Chapter 37
Early Quantum Theory and Models of the Atom

Units of Chapter 37

Wave–Particle Duality

particle **wave**

$$\begin{pmatrix} \vec{p} \\ E/c \end{pmatrix} = \frac{h}{2\pi} \begin{pmatrix} \vec{k} \\ \omega/c \end{pmatrix}$$

$$\vec{p} = \frac{h}{2\pi}\vec{k} \quad \rightarrow \quad \lambda = \frac{h}{p}$$

$$\frac{E}{c} = \frac{h}{2\pi}\frac{\omega}{c} \quad \rightarrow \quad E = hf$$

The Wave Nature of Matter

particle **wave**

$$\begin{pmatrix} \vec{p} \\ E/c \end{pmatrix} = \frac{h}{2\pi} \begin{pmatrix} \vec{k} \\ \omega/c \end{pmatrix}$$

If waves can behave like particle, then particles can behave like waves

$$\lambda = \frac{h}{p}$$ **De Broglie wavelength**

37-6 Wave–Particle Duality; the Principle of Complementarity

The principle of complementarity states that both the wave and particle aspects of light are fundamental to its nature.

Indeed, waves and particles are just our interpretations of how light behaves.

37-7 Wave Nature of Matter

Example 37-10: Wavelength of a ball.

Calculate the de Broglie wavelength of a 0.20-kg ball moving with a speed of 15 m/s.

37-7 Wave Nature of Matter

Example 37-10: Wavelength of a ball.

Calculate the de Broglie wavelength of a 0.20-kg ball moving with a speed of 15 m/s.

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{(6.6\times10^{-34}\ \mathrm{J\cdot s})}{(0.2\ \mathrm{kg})\ (15\ \mathrm{m/s})} = 2.2\times10^{-34}\ \mathrm{m}$$

The de Broglie wavelength of an ordinary object is too small to be detected.

37-7 Wave Nature of Matter

The properties of waves, such as interference and diffraction, are significant only when the size of objects or slits is not much larger than the wavelength.

If the mass is really small, the wavelength can be large enough to be measured.

37-7 Wave Nature of Matter

Example 37-11: Wavelength of an electron.

Determine the wavelength of an electron that has been accelerated through a potential difference of 100 V.

37-7 Wave Nature of Matter

Example 37-11: Wavelength of an electron.

Determine the wavelength of an electron that has been accelerated through a potential difference of 100 V.

$$\frac{1}{2}mv^2 = eV \quad \rightarrow \quad v = \sqrt{\frac{2\,(100\text{ eV})}{9.1\times10^{-31}\text{ kg}}} = 5.9\times10^{6}\text{ m/s}$$

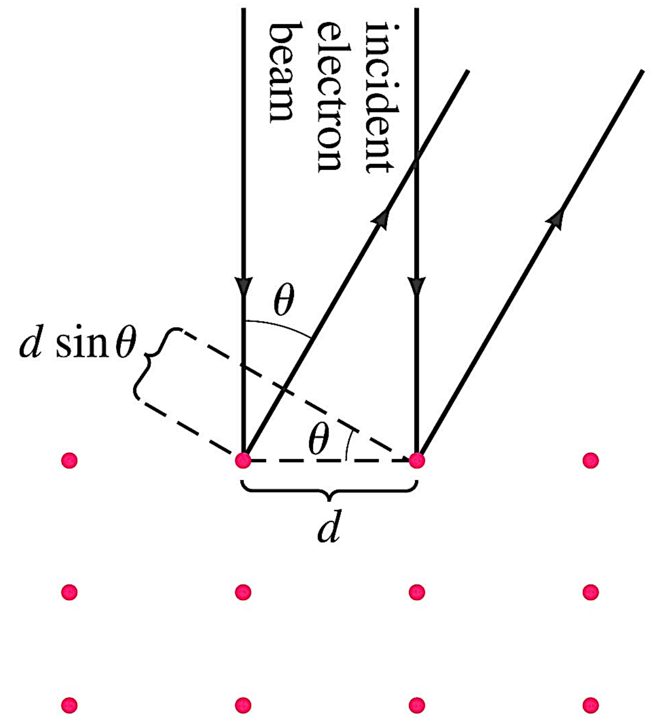
$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \rightarrow \quad \lambda = \frac{(6.6\times10^{-34}\text{ J}\cdot\text{s})}{(9.1\times10^{-31}\text{ kg})\;(5.9\times10^{6}\text{ m/s})} = 0.12\times10^{-9}\text{ m}$$

Similarly to X-ray diffraction, atom in crystals can be used to diffract electrons.

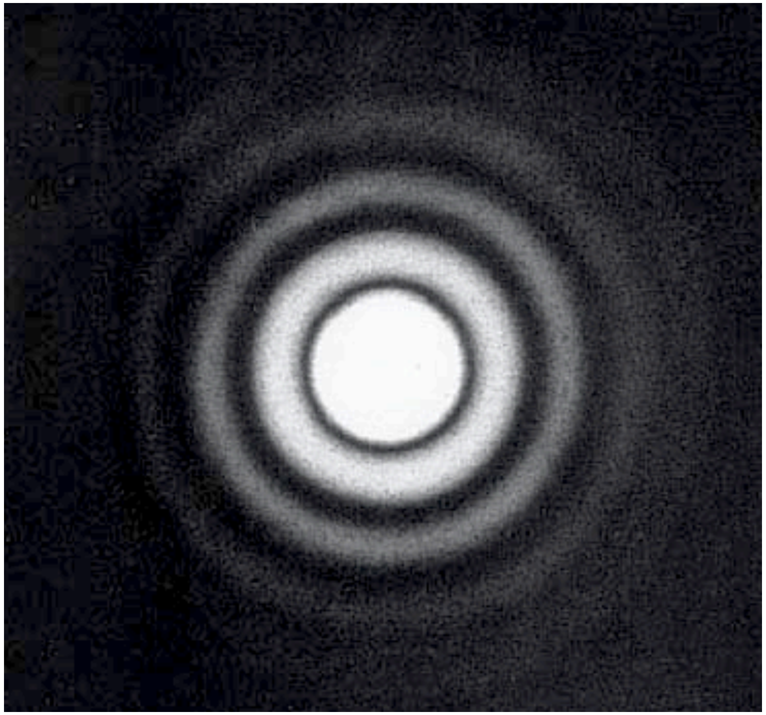
37-7 Wave Nature of Matter

Example 37-12: Electron diffraction.

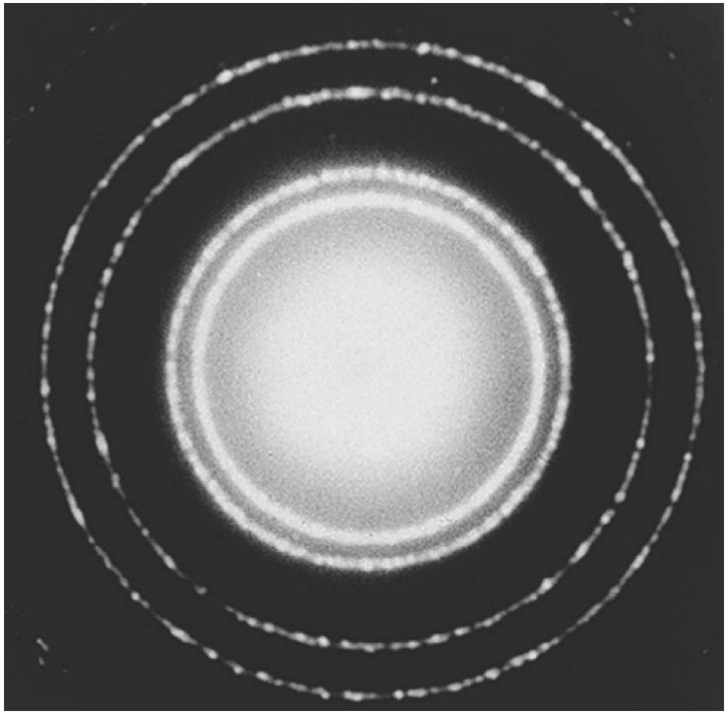
The wave nature of electrons is manifested in experiments where an electron beam interacts with the atoms on the surface of a solid. By studying the angular distribution of the diffracted electrons, one can indirectly measure the geometrical arrangement of atoms. Assume that the electrons strike perpendicular to the surface of a solid, and that their energy is low, E_K = 100 eV, so that they interact only with the surface layer of atoms. If the smallest angle at which a diffraction maximum occurs is at 24°, what is the separation d between the atoms on the surface?



