University of Rochester
Department of Physics and Astronomy
PHY 123, Spring 2012

Final Exam
SOLUTIONS

Tuesday, May 8, 2012

- Time: 2 h 30 min.

- Answer must be written in the blue book provided.

- Put your name on all blue books you use. Clearly label the problem number.

- You are only allowed to use a pen (or pencil + eraser) and one "cheat sheet" (both sides of one letter size sheet, 8.5 in × 11 in).

Answer the following multiple-choice questions:

1) (*7 points.*)

If one end of a heavy rope is attached to one end of a light rope, a wave can move from the heavy rope into the lighter one. What happens to (i) the speed of the wave? (ii) The frequency? (iii) The wavelength?

Answer the three questions above choosing from the following five possibilities:

(A)   It increases.

(B)   It decreases.

(C)   It depends on the sign of the amplitude of the wave.

(D)   It is constant.

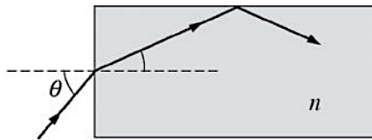(E)   It depends on the tension (F_T) of the rope.

(i) Answer **(A).** As the wave passes from the massive string to the less massive string, the wave speed will increase according to Eq. 15-2.

(ii) Answer **(D).** The frequency will remain the same.

(iii) Answer **(A).** Since $v = f\lambda$, the wavelength must increase.

2) (*6 points.*)

A model of an optical fiber is shown in the figure below. The optical fiber has an index of refraction, n, and is surrounded by free space. What angles of incidence, θ, will result in the light staying in the optical fiber?

(A) $\theta > \sin^{-1}\left(\sqrt{n^2 - 1}\right)$

(B) $\theta < \sin^{-1}\left(\sqrt{n^2 - 1}\right)$

(C) $\theta > \sin^{-1}\left(\sqrt{n^2 + 1}\right)$

(D) $\theta < \sin^{-1}\left(\sqrt{n^2 + 1}\right)$

(E) $\sin^{-1}\left(\sqrt{n^2-1}\right) < \theta < \sin^{-1}\left(\sqrt{n^2+1}\right)$

The answer is **(B)**. We solved this problem in homework 4.5.

3)   (*6 points.*)

Unpolarized light is incident on a pair of ideal linear polarizers whose transmission axes make an angle of 45° with each other. The transmitted light intensity through both polarizers is what percentage of the incident intensity?

(A) 100%

(B) 75%

(C) 50%

(D) 25%

(E) 0%

The answer is **(D)**.

This is covered in example 35-14: $I_2 = I_1 \cos^2(45°) = (0.5\ I_0) \cos^2(45°) = 0.25\ I_0$.

4)   (*6 points.*)

X rays of wavelength $\lambda = 0.250$ nm are incident on the face of a crystal at angle θ, measured from the crystal surface. The smallest angle that yields an intense reflected beam is $\theta = 14.5°$. Which of the following gives the value of the crystal interplanar spacing d? ($\sin 14.5° \approx 1/4$.)

(A)   0.125 nm

(B)   0.250 nm

(C)   0.500 nm

(D)   0.625 nm

(E)   0.750 nm

The answer is **(C)**. It is calculated by applying the Bragg equation (35-20) for $m = 1$.

5)   (*8 points.*)

In the Compton effect, a photon with energy E scatters through a 90° angle from a stationary electron of mass m. The energy of the scattered photon is

(A) E

(B) $\dfrac{E}{2}$

(C) $\dfrac{E^2}{mc^2}$

(D) $\dfrac{E^2}{E+mc^2}$

(E) $\dfrac{E\,mc^2}{E+mc^2}$

The answer is **(E).** It can be derived from Eq. 37-6 and relation: $E = hc/\lambda$.

6)   (*7 points.*)

True statements about the absorption and emission of energy by an atom include which of the following?

I. An atom can only absorb photons that have certain specific energies.

II. An atom can emit photons of any energy.

III. At very low temperature, the lines in the absorption spectrum of an atom coincide with the lines in its emission spectrum that represent transitions to the ground state.

