Exam 2

Phy122 Electricity and Magnetism
July 20 2010

Name: Solutions

You have until 11:45 to complete this exam. You are allowed one index card for formulas and notes. You may have a calculator (but it probably won't help you) but no cell phones or other electronic devices are allowed. Please ask me if any questions come up during the test. I hope you do well!

1. Multiple Choice. [30 Points] Circle the correct answers.

(a) A parallel plate capacitor has charge Q_o. The separation between the plates of the parallel-plate capacitor is then tripled, while still connected to the battery. The charge will now be:

i. 3 Q_o
ii. 1/3 Q_o
iii. 9 Q_o
iv. Q_o

$C = \frac{Q}{V} \Rightarrow Q = CV$. Also $C = \frac{\epsilon_0 A}{d}$ $\quad C' = \frac{\epsilon_0 A}{3d} = \frac{C}{3}$

So $Q' = \frac{C}{3} V = \frac{Q}{3}$

(b) What value might we assign to the dielectric constant for a good conductor?

i. 0
ii. 1
iii. 4π
iv. ∞

$C = K C_0$ Air: $K = 1$

There would be no capacitance if a conductor replaced a dielectric (or air). Thus $K=0$ would be good

(c) If the resistance of a heater (which plugs into a wall outlet) is doubled, its heating power will:

i. be doubled
ii. be halved
iii. be quadrupled
iv. remain unchanged

V is constant $\quad P = IV \Rightarrow P = \frac{V^2}{R}$
$V = IR$

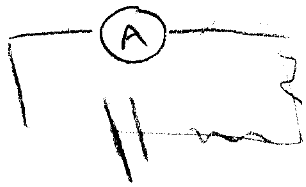
$P' = \frac{V^2}{2R} = \frac{P}{2}$

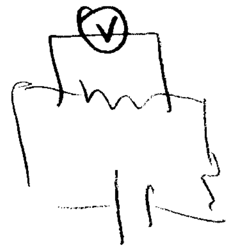
(d) You have two identical light bulbs. You hook up one to a DC power supply, with voltage of 220V, and the other to the outlet, which has a rms voltage of 220V. What do you notice?

i. The bulb with DC is brighter.
ii. The bulb with AC is brighter.
iii. They are the same brightness.
iv. It is impossible to tell.

$P = IV = \frac{V^2}{R} = \frac{V_{rms}^2}{R}$

(e) The ideal ammeter / voltmeter would have
i. infinite resistance / infinite resistance
ii. infinite resistance / zero resistance
iii. zero resistance / zero resistance
iv. zero resistance / infinite resistance





(f) Which is more likely to kill you?
i. 220 V and 1mΩ
ii. 440V and 1Ω
iii. 1000V and 1MΩ
iv. 4000V and 1Ω

$V = IR$ I is what kills

$I = \frac{V}{R}$

i) $I = \frac{220}{1\times10^{-3}} A = 220000 A$

ii) $I = 440 A$

iii) $I = \frac{1000}{1\times10^{6}} = 1\times10^{-3} A$

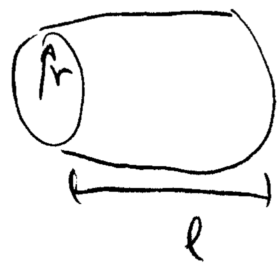
iv) $I = 4000 A$

2. [10 points] Suppose you want to make a uniform cylindrical wire resistor out of ~~1g~~ $1\times10^{-3}\,m^3$ of iron (resistivity ρ= 1×10^{-7}). If the wire is to have a resistance of 5Ω, what will be the length and radius of this wire?

bad geometry -2
hand missed
calculation -1

$R = \frac{\rho l}{A}$ use all the iron, so V (Volume) $1\times10^{-3}\,m^3 = l\pi r^2$

