University of Rochester
Department of Physics and Astronomy
Physics123, Spring 2012

Homework 9 - Solutions

Problem 9.1

Estimate what % of emitted sunlight energy is in the visible range (from 400 nm to 700 nm). Use Plank's radiation formula and numerical integration assuming that all energy is for the most part contained in the range from 20 nm to 2000 nm.

Solution 9.1

Planck's radiation formula $I(\lambda, T)$ was calculated for a temperature of 6000 K, for wavelengths from 20 nm to 2000 nm. To estimate the % of emitted sunlight that is in the visible, this ratio was calculated by numeric integration. The details are in the spreadsheet.

$$\%\,\text{visible} = \frac{\int_{400\,\text{nm}}^{700\,\text{nm}} I(\lambda, T)\,d\lambda}{\int_{20\,\text{nm}}^{2000\,\text{nm}} I(\lambda, T)\,d\lambda} = \boxed{0.42}$$

So our estimate is that 42% of emitted sunlight is in the visible wavelengths.

Problem 9.2

A photon with energy 2.28 eV is absorbed by a hydrogen atom. Find (a) the minimum *n* for a hydrogen atom that can be ionized by such a photon and (b) the speed of the electron released from the state in part (a) when it is far from the nucleus.

Solution 9.2

a) 13.6 eV / $(2)^2$ = 3.40 eV is required to ionize a hydrogen atom from state n = 2. So while the photon cannot ionize a hydrogen atom pre-excited to n = 2, it can ionize a hydrogen atom in the n = 3 level, that is, with energy – 1.51 eV.

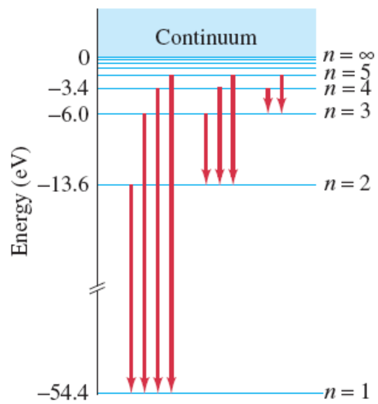
b) The electron thus freed can have kinetic energy K_e = 2.28 eV – 1.51 eV = 0.769 eV, which corresponds to $v = 5.20 \times 10^5$ m/s.

Problem 9.3

Construct the energy-level diagram (like Fig. 37-26) for (a) He^+ ion and (b) doubly ionized lithium Li^{2+}.

Solution 9.3

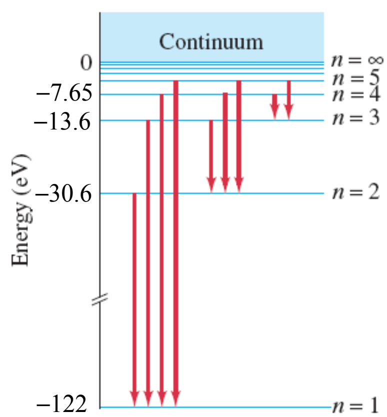
(a) Singly ionized helium is like hydrogen, except that there are two positive charges ($Z = 2$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:



$$E_n = -\frac{Z^2(13.6\,\text{eV})}{n^2} = -\frac{2^2(13.6\,\text{eV})}{n^2} = -\frac{(54.4\,\text{eV})}{n^2}$$

$$E_1 = -54.5\,\text{eV},\ E_2 = -13.6\,\text{eV},\ E_3 = -6.0\,\text{eV},\ E_4 = -3.4\,\text{eV}$$

(b) Doubly ionized lithium is like hydrogen, except that there are three positive charges ($Z = 3$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:



$$E_n = -\frac{Z^2(13.6\,\text{eV})}{n^2} = -\frac{3^2(13.6\,\text{eV})}{n^2} = -\frac{(122.4\,\text{eV})}{n^2}$$

$$E_1 = -122\,\text{eV},\ E_2 = -30.6\,\text{eV},\ E_3 = -13.6\,\text{eV},$$

$$E_4 = -7.65\,\text{eV}$$

Problem 9.4

Electron accelerated by a potential difference of 12.3 V pass through a gas of hydrogen atoms at room temperature. Using the Bohr model, what wavelengths of light will be emitted?

