Problem 6.1

Two radio antennas $S_1$ and $S_2$ emitting waves of wavelength $\lambda$ are separated by a distance $d$.

a) If $d = \lambda/2$ and the two antennas radiate $180^\circ$ out of phase with each other, find the directions (figure A) for constructive and destructive interference, and compare with the case when the sources are in phase. (These results illustrate the basis for directional antennas.) [Hint: See Giancoli, Example 34-5]

b) If $d = 175$ m and they radiate 6000 kHz waves in phase with each other, find points on the $y$-axis (figure B) where the signals from the two sources will be out of phase.

Solution 6.1

a) Since the two sources are $180^\circ$ out of phase, destructive interference will occur when the path length difference between the two sources and the receiver is 0, or an integer number of wavelengths. Since the antennae are separated by a distance of $d = \lambda/2$, the path length difference can never be greater than $\lambda/2$, so the only points of destructive interference occur when the receiver is equidistant from each antenna, that is, at $\theta_{\text{destructive}} = 0^\circ$ and $180^\circ$. Constructive interference occurs when the path difference is a half integer wavelength. Again, since the separation distance between the two antennae is $d = \lambda/2$, the maximum path length difference is $\lambda/2$, which occurs along the line through the antennae, therefore the constructive interference only occurs at $\theta_{\text{constructive}} = 90^\circ$ and $270^\circ$. As expected, these angles are reversed from those in phase, found in Giancoli, Example 34-5c.

b) The signals will be out of phase when the path difference equals an odd number of half-wavelengths. Let the 175 m distance be represented by $d$.

\[
\sqrt{y^2 + d^2} = y + \left(m + \frac{1}{2}\right)\lambda \quad \Rightarrow \quad \sqrt{y^2 + d^2} = y + \left(m + \frac{1}{2}\right)\lambda
\]

\[
y^2 + d^2 = y^2 + 2y\left(m + \frac{1}{2}\right)\lambda + \left(m + \frac{1}{2}\right)^2\lambda^2
\]

\[
y = \frac{d^2 - \left(m + \frac{1}{2}\right)^2\lambda^2}{2\left(m + \frac{1}{2}\right)\lambda}
\]
We evaluate this for the first three values of \( m \). The wavelength is
\[
\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{6.0 \times 10^6 \text{ Hz}} = 50 \text{ m}.
\]
\[
y = \frac{d^2 - \left( m + \frac{1}{2} \right)^2 \lambda^2}{2 \left( m + \frac{1}{2} \right) \lambda} = \frac{\left( 175 \text{ m} \right)^2 - \left( m + \frac{1}{2} \right)^2 \left( 50 \text{ m} \right)^2}{2 \left( m + \frac{1}{2} \right) \left( 50 \text{ m} \right)} = 600 \text{ m}, 167 \text{ m}, 60 \text{ m}, 0 \text{ m}
\]

The first three points on the \( y \)-axis where the signals are out of phase are at
\[
y = 0, \; 60 \text{ m}, \; \text{and} \; 167 \text{ m}.
\]

**Problem 6.2**

A thin metal foil of thickness \( d \) separates one end of two pieces of optically flat glass of length \( L \), as shown in figure below. When light of wavelength 670 nm is incident normally, 28 dark lines are observed (with one at each end). How thick is the metal foil? [Hint: See Giancoli, Example 34-6]

![Diagram of light passing through glass pieces separated by a thin metal foil.]

**Solution 6.2**

An incident wave that reflects from the second surface of the upper piece of glass has no phase change, so \( \phi_1 = 0 \). An incident wave that reflects from the first surface of the second piece of glass has a phase change due to both the additional path length and a phase change of \( \pi \) on reflection, so
\[
\phi_2 = \left( \frac{2t}{\lambda} \right) 2\pi + \pi.
\]

For destructive interference (dark lines), the net phase change must be an odd-integer multiple of \( \pi \), so
\[
\phi_{\text{net}} = \phi_2 - \phi_1 = \left( 2m + 1 \right) \pi, \; m = 0, 1, 2, \ldots
\]