(A)   I only

(B)   III only

(C)   I and II only

(D)   I and III only

(E)   I, II, and III

The answer is **(D).** As observed in atomic emission spectra, atoms can only emit photons of specific energy. At very low temperature the atoms are in the ground state and can only absorb from the ground level.

Problems:

Problem F.1   (*10 points.*)

Consider a forced oscillation at resonance ($\omega = \omega_0$).

(a) What is the value of the phase angle ϕ_0 in the relation $x(t) = A_0 \sin(\omega t + \phi_0)$ for the displacement?

(b) What is the displacement at a time when the driving force is a maximum and at a time when is zero?

(c) What is the phase difference (in degrees) between the driving force and the displacement in this case?

Solution F.1
We solved this problem in Recitation 2 (R2.1).

(a)   Using Eq. 14-24 in the book:

$$\phi_0 = \tan^{-1}\frac{\omega_0^2 - \omega^2}{\omega(b/m)} \quad \rightarrow \quad \text{if} \quad \omega = \omega_0, \quad \phi_0 = \tan^{-1}\frac{\omega_0^2 - \omega_0^2}{\omega(b/m)} = 0$$

(b)   With $\omega = \omega_0$, we have $F_{ext}(t) = F_0 \cos\omega_0 t$ and $x = A_0 \sin\omega_0 t$. The displacement and the driving force are one-quarter cycle ($\pi/2$ or 90°) out of phase with each other. The displacement is zero when the driving force is a maximum, and the displacement is a maximum ($+A_0$ or $-A_0$) when the driving force is zero.

(c)   As mentioned above, the phase difference is 90°.

Problem F.2   (*10 points.*)
The left-hand end of a long horizontal stretched cord oscillates transversally in simple harmonic motion with frequency $f = 0.10$ kHz and amplitude 2 cm. The cord is under a tension of 40 N and has a linear density $\mu = 4$ g/cm. At time t = 0, the end of the cord has an upward displacement of $\sqrt{2}$ cm and is falling. In SI units, determine

(a) the wavelength and

(b) the equation for the produced traveling wave.

Solution F.2

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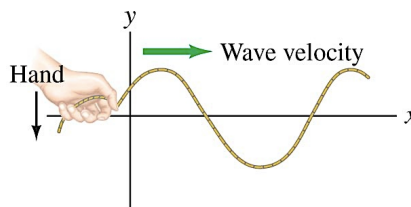
(a) With different numbers this problem is the same of Example 15-5:

$\mu = 4$ g/cm $= 0.4$ kg/m $\rightarrow$ **$v = 10$ m/s.**

$f = 0.10$ kHz $= 100$ Hz; $\lambda = v/f = (10$ m/s$) / (100$ Hz$) = 0.10$ m, **$\lambda = 0.10$ m**.

(b) Given the general form for a wave traveling to the right, in SI units:

$$D(x,t) = A\sin\left(kx - \omega t + \phi\right)$$
$$A = 0.02 \ \text{m}$$
$$k = 2\pi/\lambda = 20\pi \ \text{m}^{-1}$$
$$\omega = 2\pi f = 200\pi \ \text{rad}\cdot\text{s}^{-1}$$
$$\phi = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

Problem F.3 (*10 points.*)

A soap film of $n = 1.50$ is surrounded by air and is front illuminated by $\lambda = 480$ nm light.

(a) What is the smallest (non-zero) thickness of the soap film that would appear black?

(b) What are two other possible thicknesses for the film to appear black?

(c) If the thickness t was much less than λ (say few nanometers), why would the film also appear black?

Solution F.3

(a) An incident wave that reflects from the outer surface of the bubble has a phase change of $\phi_1 = \pi$.

An incident wave that reflects from the inner surface of the bubble has a phase change due to the additional path length, so $\phi_2 = \left(\dfrac{2t}{\lambda_{\text{film}}}\right)2\pi$. For destructive

$$\phi_1 = \pi$$
$$\phi_2 = \left(2t/\lambda_{\text{film}}\right)2\pi + 0$$
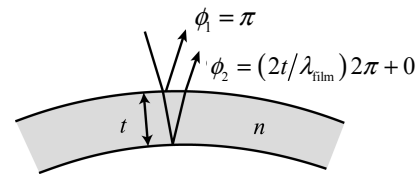
interference with a minimum non-zero thickness of bubble, the net phase change must be π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = \pi \ \rightarrow \ t = \tfrac{1}{2}\lambda_{\text{film}} = \frac{\lambda}{2n} = \frac{480\,\text{nm}}{2(1.50)} = 160 \ \text{nm}$$

(b) For the next two larger thicknesses, the net phase change would be 3π and 5π.