37-7 Wave Nature of Matter



Photon diffraction

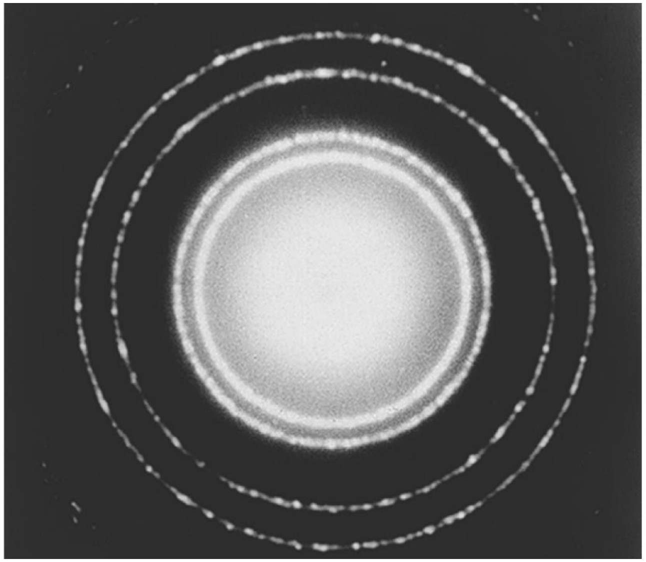


Electron diffraction

37-7 Wave Nature of Matter

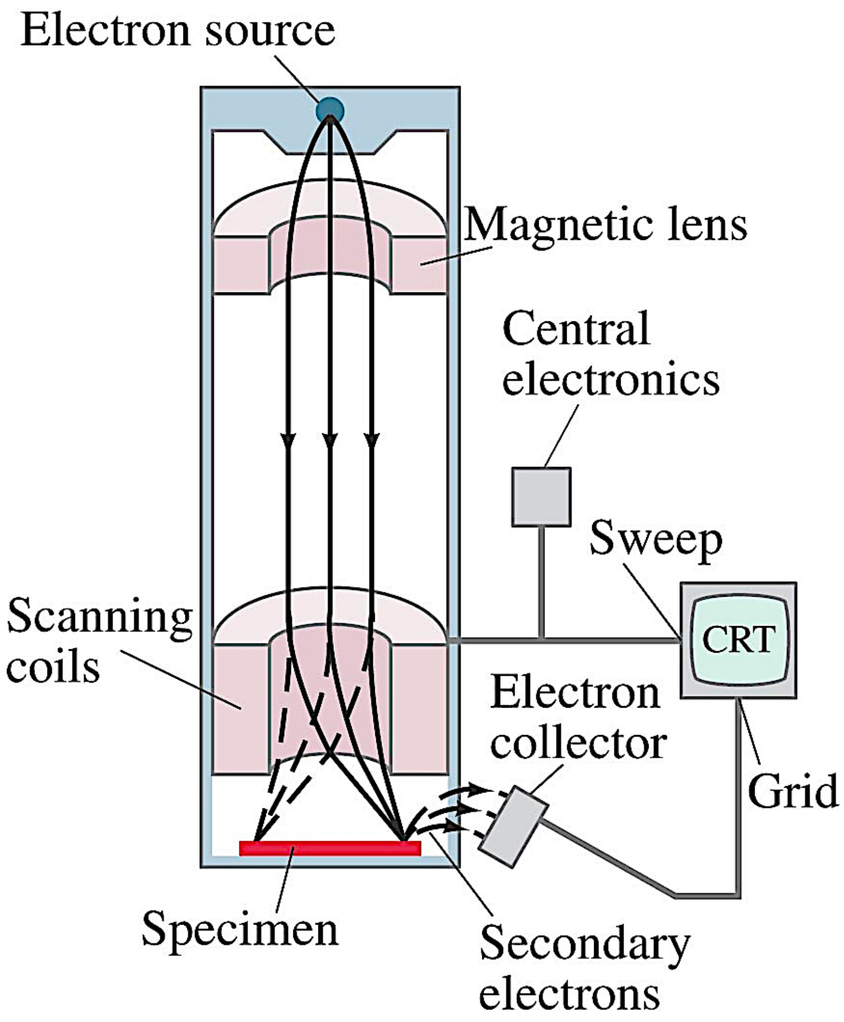
Similarly to X-ray diffraction, atom in crystals can be used to diffract electrons:

In 1927, two American physicists C. J. Davisson and L. H. Germer performed the crucial experiment that confirmed the de Broglie hypothesis.



Diffraction pattern of electrons scattered from Al foil.

37-8 Electron Microscopes



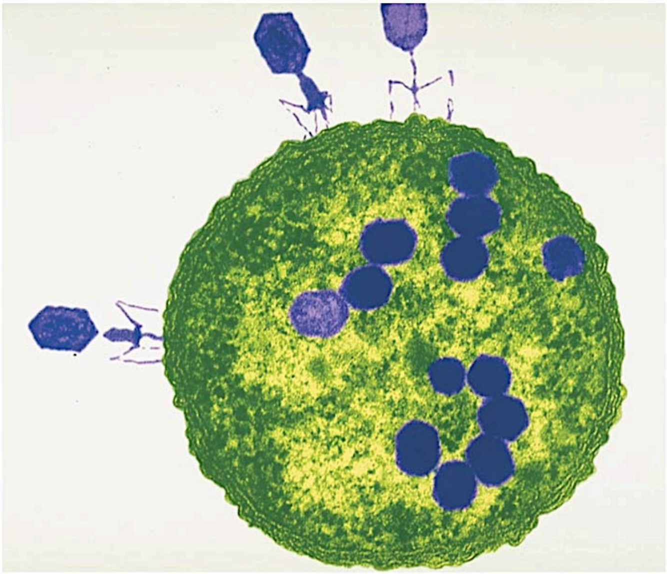
Scanning electron microscope:

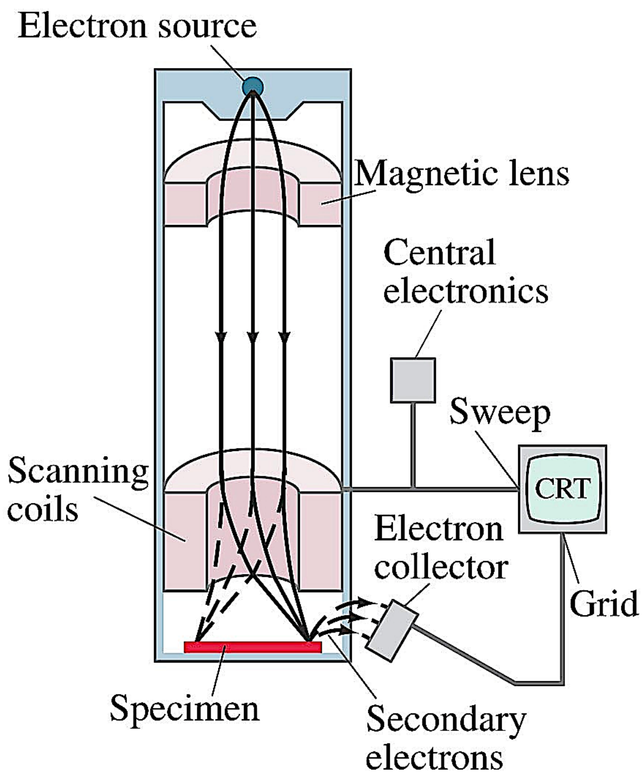
The electron beam is scanned back and forth across the object to be imaged. The electrons interact with the atoms that make up the sample producing signals that contain information about the sample's surface topography, composition, and other properties such as electrical conductivity.

