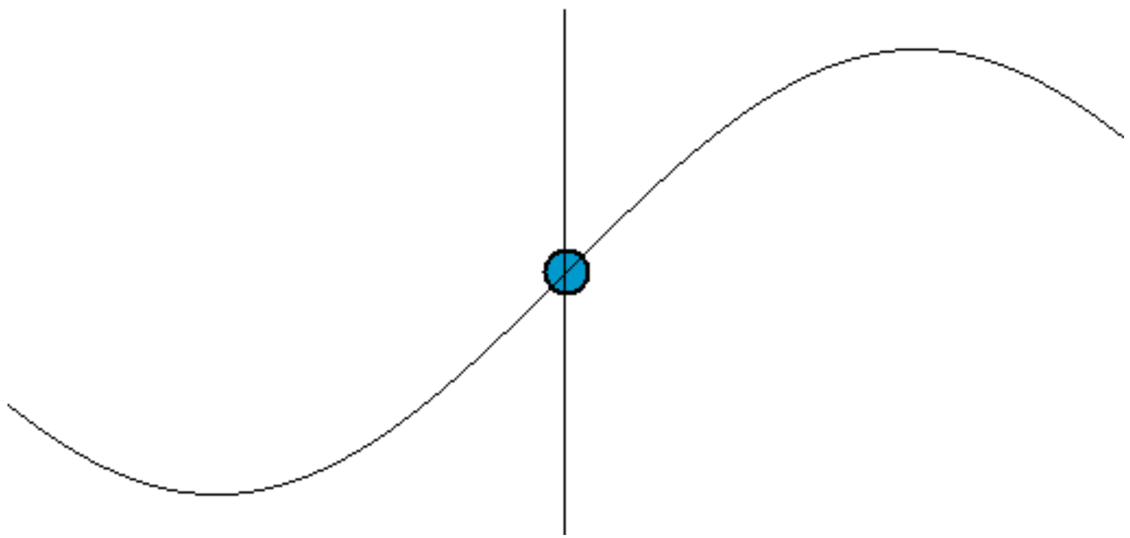
Chapter 15

Wave Motion



15-3 Energy Transported by Waves

As waves travel through a medium, the energy is transferred as vibrational energy from particle to particle. For a harmonic wave the particles move in a SHM.



$$E = \tfrac{1}{2}kA^2 = 2\pi^2 m f^2 A^2.$$

15-3 Energy Transported by Waves

The energy carried by a one-dimensional string on a full wavelength is

$$m = \mu\lambda \qquad\qquad E_\lambda = \frac{1}{2}\mu\lambda\omega^2 A^2$$

***E_λ* is the work done by the driving force per cycle.**

The average power (i.e. the power per cycle) is

$$\bar{P} = \frac{E_\lambda}{T} = E_\lambda f = \frac{1}{2}\mu\,\omega^2 A^2\, v = 2\pi^2\mu\, f^2 A^2\, v$$

15-3 Energy Transported by Waves

In three dimensions, assuming the entire medium has the same density *ρ*, we find:

$$\bar{P} = \frac{E_\lambda}{T} = E_\lambda f = \frac{1}{2}(\rho S)\,\omega^2\,A^2\,v = 2\pi^2(\rho S)\,f^2\,A^2\,v$$