$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{film}}\right)2\pi\right] - \pi = 3\pi \quad \rightarrow \quad t = \lambda_{film} = \frac{\lambda}{n} = \frac{480\,nm}{(1.50)} = 320\,nm$$

$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{film}}\right)2\pi\right] - \pi = 5\pi \quad \rightarrow \quad t = \frac{3}{2}\lambda_{film} = \frac{3}{2}\frac{\lambda}{n} = \frac{3}{2}\frac{480\,nm}{(1.50)} = 480\,nm$$

(c) If the thickness were much less than one wavelength, then there would be very little phase change introduced by additional path length, and so the two reflected waves would have a phase difference of about $\phi_1 = \pi$. This would produce destructive interference.
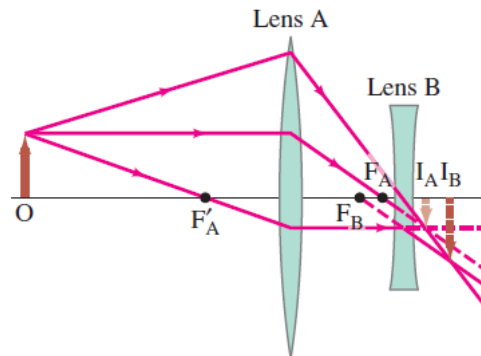
Problem F.4   (*8 points.*)

Two lenses, one converging with focal length f_A = 20 cm and one diverging with focal length f_B = -10 cm, are placed 25 cm apart. An object (*O*) is placed 60 cm in front of the converging lens (see figure). Determine

(a)   the position of the final image, and

(b)   the magnification of the final image formed by the combination of the two lenses.
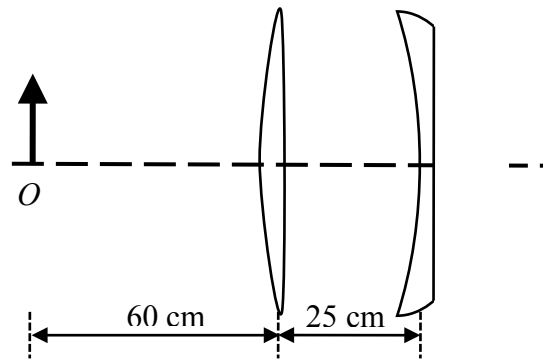
Solution F.4

(a)   The image formed by the first lens is

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \quad \rightarrow \quad d_{i1} = \frac{d_{o1}f_1}{d_{o1} - f_1} = \frac{(60.0\,\text{cm})(20.0\,\text{cm})}{(60.0\,\text{cm}) - (20.0\,\text{cm})} = 30.0\,\text{cm}$$

This image is the object for the second lens. Since this image is behind the second lens, the object distance for the second lens is negative, and so $d_{o2} = 25.0\,\text{cm} - 30.0\,\text{cm} = -5.0\,\text{cm}$. Use Eq. 33-2.

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \quad \rightarrow \quad d_{i2} = \frac{d_{o2}f_2}{d_{o2} - f_2} = \frac{(-5.0\,\text{cm})(-10.0\,\text{cm})}{(-5.0\,\text{cm}) - (-10.0\,\text{cm})} = 10\,\text{cm}$$

Thus the final image is real, and is placed **10 cm** beyond the second lens. The distance has two significant figures.

(b)   The total magnification is the product of the magnifications for the two lenses:

$$m = m_1 m_2 = \left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right) = \frac{d_{i1}d_{i2}}{d_{o1}d_{o2}} = \frac{(30.0\,\text{cm})(10.0\,\text{cm})}{(60.0\,\text{cm})(-5.0\,\text{cm})} = \boxed{-1.0\times}.$$

Problem F.5   (*7 points.*)

Yellow light (λ = 600 nm) is used to view an object under a microscope. The object lens diameter is 10.00 mm.