$= \frac{\rho l}{\pi r^2}$



$l = \frac{\pi r^2 R}{\rho}$ but $V = l\pi r^2$

so $\pi r^2 = \frac{V}{l}$

$l^2 = \frac{VR}{\rho}$

$l = \sqrt{\frac{VR}{\rho}} = \sqrt{\frac{(1\times10^{-3})(10)}{1\times10^{-7}}} = \sqrt{10^5} = 10^2\sqrt{10}\,m = 3.16\times10^2\,m = 316.2\,m$

$V = l\pi r^2$ so $l = \frac{V}{\pi r^2}$ set equal to $l = \frac{\pi r^2 R}{\rho}$

$\frac{V}{\pi r^2} = \frac{\pi r^2 R}{\rho}$

$r^4 = \frac{V\rho}{\pi^2 R}$

$r = \left(\frac{10^{-3}\cdot10^{-7}}{\pi^2\,10}\right)^{1/4} m = \left(\frac{10^{-11}}{\pi^2}\right)^{1/4} m = 1.003\times10^{-3}\,m$

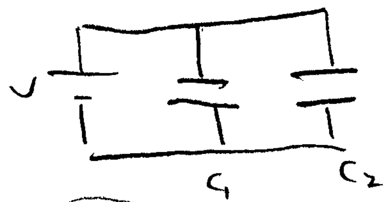
3. [20 points] Two capacitors of capacitance C_1 and C_2 are connected in parallel to a battery which maintains a potential difference V.

4/4 (a) what is the equivalent capacitance C_{eq} of the combination?

4/4 (b) What is the potential difference V_1 and V_2 across each capacitor?

4/4 (c) what is the charge Q_1 and Q_2 on each capacitor?

8/8 (d) repeat parts a, b, and c for the situation where the capacitors are in series.

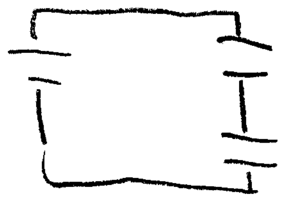


a) $C_{eq} = C_1 + C_2$

b) $V_1 = V_2 = V$

c) $Q = CV$ $\quad Q_1 = C_1 V \quad Q_2 = C_2 V$

d)



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$ 2

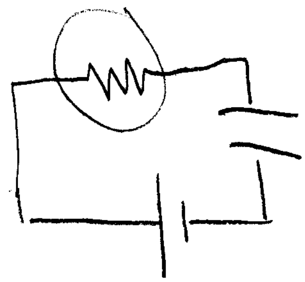
$$Q_1 = Q_2 = C_{eq} V = \frac{V C_1 C_2}{C_1 + C_2}$$ 2

$$C = \frac{Q}{V} \rightarrow V = \frac{Q}{C}$$

$$V_1 = \frac{Q_1}{C_1} = \frac{V C_2}{C_1 + C_2}$$ 2

$$V_2 = \frac{Q_2}{C_2} = \frac{V C_1}{C_1 + C_2}$$

4. [10 pts] A light bulb (with resistance R) is connected in series with an uncharged capacitor (capacitance C) and a battery (voltage V). At time t=0, a switch is closed and the circuit is complete.
   (a) Write an expression (equation) that describes how the power out of the bulb changes with time.
   (b) Is the light getting brighter or dimmer?



a)