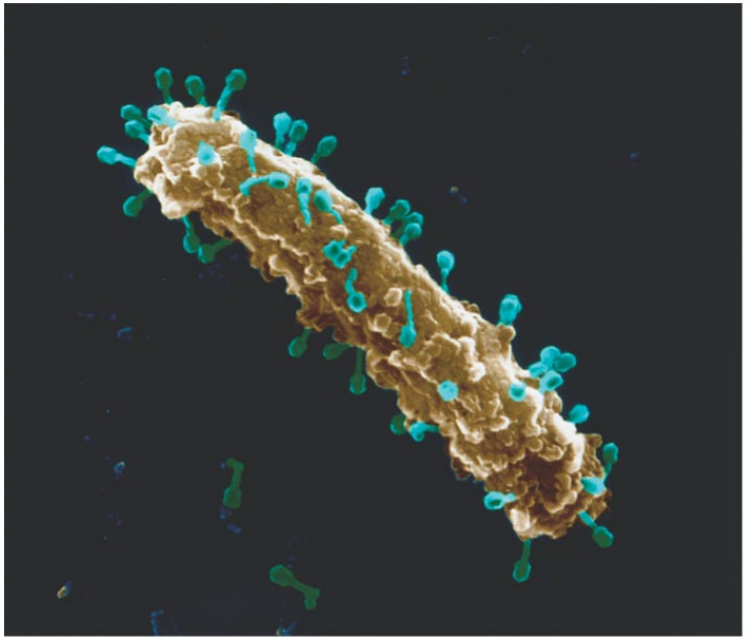
37-8 Electron Microscopes

Electron accelerated by high voltages (100 kV) have wavelengths of about 4 pm.

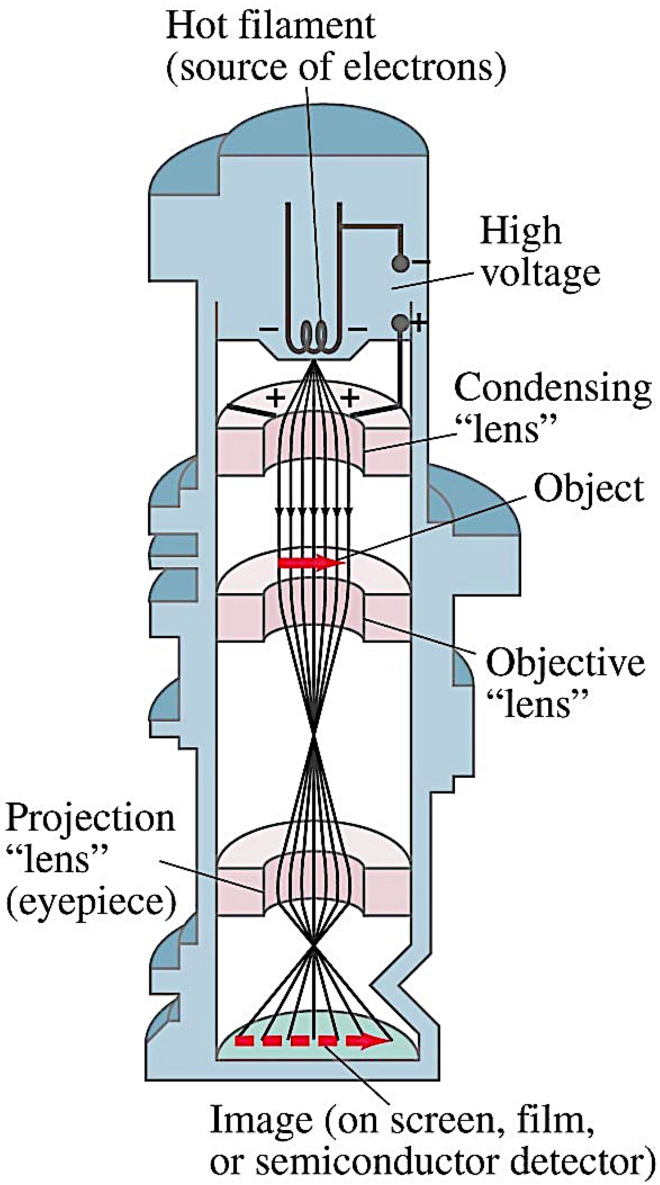
Resolution is limited by aberrations in the magnetic lenses.







37-8 Electron Microscopes



Transmission electron microscope:

is a microscopy technique whereby a beam of electrons is transmitted through an ultra thin specimen, interacting with the specimen as it passes through. An image is formed from the interaction of the electrons transmitted through the specimen.

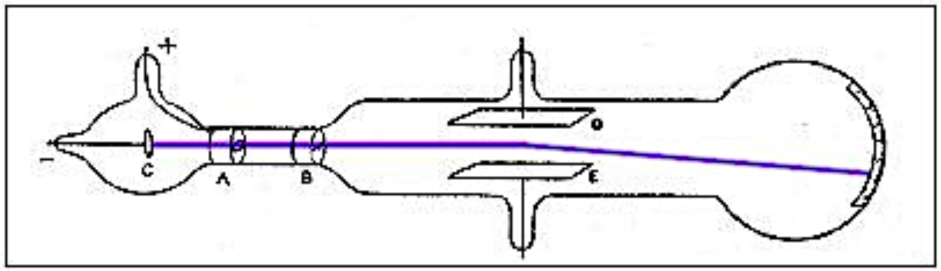
37-8 Electron Microscopes

The wavelength of electrons will vary with energy, but is still quite short. This makes electrons useful for imaging – remember that the smallest object that can be resolved is about one wavelength.

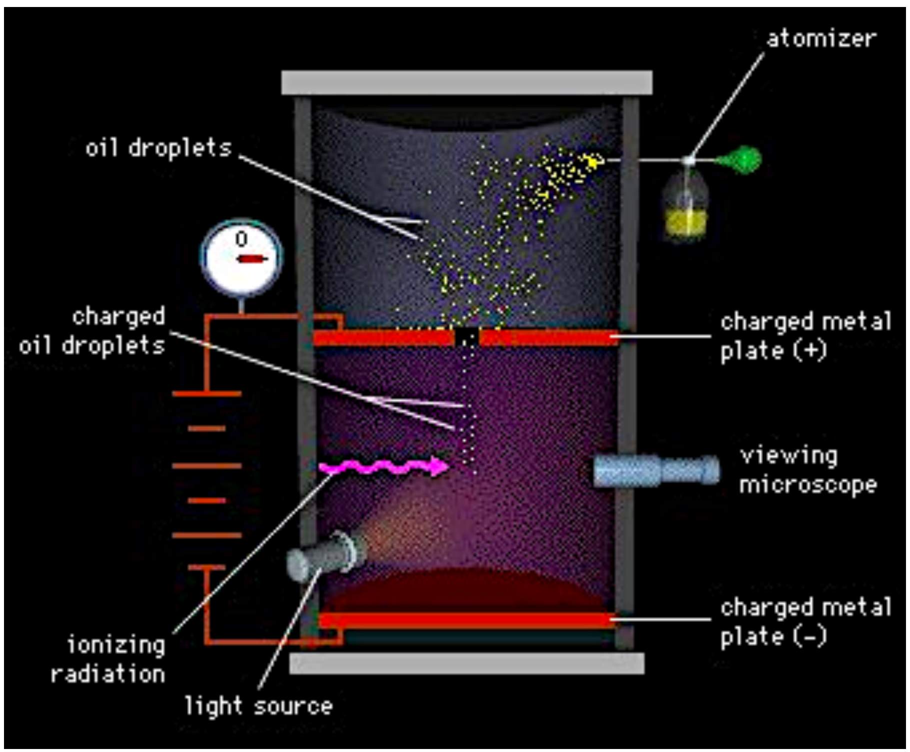
Electrons used in electron microscopes have wavelengths of about 4 pm.

What is an electron?

"*A logical construction*" – B. Russell



Experimenting with cathode rays in 1897, J.J. Thomson had discovered negatively charged 'corpuscles', as he called them, with a charge to mass ratio 1840 times that of a hydrogen ion.