(a) What is the limiting angle of resolution (i.e. angle of diffraction)?

(b) Suppose it is possible to use visible light of any wavelength. What color should you choose to give the smallest possible angle of resolution, and what is this angle?

(c) Suppose water fills the space between the object and the objective. What effect does this change have on the resolving power when yellow light is used?

Solution F.5

(a)

$$\theta_{min} = 1.22\frac{\lambda}{D} = 1.22\frac{(600\times10^{-9}\ \text{m})}{(10^{-2}\ \text{m})} = 6(1.22)\times10^{-5}\ \text{rad}$$

(b) For a smaller angle of diffraction we choose the smallest visible wavelength (violet) at 400 nm, to obtain

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$$\theta_{min} = 1.22\frac{\lambda}{D} = 1.22\frac{\left(400\times10^{-9}\text{ m}\right)}{\left(10^{-2}\text{ m}\right)} = 4(1.22)\times10^{-5}\text{ rad}$$

(c) The main effect is the shortening of the wavelength in water, $\lambda_w = \lambda/n$. Thus the resolving power (RP) will be smaller, that is, better resolution. The minimum resolvable angle will be

$$\theta_{min} = 1.22\frac{\lambda_w}{D} = 1.22\frac{\left(600\times10^{-9}\text{ m}\right)/1.33}{\left(10^{-2}\text{ m}\right)} = 6\left(\frac{1.22}{1.33}\right)\times10^{-5}\text{ rad}$$

Problem F.6   (*15 points.*)

An electron is accelerated through a potential difference of V. Assume that the electron remains non-relativistic.

(a)   Derive the formula for the wavelength of the electron as function of V.

(b)   The same electron strikes perpendicular to the surface of a solid (see figure) interacting only with the surface layer of atoms. If the smallest angle at which a diffraction maximum occurs is at θ_m, what is the separation d between the atoms on the surface?
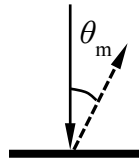
If $V = 11.4$ V and the electron passes through a gas of hydrogen atoms at room temperature.

(c)   Using the Bohr model, what wavelengths of light will be emitted? (*Remember that for hydrogen, $E_1 = -13.6$ eV, $E_2 = -3.40$ eV, $E_3 = -1.51$ eV, $E_4 = -0.85$ eV.*)

(d)   What quantum numbers (n, l, m_l, m_s) characterize the initial and final atomic states to which the atom is "dipole-allowed" to jump with the emission of a photon.

(e)   [*Extra points*: After all possible emissions, what is the wave function representing the final state reached by the hydrogen atoms? (Do not write it explicitly, but use the notation $\psi_{nlm_l m_s}$.)

Solution F.6

(a) This is solved in Example 37-11:

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$$

(b) This is solved in Example 37-12:

$$d = \frac{\lambda}{\sin\theta_m} = \frac{h}{\sqrt{2meV}} \frac{1}{\sin\theta_m}$$

(c)

$$\lambda = \frac{1240 \text{ eV nm}}{E_2 - E_1} = \frac{1240 \text{ eV nm}}{10.2 \text{ eV}} = 122 \text{ nm},$$

which is in the UV region of the EM spectrum.

(d)
The dipole-allowed transitions are all **2p** $\rightarrow$ **1s** (which obey to the selection rules $\Delta l = \pm 1$ and $\Delta m_l = 0, \pm 1$), that is,

- six initial states with the same energy: **(2, 1, 0, ±1/2); (2, 1, 1, ±1/2); (2; 1; -1, ±1/2).**
- two final states with the same energy: **(1, 0, 0, ±1/2)**.

(e) (extra)
The final state is the ground state of the hydrogen. The wave function representing the ground state is a <u>linear combination</u> of the wave functions with (1, 0, 0, ±1/2):

$$\psi = C_1\psi_{100\,1/2} + C_2\psi_{100-1/2}, \text{ where } |C_1|^2 = |C_2|^2 = 1/2.$$