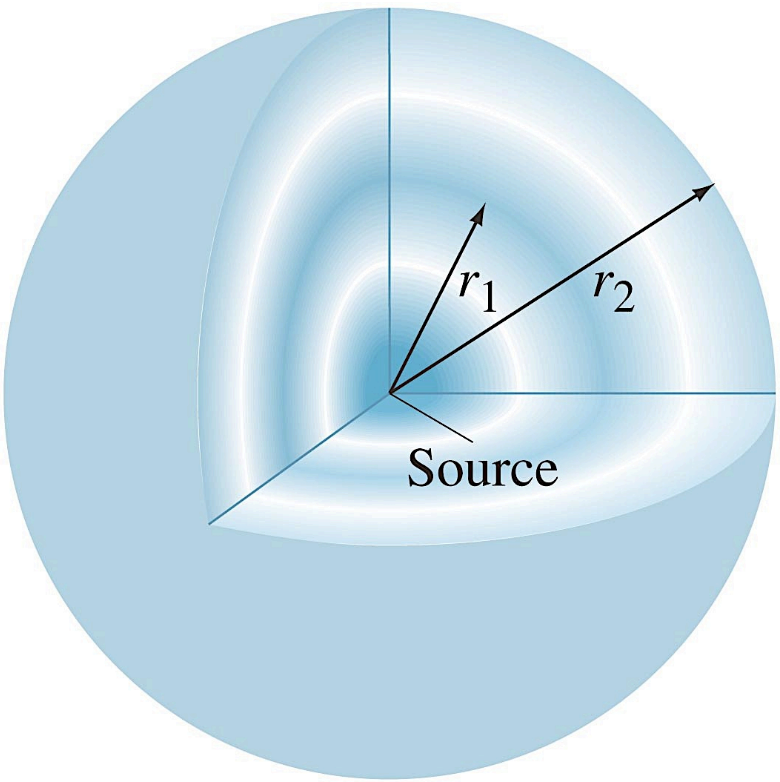
and the intensity

$$I = \frac{\bar{P}}{S} = 2\pi^2 v\rho f^2 A^2.$$

Therefore, the intensity is proportional to the square of the frequency and to the square of the amplitude.

15-3 Energy Transported by Waves

If a wave is able to spread out three-dimensionally from its source, and the medium is uniform, the wave is spherical.



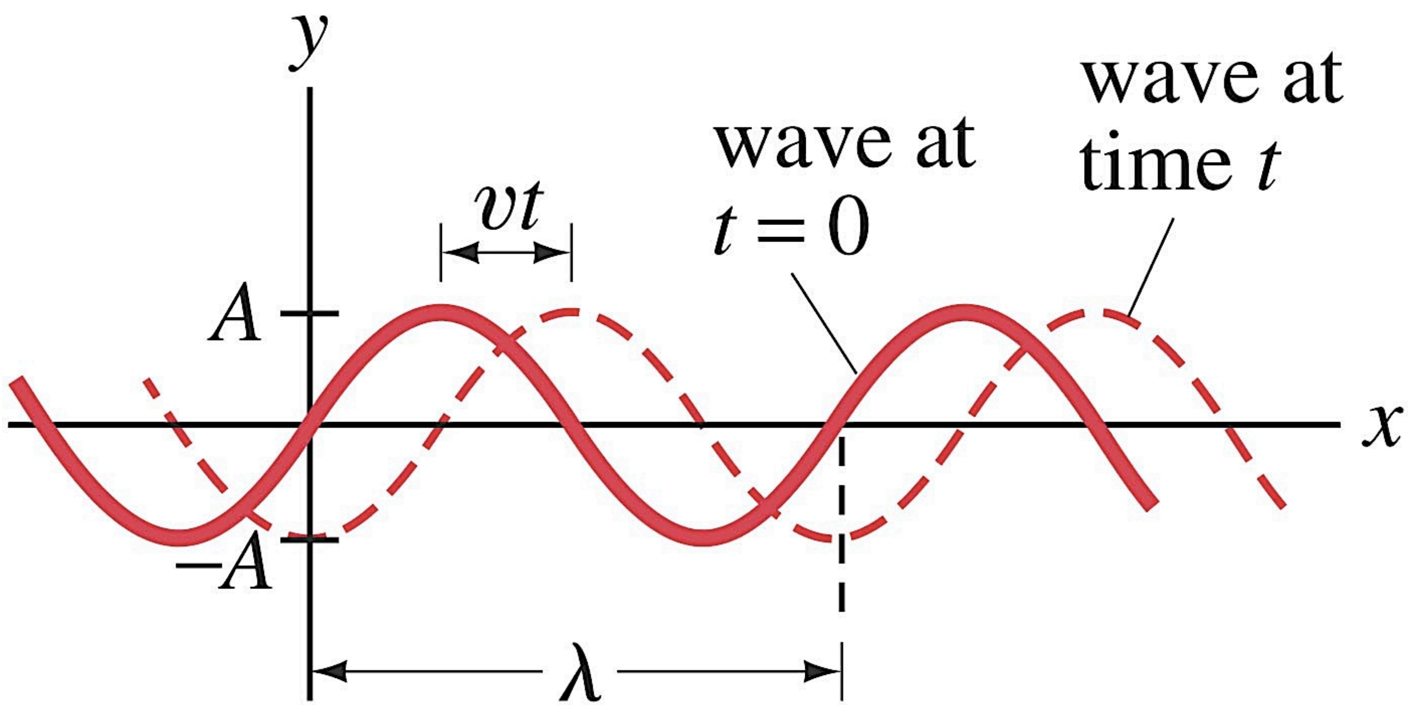
Just from geometrical considerations, as long as the power output is constant, we see:

