Problem 7.1
Young’s double-slit experiment underlines the *instrument landing system* used to guide aircrafts to safe landing when visibility is poor. Two radio antennas are separated by a runway of length $d = 40$ m (see figure below). The antennas broadcast coherent radio waves at 30.0 MHz. The pilot locks onto the strong signal radiated along an interference maximum and steers the plane to keep the received signal strong. If the pilot has found the central maximum, the plane will have precisely the correct heading to land when it reaches the runway (trajectory A).

(a) Suppose the plane is mistakenly flying along the first side maximum. How far to the side of the runway centerline will the plane be when it is 2 km from the antennas, measured along its direction off travel?

(b) It is possible to tell the pilot that she is on the wrong maximum by sending out two signals from each antenna and equipping the aircraft with a two-channel receiver. Explain how this two-frequency system would work and why it would not necessarily work if the frequencies were related by an integer ratio.

Solution 7.1
(a) $\lambda = c/f = 10.0$ m. The first side maximum is at an angle given by $d \sin \theta = (1) \lambda$, with $d = 40$ m $\Rightarrow y = L \sin \theta = (2000$ m) $(1/4) = 500$ m.

(b) The intent is to inform the pilot about the signal that corresponds to the central maximum. The signal with $\lambda = 10.0$ m would show maxima at $0$, $14.5^\circ$, $30.0^\circ$, $48.6^\circ$, and $90^\circ$. A signal of wavelength, say, $11.23$ m would show maxima at $0$, $16.3^\circ$, $34.2^\circ$, $57.3^\circ$. the only value in common is $0$.
A strong signal for both frequencies would indicate that the airplane was traveling along the central maximum; thus, straight on the runway. If $\lambda_1$ and $\lambda_2$ were related by a ratio of small integers in $\lambda_1/\lambda_2 = n_1/n_2$, then the equations $d \sin \theta = n_2 \lambda_1$ and $d \sin \theta = n_1 \lambda_2$ would both be satisfied for the same nonzero angle. Thus the pilot could approach on an inappropriate bearing, and run off the runway immediately after touchdown.

Problem 7.2
Two bright lights of wavelength 600 nm are positioned 0.30 m apart and 16.09 km away from a telescope having a lens with diameter of 5 cm.

(a) Is the telescope able to resolve the two lights?

(b) A circular aperture with varying diameter is placed in front of the microscope. Find the diameter of the aperture that make the two lights just resolved according to Rayleigh criterion.

Solution 7.2

(a) In order to resolve the two light sources, we must be able to differentiate the two diffraction patterns on the objective of the telescope. Let us first see whether the 5 cm lens is capable of resolving the two lights. Its angular resolution is \( \Delta \theta \approx \frac{1.22 \lambda}{D} \approx 1.4 \times 10^{-5} \text{ rad} \).

The angular separation of 0.30 m at a distance of 16.09 km is about \( 1.9 \times 10^{-5} \text{ rad} \). Thus the telescope will be able to resolve the two sources of light.

(b) If we now place a slit in front of the lens, whose width is less than 5 cm, it will become more difficult to resolve the two lights. We require \( 1.9 \times 10^{-5} \text{ rad} > \frac{1.22 \lambda}{D} \).

Hence, \( D > 3.8 \text{ cm} \).

Problem 7.3

The full-width at half-maximum (FWHM) of the central peak for single-slit diffraction is defined as the angle \( \Delta \theta \) between the two points on either side of center where the intensity is 0.5\( I_0 \). (a) Determine \( \Delta \theta \) in terms of \( \frac{\lambda}{D} \). Use graphs or a spreadsheet to solve \( \sin \alpha = \alpha/\sqrt{2} \). (b) Determine \( \Delta \theta \) (in degrees) for \( D = \lambda \) and for \( D = 100 \lambda \).

Solution 7.3

(a) The intensity of the diffraction pattern is given by Eqs. 35-6 and 35-7. We want to find the angle where \( I = \frac{1}{2} I_0 \). Doubling this angle will give the desired \( \Delta \theta \).

\[
I_0 = I_0 \left( \sin \frac{\beta/2}{\beta/2} \right)^2 = \frac{1}{4} I_0 \quad \rightarrow \quad \sin \beta/2 = \frac{\beta/2}{\sqrt{2}} \quad \text{or} \quad \sin \alpha = \frac{\alpha}{\sqrt{2}}, \quad \text{with} \quad \alpha = \frac{1}{2} \beta
\]

This equation must be solved numerically. A spreadsheet was developed to find the non-zero values of \( \alpha \) that satisfy \( \sin \alpha - \frac{\alpha}{\sqrt{2}} = 0 \). It is apparent from this expression that there will be no solutions for \( \alpha > \sqrt{2} \). The only non-zero value is \( \alpha = 1.392 \). Now use Eq. 35-6 to find \( \theta \).