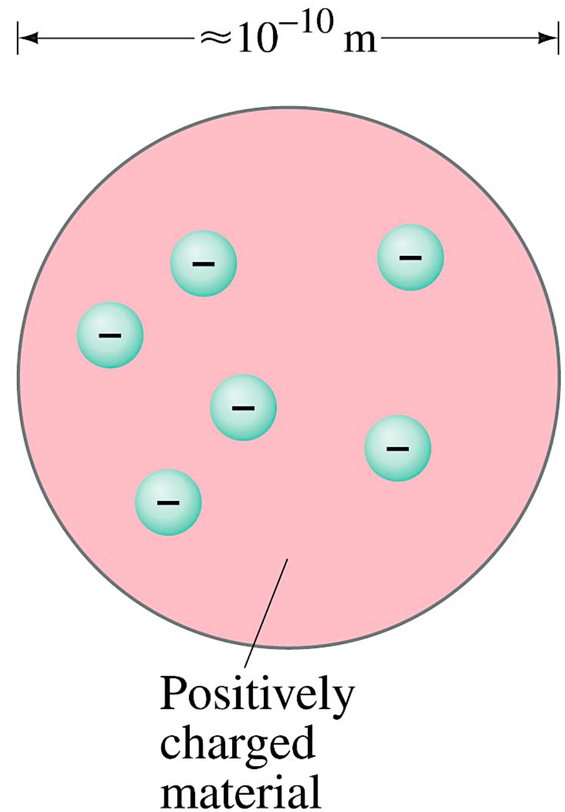


In 1913, Robert A. Millikan

measured the charge of an electron, one of the fundamental physical constants. His experiment measured the force on tiny charged droplets of oil suspended against gravity between two metal electrodes.

37-9 Early Models of the Atom

It was known that atoms were electrically neutral, but that they could become charged, implying that there were positive and negative charges and that some of them could be removed.



One popular atomic model was the "plum-pudding" model:

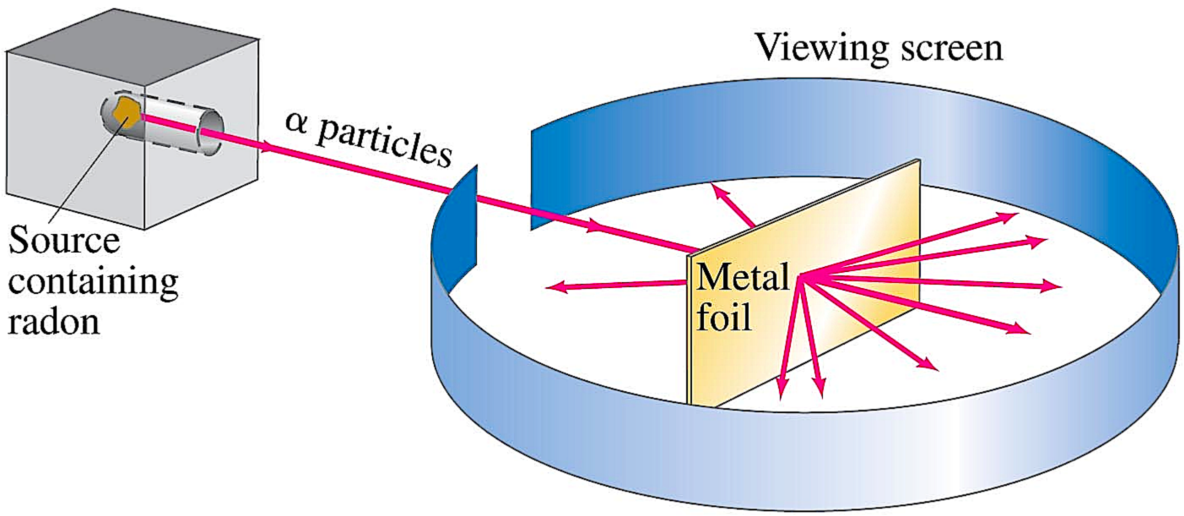
37-9 Early Models of the Atom

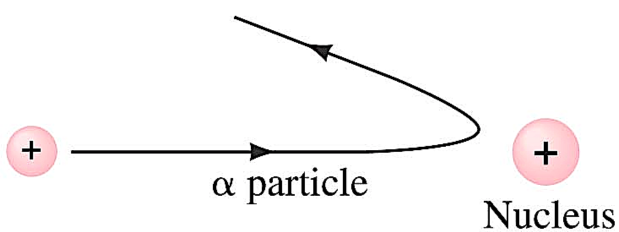
This model had the atom consisting of a bulk positive charge, with negative electrons buried throughout.

Rutherford did an experiment that showed that the positively charged nucleus must be extremely small compared to the rest of the atom. He scattered alpha particles – helium nuclei – from a metal foil and observed the scattering angle. He found that some of the angles were far larger than the plum-pudding model would allow.

37-9 Early Models of the Atom

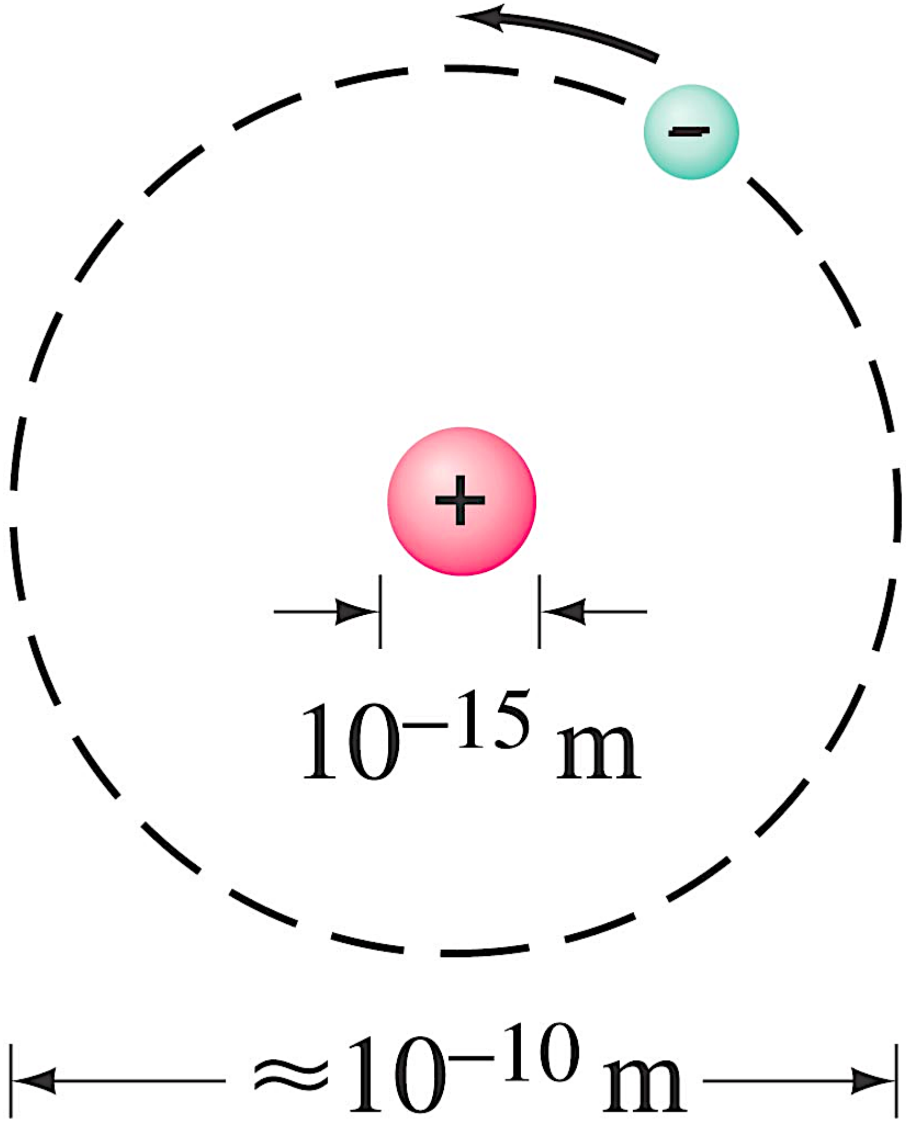
In 1911, Rutherford experiments showed that the only way to account for the large angles was to assume that **all the positive charge was contained within a tiny volume** – now we know that the radius of the nucleus is 1/10,000 that of the atom.





