Exam 1

Phy122 Electricity and Magnetism
July 8 2010

Name: Solutions

You have until 11:45 to complete this exam. You are allowed one index card for formulas and notes. You may have a calculator (but it probably won't help you) but no cell phones or other electronic devices are allowed. Please ask me if any questions come up during the test. I hope you do well!

1. **Multiple Choice**. [20 Points]
   Circle the correct answers.
   (a) A point charge is inside of a spherical surface. If the sphere is replaced by a cube, the flux will:
      i. increase.
      ii. decrease.
      iii. stay the same.
      iv. increase or decrease depending on the size of the cube.

   flux depends on $\vec{E}\cdot\vec{A}$, which is constant here.
   Think Gauss' law. $\oint \vec{E}\cdot d\vec{A} = \boxed{\frac{Q_{en}}{\epsilon_0} = \Phi}$
   constant

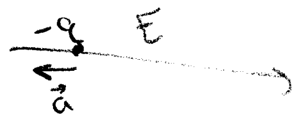
   (b) A wire carries a large amount of charge Q distributed along its length. The best way to decrease the electric field outside it would be to encase it in a cylindrical
      i. conducting shell of radius R and neutral charge
      ii. non-conducting shell of radius R and neutral charge
      iii. conducting shell of radius R and charge -Q ← the field outside will be zero!
      iv. conducting shell of radius R and charge Q/R

   (c) Three point charges sit on the corners of an equilateral triangle. The arrangement can be in equilibrium if:
      i. The charges are Q, Q, and -2Q
      ii. The charges are -Q, -Q, and Q^2
      iii. The charges are Q, 2Q, and -5Q
      iv. There is no way the arrangement can be in equilibrium.

   Think workshop problem - the charges need to be on a straight line

   (d) A small negative test charge is on an electric field line which points to the right. The direction of the acceleration of the charge will be
      i. to the right
      ii. to the left
      iii. up
      iv. down

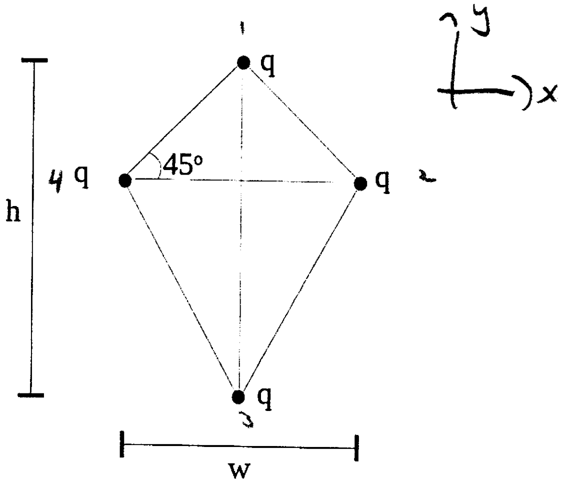
   "positive charge" would follow E-field lines, (since $\vec{E} = k\frac{q}{r^2}\hat{r}$) but a negative charge goes in the opposite direction



$\vec{F} = q\vec{E} = m\vec{a}$

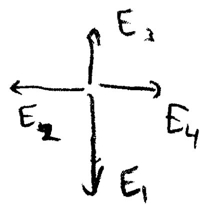
if q is negative, $\vec{F}$ points in opposite direction of $\vec{E}$.

2. **The Superposition Kite** [30 points]
Four positive charges (each with charge q) are arranged fixed at each point of the kite shown. The kite has height *h* and width *w*, and the crossbar is attached so that the angles between it and the upper portions of the kite are 45 degrees, as shown.
(a) Find the electric field in the center of the kite, where the cross-bars meet.
(b) Find the electric force on the top charge caused by the other three.
Note: don't worry about simplifying your expressions – just get them in terms of the variables given.