\[
\beta = \frac{2\pi}{\lambda} D \sin \theta \quad \rightarrow \quad \theta = \sin^{-1} \frac{\lambda \beta}{2\pi D} = \sin^{-1} \frac{2\lambda \alpha}{2\pi D} = \sin^{-1} \frac{\lambda (1.392)}{\pi D} ;
\]

\[
\Delta \theta = 2\theta = 2 \sin^{-1} \frac{\lambda (1.392)}{\pi D};
\]

(b) For \( D = \lambda \):

\[
\Delta \theta = 2 \sin^{-1} \frac{\lambda (1.392)}{\pi D} = 2 \sin^{-1} \frac{(1.392)}{\pi} = 52.6^\circ
\]
For \( D = 100\lambda \): \( \Delta \theta = 2\sin^{-1}\left(\frac{\lambda (1.392)}{\pi D}\right) = 2\sin^{-1}\left(\frac{1.392}{100\pi}\right) = 0.508^\circ \).

Problem 7.4

Light from a laser strikes a diffraction grating with 5310 grooves/cm. The central and first order principal maxima are separated by 0.488 m on a wall 1.72 m from the grating. Determine the wavelength of the laser light.

Solution 7.4

The principal maxima are defined by \( m\lambda = d\sin\theta \). For \( m = 1 \), \( \lambda = d\sin\theta \).

The angle can be determined by \( \tan\theta = \frac{0.488 \text{ m}}{1.72 \text{ m}} \rightarrow \theta = 15.8^\circ \rightarrow \sin\theta = 0.273 \). The distance between gratings slits equals \( d = (1/5310) \text{ cm} = 1.88 \mu\text{m} \). The wavelength is then given by \( \lambda = d\sin\theta = 514 \text{ nm} \).

Problem 7.5

In a diffraction experiment the first-order diffraction maximum is observed at 12.6° for a crystal having a spacing between planes of atoms of 0.250 nm.

(a) What wavelength is used to observe this first-order pattern?

(b) How many orders can be observed for this crystal at this wavelength?

Solution 7.5

(a) Using Bragg’s equation (35-20) with \( m = 1 \),
\[ \lambda = 2d\sin\theta = 2 (0.25 \text{ nm}) \sin 12.6^\circ = 0.109 \text{ nm} \]

(b) \( m\lambda/2d = \sin\theta \leq 1 \rightarrow m \leq 4.59 \). The order-number must be an integer, so the largest value \( m \) can have is 4, that is, four orders can be observed.

Problem 7.6

Iridescence (also known as goniochromism) is generally known as the property of certain surfaces that appear to change color as the angle of view changes (as may be seen commonly in soap bubbles, butterfly wings, and sea shells). An example of iridescent material is the peacock feathers. The surface of one microscopic barbule is composed of transparent keratin (what makes your fingernails) that supports rods of dark brown melanin (dark pigments) in a periodic structure. In a portion of the feather that can appear turquoise (blue-green), assume the melanin rods are uniformly separated by 250 nm, with air between them.

(a) Explain how this structure can appear turquoise when it contains no blue or green pigments.

(b) Explain how it can also appear violet if light falls on it in a different direction.
(c) Why the portion of the feather with 250 nm separation cannot appear yellow or red?

Figure.
Scanning electron microscope images of barbule structures. (A) Transverse cross section of the green barbule. The outer cortex layer contains a periodic structure. The central part is the medullar layer. Transverse cross section of the cortex under higher magnification is shown in two different barbules (B) and (C). The surface of the cortex is a thin keratin layer. Beneath the surface keratin layer, there is a periodic structure made up of an array of melanin rods connected by keratin. The remaining hollows are air holes (dark gray). Melanin rods are parallel to the cortex surface. The melanin rods embedded in the surface keratin layer can be clearly seen. (D) Longitudinal cross section of the barbule with the surface keratin layer removed.

Solution 7.6
(a) Bragg’s law applies to the space lattice of melanin rods. Consider the planes \( d = 250 \) nm apart. For light at near-normal incidence, strong reflection happens for the wavelength given by \( m\lambda = 2d \sin \theta \). The longest wavelength reflected strongly correspond to \( m = 1 \).
\[
2 \cdot 250 \cdot \sin 90^\circ = \lambda = 500 \text{ nm}, \text{ this is the blue-green color.}
\]
(b) For light incident at grazing angle 60\(^\circ\), \( \lambda = 433 \text{ nm} \). This is the violet.
(c) The longest wavelength that can be reflected with extra strength by these melanin rods is the one we computed first, 500 nm. If melanin rods were farther apart (say 320 nm) they could reflect red with constructive interference.