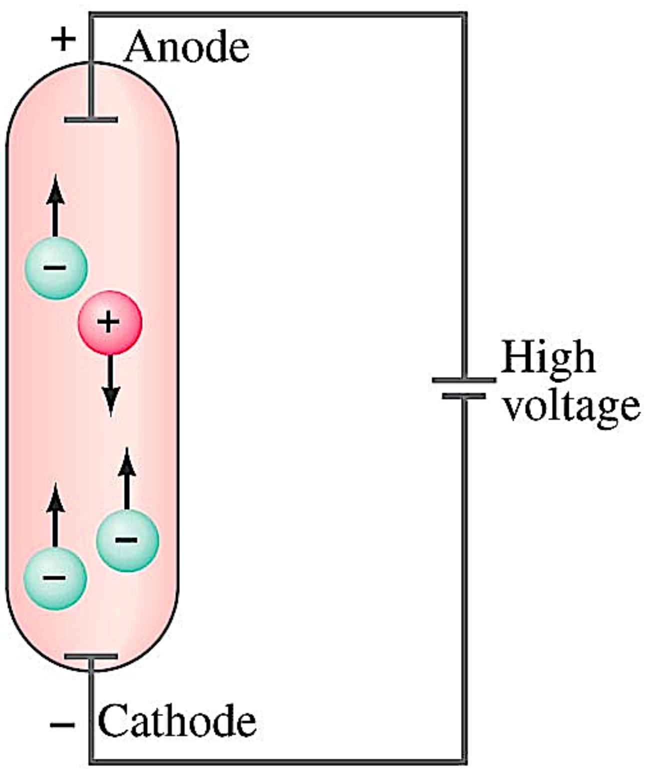
37-9 Early Models of the Atom

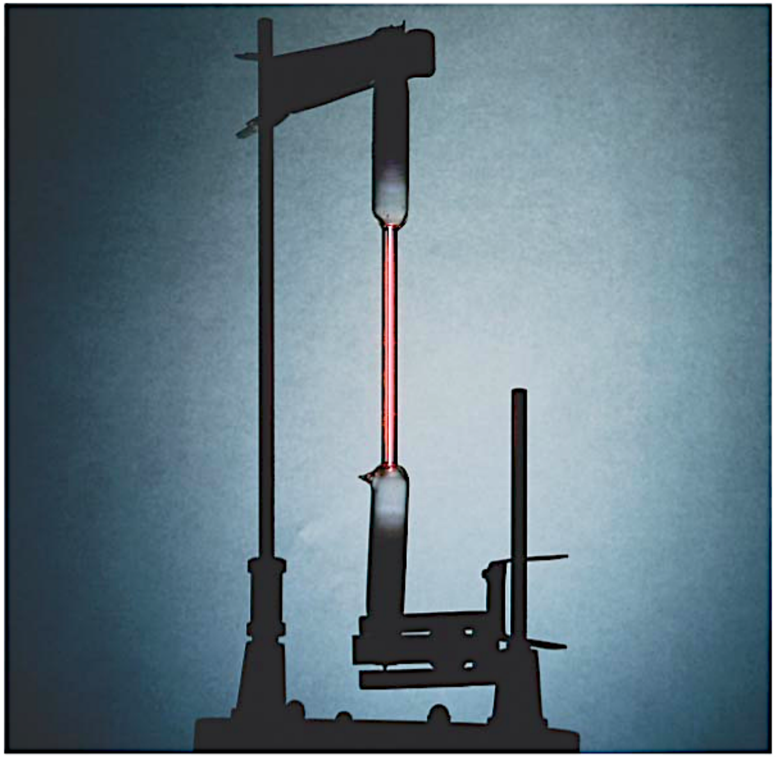
Therefore, Rutherford's "planetary" model of the atom is mostly empty space:



37-10 Atomic Spectra: Key to the Structure of the Atom

Differently than *thermal light*, a very thin gas heated in a discharge tube emits light only at characteristic frequencies.





37-10 Atomic Spectra: Key to the Structure of the Atom

An atomic spectrum is a line spectrum – only certain frequencies appear. If white light passes through such a gas, it absorbs at those same frequencies.

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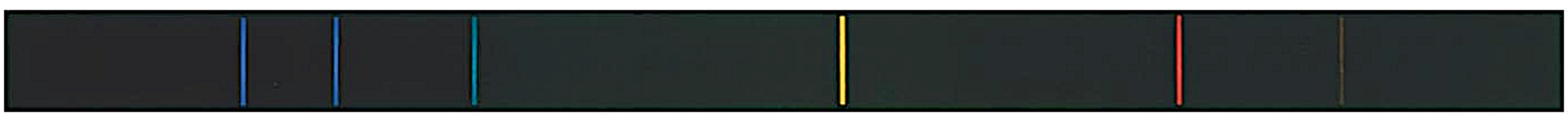
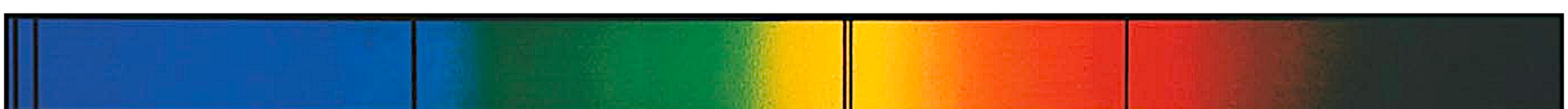


He



***solar absorption* spectrum**

37-10 Atomic Spectra: Key to the Structure of the Atom

Any theory of atomic structure must be able to explain why atoms emit light only of discrete wavelengths, and it should be able to predict what these wavelengths are.

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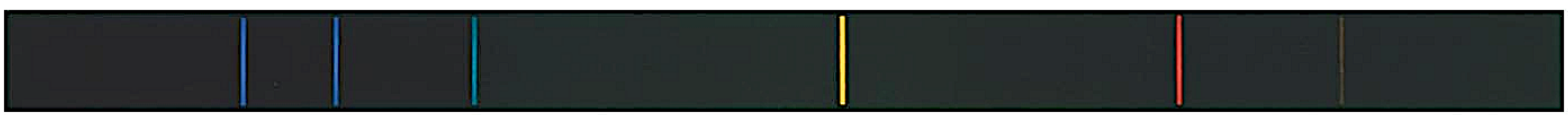
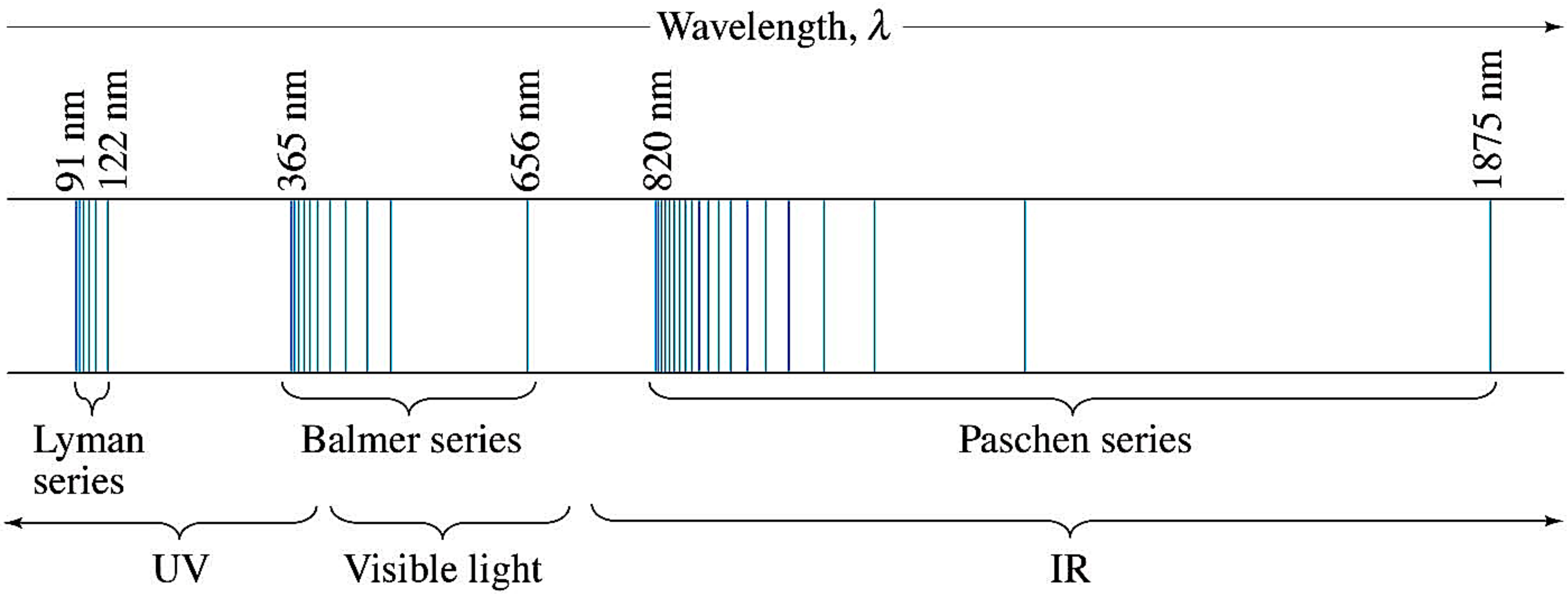


He

solar absorption spectrum

37-10 Atomic Spectra: Key to the Structure of the Atom

A portion of the complete spectrum of hydrogen is shown here. The lines cannot be explained by the Rutherford theory.