$$I \propto \frac{1}{r^2}.$$

15-4 Mathematical Representation of a Traveling Wave

Suppose the shape of a wave is given by:

$$D(x) = A \sin \frac{2\pi}{\lambda} x.$$



15-4 Mathematical Representation of a Traveling Wave

After a time ***t***, the wave crest has traveled a distance ***vt***, so we write:

$$D(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right].$$

Or: $$D(x,t) = A\sin(kx - \omega t),$$

with $\omega = 2\pi f$, $k = \frac{2\pi}{\lambda}.$

Example:

A transverse wave travels to the right along a string with a speed 2.0 m/s. At $t = 0$ the shape of wave is given by the function

$$D(x,t) = -0.2\cos(\pi x)$$

a) Is it a harmonic wave?
b) Determine amplitude, wave number, wavelength.
c) Determine a formula for the wave at any time t assuming there are no frictional losses.
d) Determine the angular frequency and frequency.

Example:

A transverse wave travels to the right along a string with a speed 2.0 m/s. At *t = 0* the shape of wave is given by the function

$$D(x,t) = -0.2\cos(\pi x)$$

a) Is it a harmonic wave?

Yes. It is easy to plot the wave. Rewriting:

$$D(x,t) = -0.2\cos(\pi x) = 0.2\sin(\pi x - \frac{\pi}{2})$$

$$= 0.2\sin(\frac{2\pi}{2}x - \frac{\pi}{2})$$

Example:

A transverse wave travels to the right along a string with a speed 2.0 m/s. At $t = 0$ the shape of wave is given by the function

$$D(x,t) = -0.2\cos(\pi x)$$

a) Is it a harmonic wave?
b) Determine amplitude, wave number, wavelength.

$$D(x,t) = 0.2\sin(\frac{2\pi}{2}x - \frac{\pi}{2})$$

$$A = 0.2\text{ m};\quad k = \pi\text{ m}^{-1};\quad \lambda = 2\text{ m}$$

Example:

A transverse wave travels to the right along a string with a speed 2.0 m/s. At $t = 0$ the shape of wave is given by the function

$$D(x,t) = -0.2\cos(\pi x)$$

a) Is it a harmonic wave?
b) Determine amplitude, wave number, wavelength.
c) Determine a formula for the wave at any time t assuming there are no frictional losses.
d) Determine the angular frequency and frequency.

$$D(x,t) = 0.2\sin(\frac{2\pi}{2}x - vkt - \frac{\pi}{2})$$

$$\omega = vk = 2\pi \text{ rad/s}; \qquad f = \frac{\omega}{2\pi} = 1 \text{ Hz}$$

One-Dimensional **Wave Equation**

$$\frac{\partial^2 D}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2} = 0$$

General Solution $D(x,t) = f(x - vt) + g(x + vt)$

Harmonic Wave $D(x,t) = A\sin(kx - \omega t)$

Example 15-6:

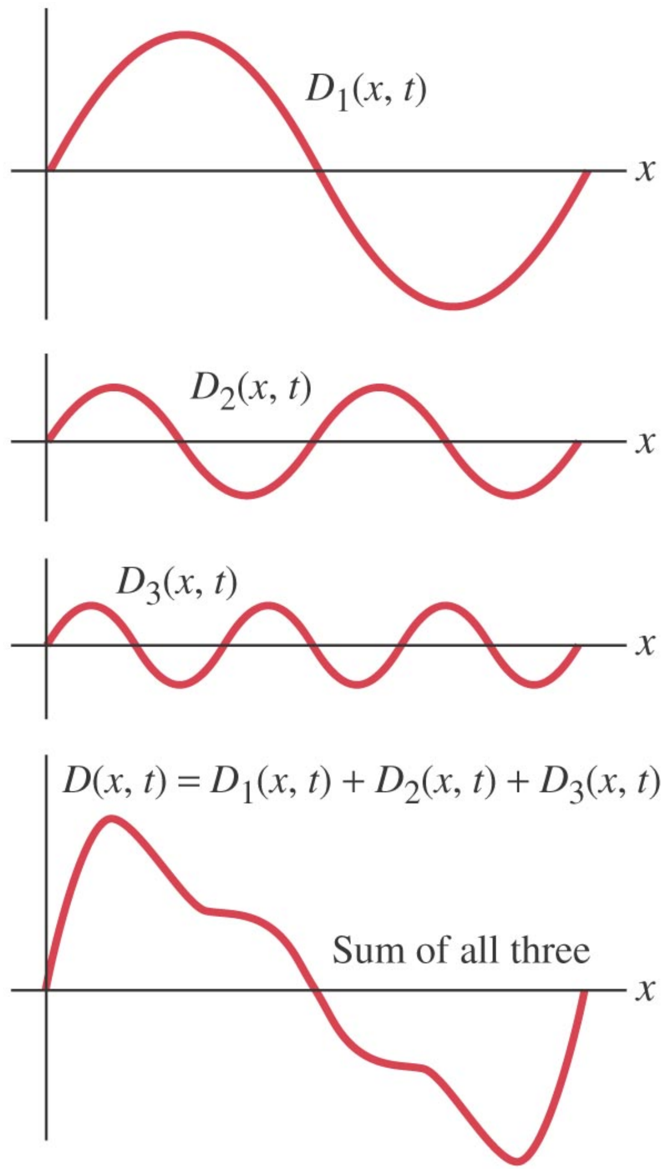
Verify that the harmonic wave satisfies the wave equation

$$\frac{\partial^2 D}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2} = 0$$

$$D(x,t) = A\sin(kx - \omega t)$$

15-6 The Principle of Superposition

The Wave Equation is Linear
=> Superposition Principle



Fourier's theorem:

Any complex periodic wave can be written as the sum of sinusoidal waves of different amplitudes, frequencies, and phases.

15-6 The Principle of Superposition

Conceptual Example 15-7:
Making a square wave.