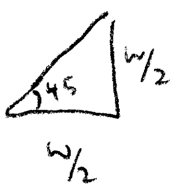
(a) Label charges

at the center:



Since the charges are equal, the x-components cancel out.

find distances for $E = \frac{kq}{r^2}$



So distance to q_3 is $h - \frac{w}{2}$

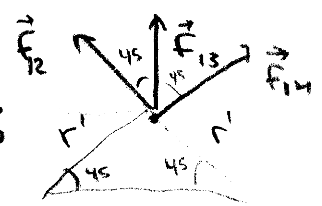
" " q_1 is $\frac{w}{2}$

$$\vec{E} = \vec{E}_x + \vec{E}_y$$

$$= \frac{kq}{(w/2)^2}(-\hat{j}) + \frac{kq}{(h-\frac{w}{2})^2}\hat{j}$$

$$\boxed{\vec{E} = kq\left(-\frac{4}{w^2} + \frac{1}{(h-w/2)^2}\right)\hat{j}}$$

(b) I start with a drawing of the vectors:



I can tell the x-components cancel, but I'll work it out for fun :)

$\theta = 45°$

$$\vec{F}_{net} = \vec{F}_x + \vec{F}_y$$

$$= F_{14}\sin\theta\,\hat{i} + F_{12}\sin\theta(-\hat{i}) + F_{14}\cos\theta\,\hat{j} + F_{12}\cos\theta\,\hat{j} + F_{13}\hat{j}$$

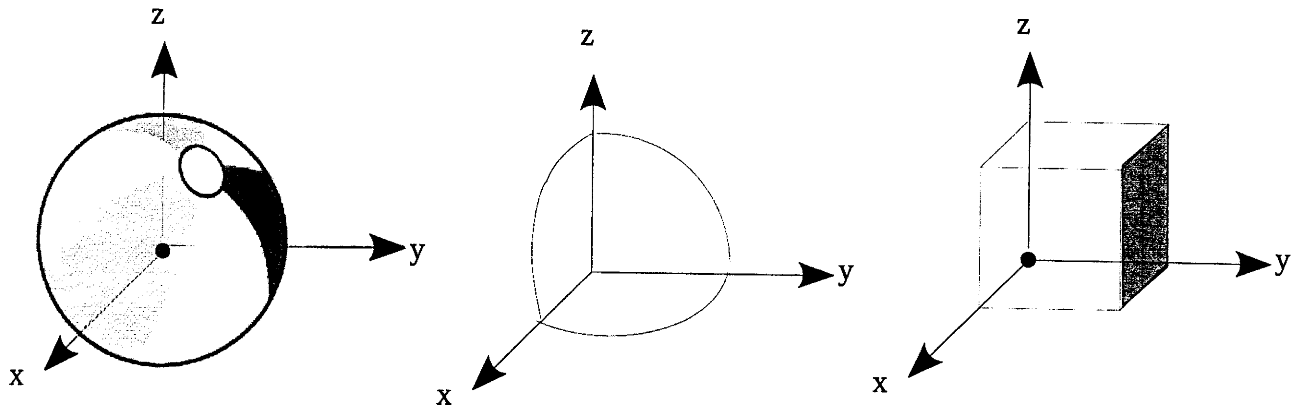
$$= \cancel{\frac{kqq}{r'^2}\frac{1}{\sqrt{2}}\hat{i}} + \cancel{\frac{kqq}{r'^2}\frac{1}{\sqrt{2}}(-\hat{i})} + \frac{kqq}{r'^2}\frac{1}{\sqrt{2}}\hat{j} + \frac{kqq}{r'^2}\frac{1}{\sqrt{2}}\hat{j} + \frac{kqq}{h^2}\hat{j}$$

$$r'^2 = \left(\frac{w}{2}\right)^2 + \left(\frac{w}{2}\right)^2 = \frac{w^2}{2}$$

$$= \left(\frac{2kq^2}{r'^2}\frac{1}{\sqrt{2}} + \frac{kq^2}{h^2}\right)\hat{j}$$

$$\boxed{\vec{F}_{net} = \left(\frac{4kq^2}{\sqrt{2}\,w^2} + \frac{kq^2}{h^2}\right)\hat{j}}$$

3. **Flux in the Summertime** [20 points]
A point charge q sits at the origin.
(a) Calculate the flux through one panel of a beach ball centered at the origin, shown in (a). (Ignore the little top circle and assume panels come to a point)
(b) Calculate the flux through the portion of a spherical surface in the first octant that has its center at the origin, shown in (b).
(c) Calculate the flux through the shaded side of the ice cube shown in (c).



(a) (b) (c)

(a) The flux through a sphere due to a charge at the origin is
$\Phi = \frac{q}{\epsilon_0}$. As we showed in class, the shape doesn't have to be a sphere; it can be anything so long as it encloses the charge. This is the beauty of Gauss' law. The beach ball has 6 panels, so $\frac{1}{6}$ of the total flux goes through each panel.