37-10 Atomic Spectra: Key to the Structure of the Atom

The wavelengths of electrons emitted from hydrogen have a regular pattern:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad n = 3, 4, 5 \ldots$$

This is called the Balmer series. R is the Rydberg constant:

$$R = 1.0974 \times 10^7 \text{ m}^{-1}$$

37-10 Atomic Spectra: Key to the Structure of the Atom

Other series include the Lyman series:

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad n = 2,3,4,5\ldots$$

and the Paschen series:

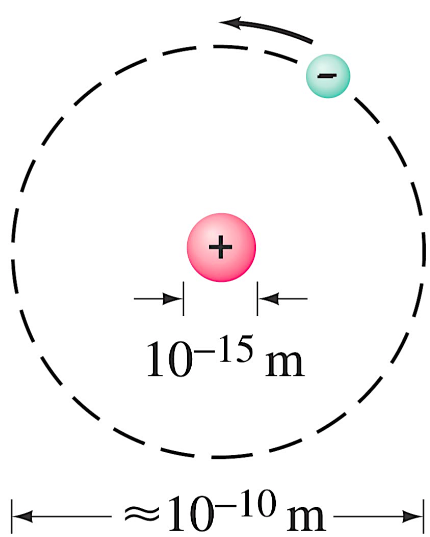
$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \qquad n = 4,5\ldots$$

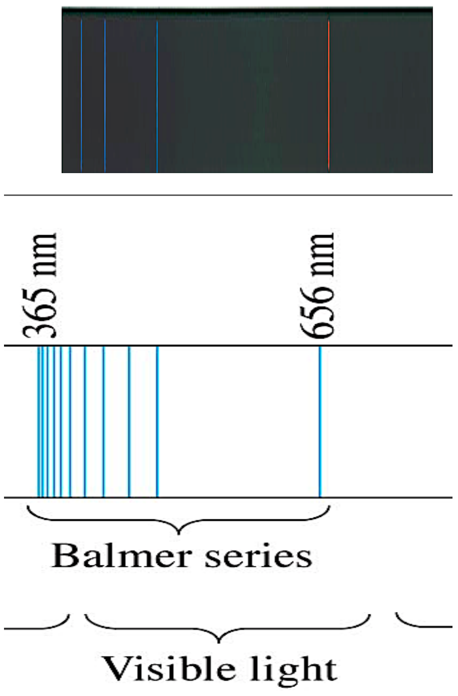
From the Rutherford Model to the Bohr Model

The two main difficulties with the Rutherford Model are:

1) It predicts that light of a continuous range will be emitted.

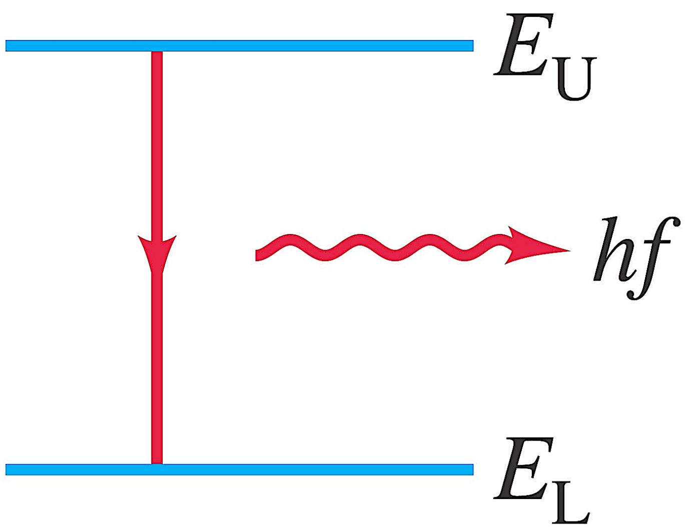
1) It predicts that atoms are unstable.





37-11 The Bohr Model

Bohr was convinced that Rutherford Model was valid, but it needed to be modified. In 1912, he proposed that the possible energy states for atomic electrons were quantized – only certain values were possible. Then the spectrum could be explained as transitions from one level to another.

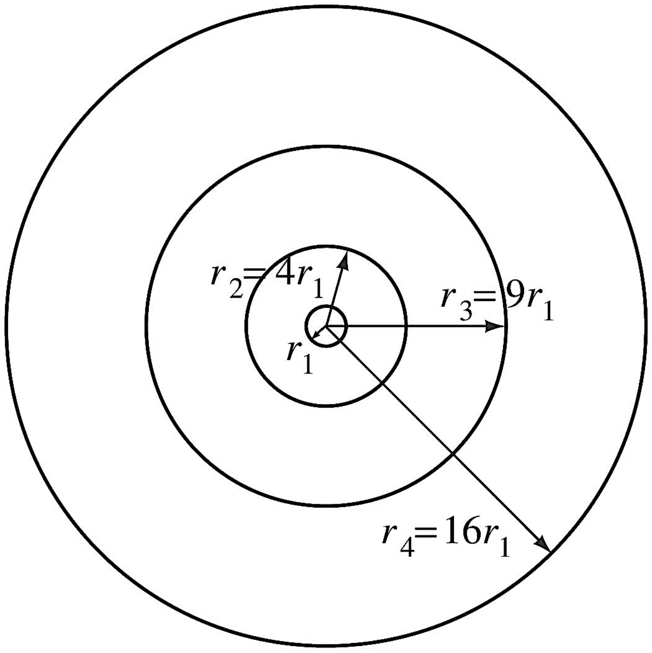


37-11 The Bohr Model

Bohr found that to explain the spectral line series also the angular momentum had to be quantized:

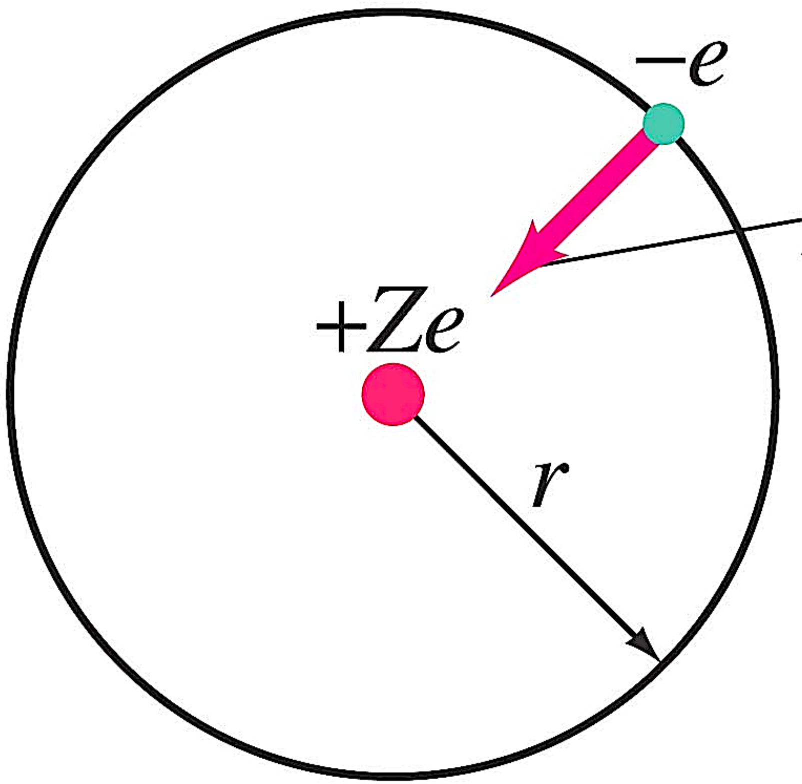
Bohr's quantum condition:

$$L = mvr_n = n\frac{h}{2\pi}, \qquad n = 1,2,3,\ldots$$



37-11 The Bohr Model

An electron is held in orbit by the Coulomb force:



$$F = \frac{1}{4\pi\epsilon_0}\frac{(Ze)(e)}{r^2}$$

37-11 The Bohr Model

Using the Coulomb force, we can calculate the radii of the orbits.