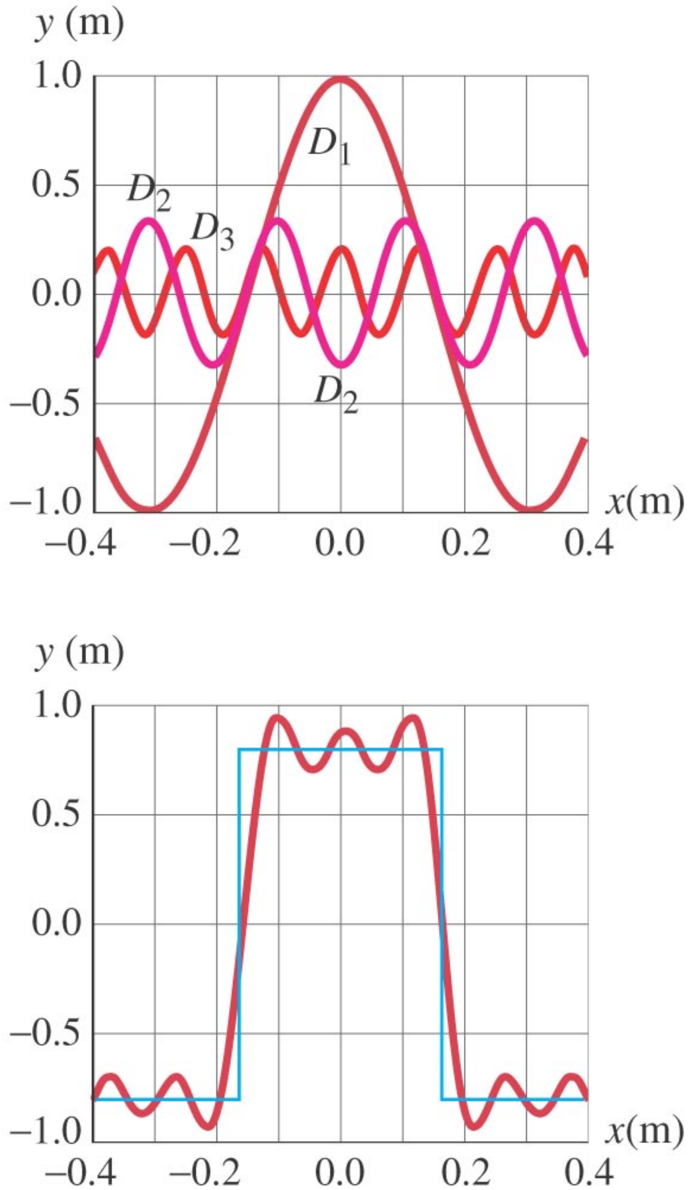
At $t = 0$, three waves are given by

$D_1 = A \cos kx,$

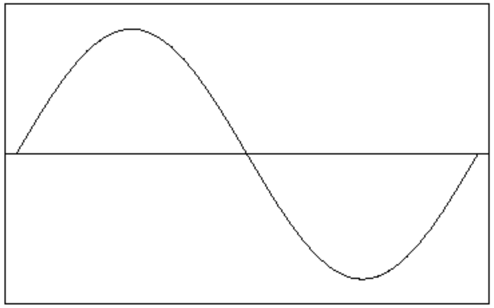
$D_2 = -\frac{1}{3} A \cos 3kx,$

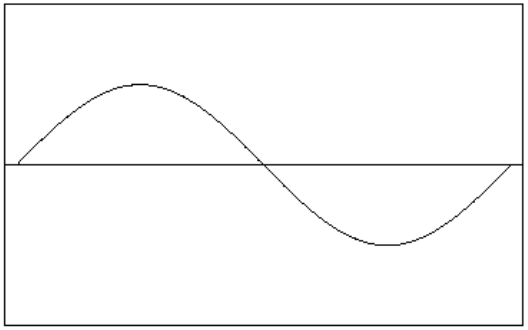
$D_3 = \frac{1}{5} A \cos 5kx.$

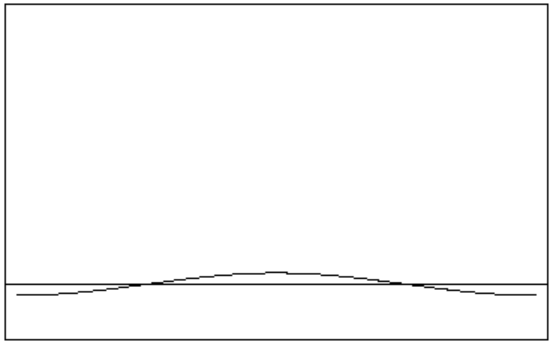
(These three waves are the first three Fourier components of a "square wave.")



The Wave Equation is Linear => Superposition Principle







The Wave Equation is Linear => Superposition Principle