$\boxed{\Phi_{panel} = \frac{1}{6}\Phi = \frac{q}{6\epsilon_0}}$ if I substitute in $k = \frac{1}{4\pi\epsilon_0}$, $\boxed{\Phi_{panel} = \frac{2\pi k q}{3}}$

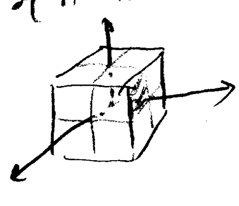
(b) This time we are considering $\frac{1}{8}$ of a sphere, since the coordinate system has 8 sections and we are considering just one.

$\boxed{\Phi = \frac{1}{8}\frac{q}{\epsilon_0} = \frac{kq\pi}{2}}$

(c) We just found the flux through the first octant. Now we see that the cube has three out-ward facing surfaces. One of them is the shaded side, so we just divide our answer above by 3!

$\boxed{\Phi = \frac{1}{24}\frac{q}{\epsilon_0} = \frac{kq\pi}{6}}$

* another way to think of it is to imagine a large cube made of 8 cubes centered at the origin.
The total flux through it is $\frac{q}{\epsilon_0}$. There are $4 \cdot 6 = 24$ faces like the shaded one, so $\Phi = \frac{1}{24}\frac{q}{\epsilon_0}$ ✓



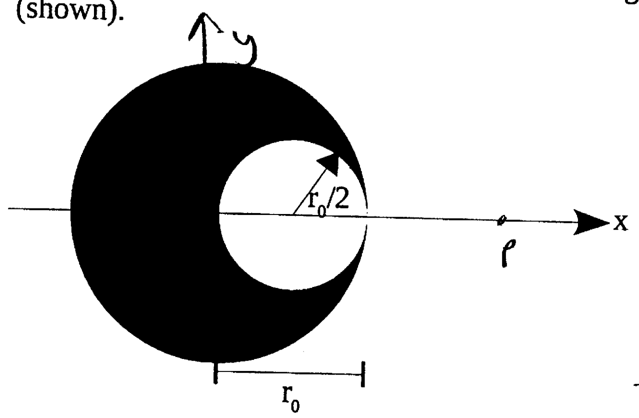
4. **Fun with Spheres** [30 Points]

(a) Consider a nonconducting sphere of radius r_0 and with a charge Q uniformly distributed throughout its volume.

  i. Describe the symmetry and draw the appropriate Gaussian surfaces you would use to find the field outside and inside the sphere.

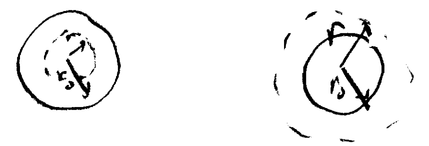
  ii. Use Gauss' law to find the field outside the sphere due to the sphere ($r>r_0$).

(b) Consider the situation pictured below: A solid nonconducting sphere has a spherical section cut out of it. The section cut out has radius $r_0/2$ and its center is located $r_0/2$ from the center of the large sphere. Find the field due to the charge distribution on a point along the x-axis (shown).



a) i The symmetry of a sphere is spherical; the electric field must therefore point radially in all directions

for $r<r_0$ for $r>r_0$



ii) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$

$Q_{encl} = Q$ (given)

$\oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\int dA = E\,4\pi r^2$ ← this is E for a point charge, which is as I would expect.

$E\,4\pi r^2 = \frac{Q}{\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{Q}{4\pi r^2\epsilon_0}\hat{r}}$

b) I can use the principle of superposition to solve this problem.

$\vec{E} = \vec{E}_{large} - \vec{E}_{small}$

I just found $\vec{E}_{large}$! I will let $r=x$, since I am now concerned w/ the x-axis. I pick my origin to be at the center of the big sphere. $\vec{E}_{large} = \frac{Q}{4\pi\epsilon_0 x^2}\hat{i}$

$\vec{E}_{small}$ is almost the same. It is centered a distance $x-\frac{r_0}{2}$ from p.

Its total charge is some fraction of the larger one;

$Q_{small} = Q\frac{V_{small}}{V_{large}} = \frac{\frac{4}{3}\pi\left(\frac{r_0}{2}\right)^3}{\frac{4}{3}\pi(r_0)^3}Q = \frac{Q}{8}$ So $\vec{E}_{small} = \frac{Q/8}{4\pi\epsilon_0(x-r_0/2)^2}\hat{i}$

Thus $\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{x^2} - \frac{1}{8(x-r_0/2)^2}\right]\hat{i}}$