Solution 9.4

The potential difference gives the electrons a kinetic energy of 12.3 eV, so it is possible to provide this much energy to the hydrogen atom through collisions. From the ground state, the maximum energy of the atom is $-13.6\text{ eV} + 12.3\text{ eV} = -1.3\text{ eV}$. From the energy level diagram, Figure 37-26, we see that this means the atom could be excited to the $n = 3$ state, so the possible transitions when the atom returns to the ground state are $n = 3$ to $n = 2$, $n = 3$ to $n = 1$, and $n = 2$ to $n = 1$. We calculate the wavelengths from the equation above Eq. 37-15.

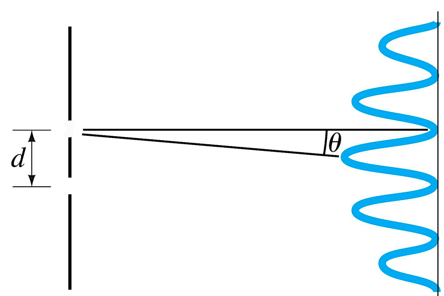
$$\lambda_{3\to2} = \frac{hc}{(E_3 - E_2)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-1.5\text{ eV} - (-3.4\text{ eV})]} = \boxed{650\,\text{nm}}$$

$$\lambda_{3\to1} = \frac{hc}{(E_3 - E_1)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-1.5\text{ eV} - (-13.6\text{ eV})]} = \boxed{102\,\text{nm}}$$

$$\lambda_{2\to1} = \frac{hc}{(E_2 - E_1)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-3.4\text{ eV} - (-13.6\text{ eV})]} = \boxed{122\,\text{nm}}$$

Problem 9.5

In a double-slit experiment on electrons (or photons), suppose that we use indicators to determine which slit each electron went through. These indicators must tell us the y coordinate to within $d/2$, where d is the distance between slits. Use the uncertainty principle to show that the interference pattern will be destroyed. [*Note*: First show that the angle θ between maxima and minima of the interference pattern is given by $\lambda/(2d)$. Let us assume that the electron has an initial x momentum , p_x, and that the angles are small, then we replace *sin* θ by θ.]



Solution 9.5

Let us assume that the electron has an initial x momentum p_x, so that it has a wavelength of $\lambda = h/p_x$. The maxima of the double-slit interference pattern occur at locations satisfying Eq. 34-2a, $d\sin\theta = m\lambda,\ m = 0,1,2,\ldots$. If the angles are small, then we replace $\sin\theta$ by θ, and so the maxima are given by $\theta = m\lambda/d$. The angular separation of the maxima is then $\Delta\theta = \lambda/d$, and the angular separation between a maximum and the adjacent minimum is $\Delta\theta = \lambda/2d$. The separation of a maximum and the adjacent minimum on the screen is then $\Delta y_{\text{screen}} = \lambda l/2d$, where l is the distance from the slits to the detection screen. This means that many electrons hit the screen at a maximum position, and very few electrons hit the screen a distance $\lambda l/2d$ to either side of that maximum position.

If the particular slit that an electron passes through is known, then Δy for the electrons at the location of the slits is $d/2$. The uncertainty principle says $\Delta p_y \geq \frac{\mathrm{h}}{\Delta y_{\text{slits}}} = \frac{\mathrm{h}}{\frac{1}{2}d} = \frac{h}{\pi d}$. We assume that p_y for the electron must be at least that big. Because of this uncertainty in y momentum, the electron has an uncertainty in its location on the screen, as $\frac{\Delta y_{\text{screen}}}{l} = \frac{\Delta p_y}{p_x} \rightarrow \Delta y_{\text{screen}} = l\,\frac{\frac{h}{\pi d}}{\frac{h}{\lambda}} = \frac{\lambda l}{\pi d}$. Since this is about the same size as the separation between maxima and minima, the interference pattern will be "destroyed." The electrons will not be grouped near the maxima locations. They will instead be "spread out" on the screen, and no interference pattern will be visible.