$$F = m\frac{v^2}{r_n}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{(r_n)^2} = m\frac{v^2}{r_n}$$

$$v = n\frac{h}{2\pi}\frac{1}{mr_n}$$

Bohr's radius (r_1):

$$r_1 = \frac{\varepsilon_0}{\pi}\frac{h}{me^2} = 0.529 \times 10^{-10}\,\mathrm{m}$$

$$r_n = \frac{n^2}{Z}(0.529 \times 10^{-10}\,\mathrm{m}), \qquad n = 1,2,3,\ldots$$

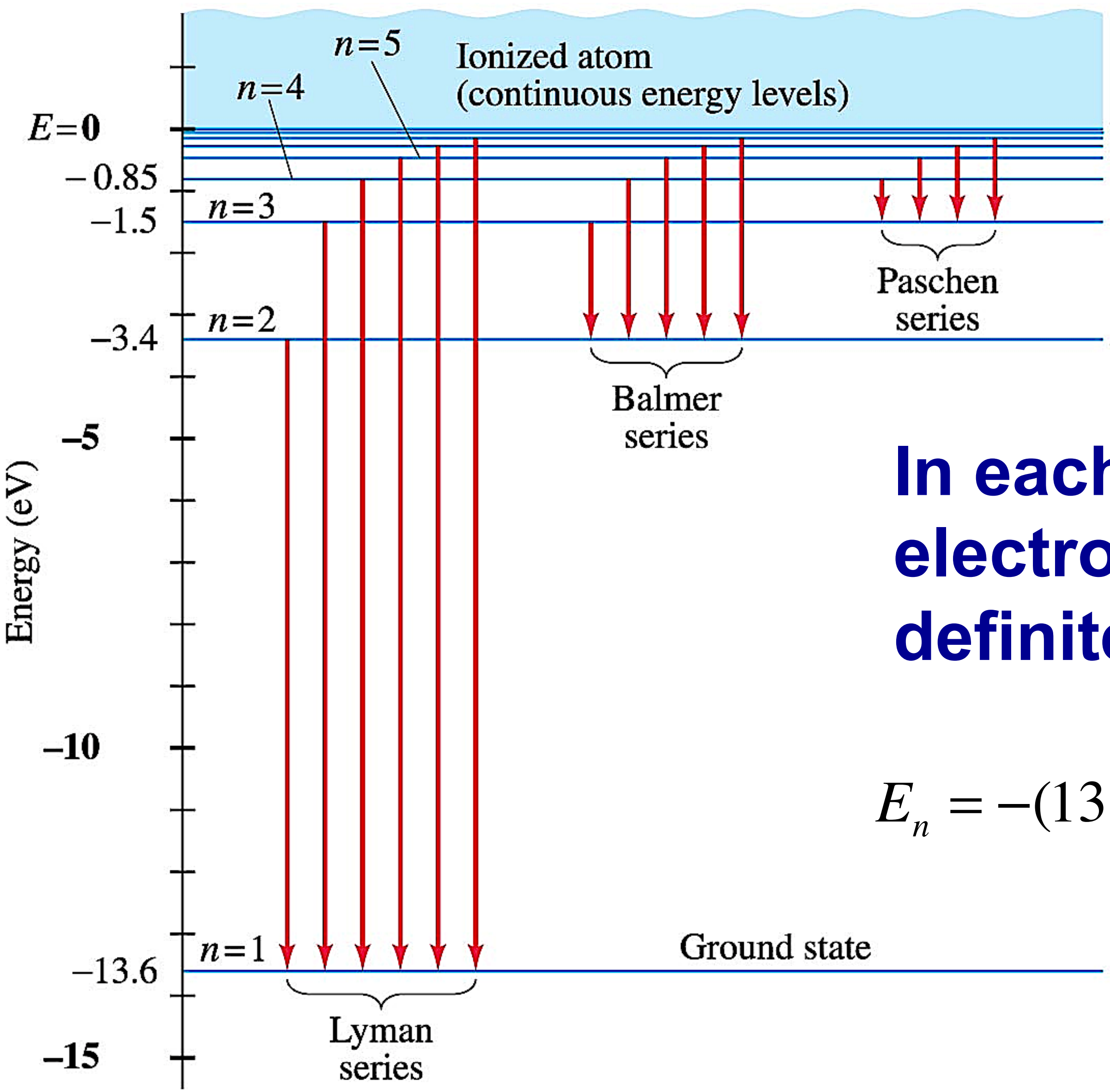
37-11 The Bohr Model

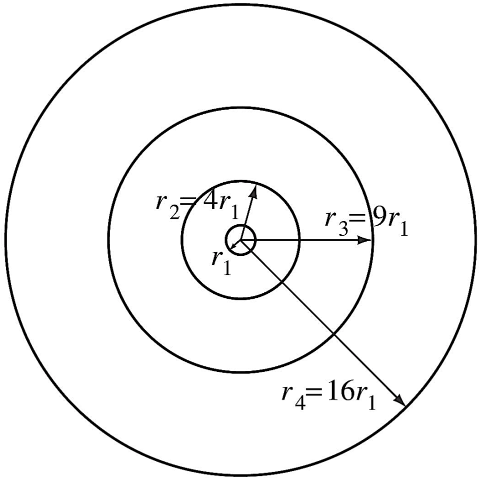
In each of its possible orbits, the electron would have a definite energy:

$$
\begin{cases}
E_n = E_K + U = \dfrac{1}{2}mv^2 - \dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze^2}{r_n} \\[2ex]
r_n = \dfrac{n^2}{Z}(0.529\times 10^{-10}\,\mathrm{m}), \qquad n = 1,2,3,\ldots \\[2ex]
v_n = n\dfrac{h}{2\pi}\dfrac{1}{mr_n}
\end{cases}
$$

$$E_n = -(13.6\ \mathrm{eV})\frac{Z^2}{n^2}, \qquad n = 1,2,3,\ldots$$

37-11 The Bohr Model





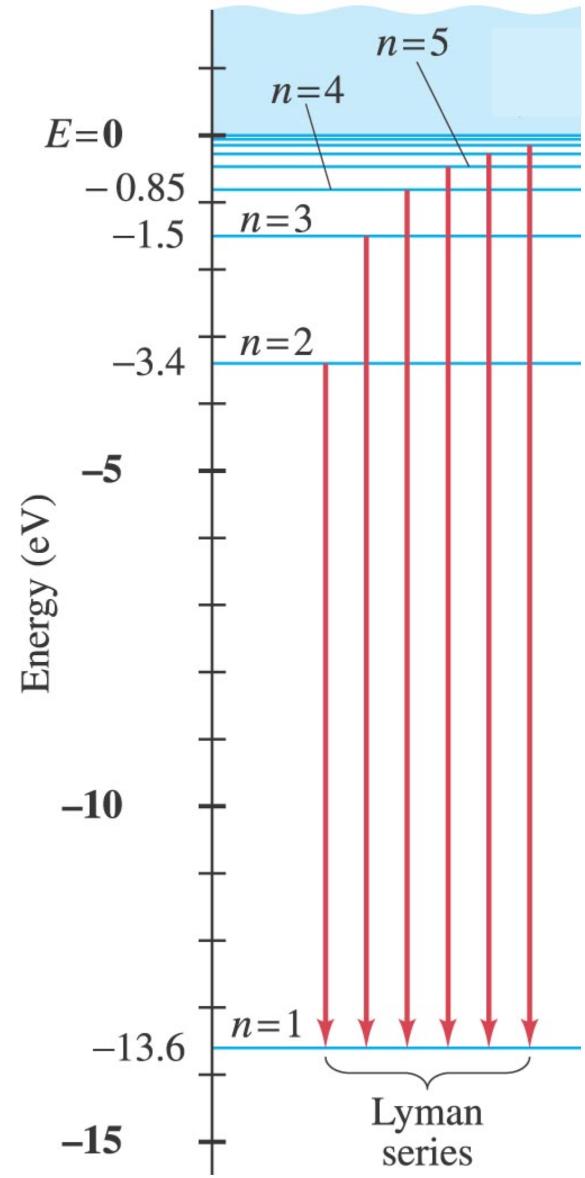
In each of its orbits, the electron would have a definite energy:

$$E_n = -(13.6\ \text{eV})\frac{Z^2}{n^2}, \qquad n = 1, 2, 3, \ldots$$

37-11 The Bohr Model

Example 37-13: Wavelength of a Lyman line.