$$P = \underset{\text{changes}}{I}\,\underset{\text{changes}}{V} \qquad V = IR$$

$$P = I^2 R$$

$$I = I_0 e^{-t/RC} \qquad I_0 = I(t=0) = \frac{V}{R}$$

/6

$$I = \frac{V}{R} e^{-t/RC}$$

-2 if used V_{cap} instead of V_{bulb}

$$P(t) = \left(\frac{V}{R}\right)^2 R\, e^{-2t/RC}$$

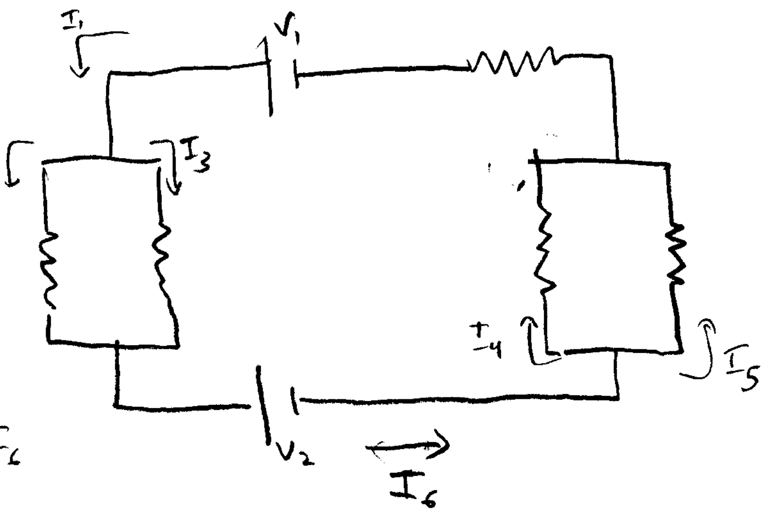
$$= \frac{V^2}{R} e^{-2t/RC}$$

b) The light is getting dimmer. At $t \Rightarrow \infty$, $P(t) = 0$

/4 points for correct analysis of result in a) and for correctness

$$\frac{V_1 - V_2}{K/2} = \frac{V_1 - V_2}{(r - [illegible])}$$

5. [30 points] Consider the circuit shown below. All of the resistors have the same resistance, R. The batteries have voltages V_1 and V_2 as shown.
   - (a) This problem can be greatly simplified. Find a relation between the currents (hint: symmetry and conservation of charge might be useful) so that there are only two distinct currents. Call these currents I_a and I_b and draw the circuit again, with these currents labeled. 10
   - (b) Use Kirchhoff's rules to set up the equations you will need to find these two unknown currents. (note: if you didn't get part (a), just solve using all 6 branches. It will be a mess but at least I can give you points for understanding Kirchhoff's rules) 10
   - (c) Solve the system of equations and find the potential difference across the top resistor. 10



a) Consider one of the smaller loops

$I = \frac{V}{R}$ both V and R are the same,

so the current through each branch must be equal, therefore

$I_2 = I_3$ and $I_4 = I_5$

Also $I_2 = I_3 = \frac{1}{2} I_1$ $\quad I_4 = I_5 = \frac{1}{2} I_6$

Now consider conservation of charge.