Because \( m = 0 \) corresponds to the left edge of the diagram, the 28th dark line corresponds to \( m = 27 \). The 28th dark line also has a gap thickness of \( d \).
\[ \phi_{\text{out}} = \phi_2 - \phi_1 = \left( \frac{2\pi}{\lambda} \right) 2\pi + \pi \rightarrow 0 = (2m + 1)\pi \rightarrow t = \frac{1}{2} m\lambda \rightarrow \\
\]
\[ d = \frac{1}{2} (27)(670\text{ nm}) = 9045\text{ nm} = 9.0\mu\text{m} \]

**Problem 6.3**

In a compact disc (CD), digital information is stored as a sequence of raised surfaces called “pits” and recessed surfaces called “lands”. Both pits and lands are highly reflective and are embedded in a thick plastic material with index of refraction \( n = 1.55 \) (see figure below). As a 780-nm wavelength (in air) laser scans across the pit-land sequence, the transition between a neighboring pit and land is sensed by monitoring the intensity of reflected laser light from the CD. At the moment when half the width of the laser beam is reflected from the pit and the other half from the land, we want the two reflected halves of the beam to be 180° out of phase with each other. What should be the (minimum) height difference \( t \) between a pit and land? [When this light enters a detector, cancellation of the two out-of-phase halves of the beam produces a minimum detector output.]

![Diagram of CD with laser beam and pit/land sequence]

**Solution 6.3**

In order for the two reflected halves of the beam to be 180° out of phase with each other, the minimum path difference \((2t)\) should be \( \frac{1}{2} \lambda \) in the plastic. Notice that there is no net phase difference between the two halves of the beam due to reflection, because both halves reflect from the same material.

\[ 2t = \frac{1}{2} \frac{\lambda}{n} \rightarrow t = \frac{\lambda}{4n} = \frac{780\text{ nm}}{4(1.55)} = 126\text{ nm} \]

**Problem 6.4**

Solar cells are often coated with a transparent, thin film of silicon monoxide (SiO, \( n = 1.45 \)) to minimize reflection losses from the surface. Suppose a silicon (Si, \( n = 3.5 \)) thick
layer is coated with a thin film of SiO for this purpose. Briefly explain the mechanism that allows us to minimize the reflection. Determine the minimum film thickness that produces the least reflection at a wavelength of 550 nm.

Solution 6.4

(This problem is similar to Giancoli, Examples 34-7 and 34-8.)

The reflected light is a minimum when rays 1 and 2 meet the condition of destructive interference. In this situation, both rays undergo a 180° phase change upon reflection: ray 1 from the upper air/SiO surface and ray 2 from the lower SiO/Si surface. (In Giancoli Chapter 34-5 we learnt that a beam of light reflected by a material with index of refraction greater than that of the material in which it is traveling, changes phase by 180°). Therefore the net change in phase due to reflection is zero. The condition for a reflection minimum requires a path difference of \( \frac{\lambda n}{2} \), where \( \lambda n \) is the wavelength of the light in SiO. Hence, \( 2d = \frac{\lambda}{2n} \), where \( n \) is the SiO refractive index and \( \lambda \) the wavelength in air. Then

\[
d = \frac{\lambda}{4n} = \frac{550 \text{ nm}}{4(1.45)} \approx 95 \text{ nm} = 95 \times 10^{-9} \text{ m}
\]

Problem 6.5

What is the wavelength of the light entering a Michelson interferometer if 384 bright fringes are counted when the movable mirror moves 125 \( \mu \text{m} \).

Solution 6.5

From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of
\( \lambda \), and so corresponds to a mirror motion of \( \frac{1}{2} \lambda \). Let \( N \) be the number of fringe shifts produced by a mirror movement of \( \Delta x \).

\[
N = \frac{\Delta x}{\frac{1}{2} \lambda} \quad \Rightarrow \quad \lambda = \frac{2 \Delta x}{N} = \frac{2(1.25 \times 10^{-4} \text{ m})}{384} = 6.51 \times 10^{-7} \text{ m} = 651 \text{ nm} = 651 \times 10^{-9} \text{ m}.
\]