Use this figure to determine the wavelength of the first Lyman line, the transition from n = 2 to n = 1. In what region of the electromagnetic spectrum does this lie?



37-11 The Bohr Model

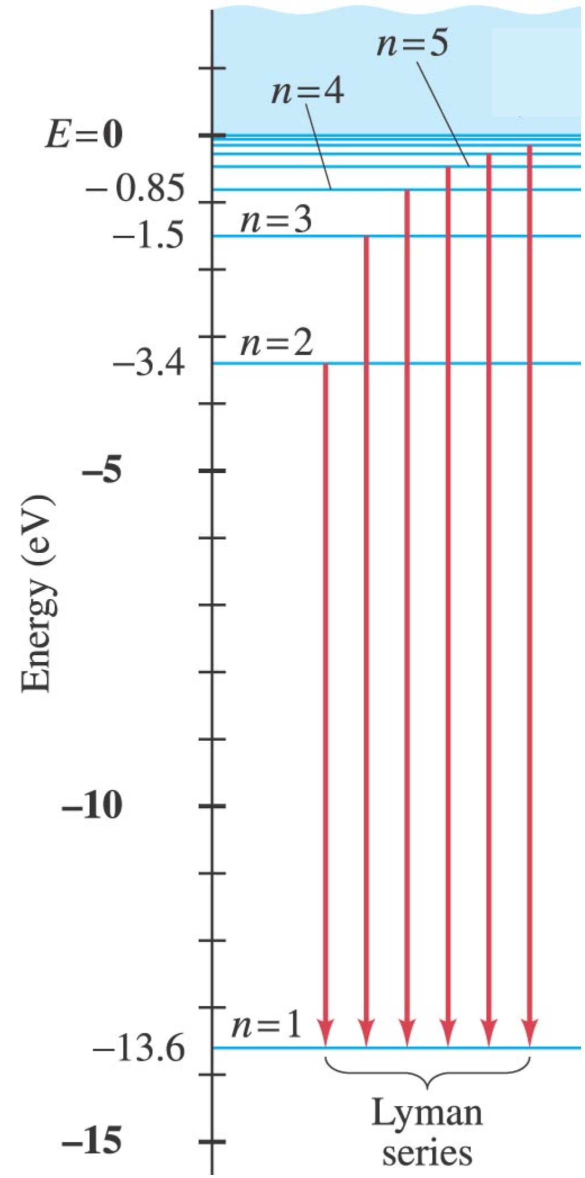
Example 37-14: Wavelength of a Balmer line.

Determine the wavelength of light emitted when a hydrogen atom makes a transition from the $n = 6$ to the $n = 2$ energy level according to the Bohr model.

37-11 The Bohr Model

Example 37-15: Absorption wavelength.

Use this figure to determine the maximum wavelength that hydrogen in its ground state can absorb. What would be the next smaller wavelength that would work?



37-11 The Bohr Model

Example 37-16: He^+ ionization energy.

(a) Use the Bohr model to determine the ionization energy of the He^+ ion, which has a single electron. (b) Also calculate the maximum wavelength a photon can have to cause ionization.

37-11 The Bohr Model

Conceptual Example 37-17: Hydrogen at 20°C.

Estimate the average kinetic energy of whole hydrogen atoms (not just the electrons) at room temperature, and use the result to explain why nearly all H atoms are in the ground state at room temperature, and hence emit no light.

37-11 The Bohr Model

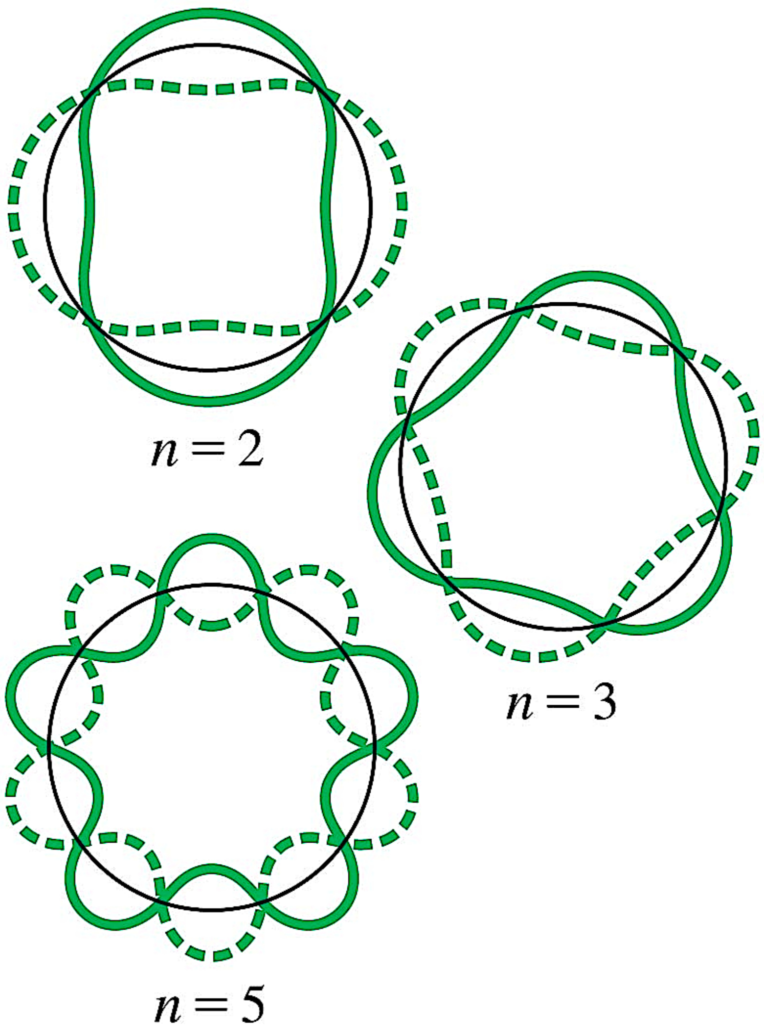
The **correspondence principle** applies here as well – when the differences between quantum levels are small compared to the energies, they should be imperceptible.

37-12 de Broglie's Hypothesis Applied to Atoms

De Broglie's hypothesis is the one associating a wavelength with the momentum of a particle. He proposed that only those orbits where the wave would be a circular standing wave will occur. This yields the same relation that Bohr had proposed.

In addition, it makes more reasonable the fact that the electrons do not radiate, as one would otherwise expect from an accelerating charge.

37-12 de Broglie's Hypothesis Applied to Atoms



These are circular standing waves for n = 2, 3, and 5.

Summary of Chapter 37

- **Planck's hypothesis: molecular oscillation energies are quantized:**

$$E = n\,h\,f \qquad\qquad n = 1, 2, 3, \ldots$$

$$h = 6.6 \times 10^{-34}\,\mathrm{J \cdot s}$$

- **Light can be considered to consist of photons, each of energy**

$$E = hf$$

- **Photoelectric effect: incident photons knock electrons out of material.**

Summary of Chapter 37

- **Compton effect and pair production also support photon theory.**
- **Wave–particle duality – both light and matter have both wave and particle properties.**
- **Wavelength of an object:**

$$\lambda = \frac{h}{p}$$

Summary of Chapter 37

- Principle of complementarity: both wave and particle properties are necessary for complete understanding.

- Rutherford showed that atom has tiny nucleus.

- Line spectra are explained by electrons having only certain specific orbits.

- Ground state has the lowest energy; the others are called excited states.

$$E_n = -(13.6\ \mathrm{eV})\frac{Z^2}{n^2}, \qquad n = 1,2,3,\ldots$$