For the junction on top of the left loop

$$I_1 = I_2 + I_3$$

for the junction at the bottom of the same loop

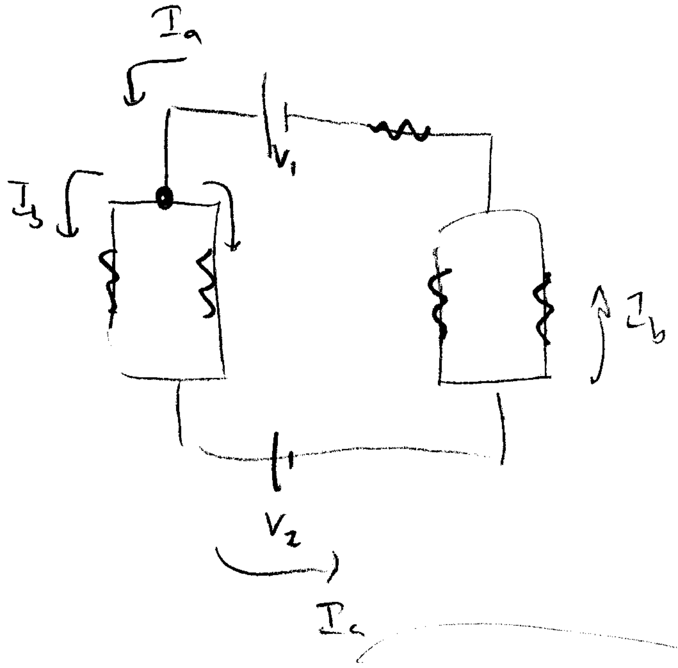
$$I_2 + I_3 = I_6$$

Therefore $I_1 = I_2 + I_3 = I_6$

$$I_1 = I_6$$

So $I_2 = I_3 = I_4 = I_5 = \frac{1}{2} I_1 = \frac{1}{2} I_6 = I_b$

$$I_1 = I_6 = I_a$$



Junction: $I_a = I_b + I_b = 2I_b$ ①

loop: take very outermost loop:

② $V_1 - I_b R - V_2 - I_b R - I_a R = 0$

$V_1 - V_2 - 2I_b R - I_a R = 0$

c) but $I_a = 2I_b$

$V_1 - V_2 - I_a R - I_a R = 0$

$V_1 - V_2 - 2 R I_a = 0$

$$\boxed{I_a = \frac{V_1 - V_2}{2R}}$$

$$I_b = \frac{I_a}{2} = \frac{V_1 - V_2}{4R}$$

$V = IR$

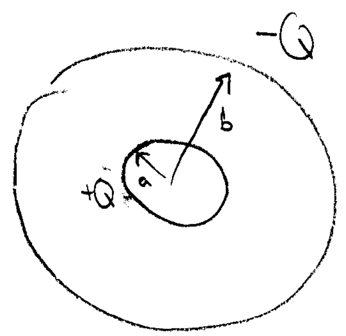
$$V = I_a R = \boxed{\frac{V_1 - V_2}{2}}$$

Bonus Questions: These bonus questions will add on to this test score. If you happen to get over 100, then those points can go toward your other test score.

1. [10 points] A spherical capacitor consists of a spherical conducting shell of radius b and charge -Q that is concentric with a smaller conducting sphere of radius a and charge +Q. Show that its capacitance is $C = \dfrac{ab}{k(b-a)}$ (where k is the electric constant in Coulombs law)

2. [5 points] A cube of wire has 12 resistors: one on each "edge", as shown. All of the resistors have the same resistance $\boldsymbol{R}$. A current $\boldsymbol{I}$ flows through the cube, 'entering' at one corner and going out the far opposite corner. The current through the a given resistor can have one of two values (by symmetry). What are those two values?

1



$$C = \frac{Q}{V}$$

$$V = ? \quad V = |\Delta V| = |V_{ab}| = \left| -\int_a^b \vec{E} \cdot d\vec{l} \right|$$

$$E = \frac{kQ}{r^2}$$

$$V = \int_a^b \frac{kQ}{r^2} \, dr$$

$$= kQ \left[-\frac{1}{r} \right]_a^b$$

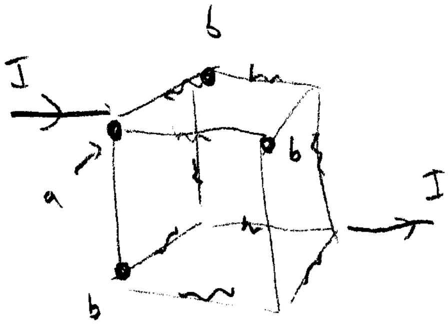
$$= kQ \left[\frac{1}{a} - \frac{1}{b} \right]$$

$$= kQ \left[\frac{b-a}{ab} \right]$$

$$C = \frac{Q}{V} = \frac{\cancel{Q}}{k\cancel{Q}\left[\frac{b-a}{ab}\right]}$$

$$\boxed{C = \frac{ab}{k(b-a)}}$$

2.



I splits into three parts at the first junction a

Each part has equal resistance

so the current through these branches must be $\frac{I}{3}$

At the following junctions it splits into two equal currents,

so the current through those must be $\frac{\frac{I}{3}}{2} = \frac{I}{6}$

In the last branches, equal branches must recombine to be

I , so the current is once again $\frac{I}{3}$