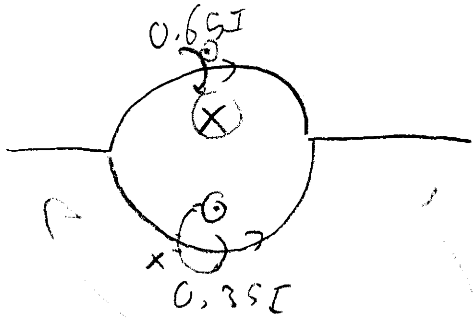
HW10

28-35



$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2}$$

$d\vec{l} \times \hat{r} = 0$ on the outside segments

top and bottom magnitudes subtract, since B-field is in op. directions, by RHR.

$$|\vec{B}| = \frac{\mu_0}{4\pi}\left[I_1 \int_{top} \frac{ds\,(1)}{R^2} + I_2 \int_{bottom} \frac{ds}{R^2} \right]$$

$$= \frac{\mu_0}{4\cancel{\pi}}\left[\frac{I}{R^2} \cdot \frac{\cancel{2}\cancel{\pi} R}{\cancel{2}} + I_2 \frac{\cancel{2}\cancel{\pi} \cancel{R}}{\cancel{2}} \right]$$

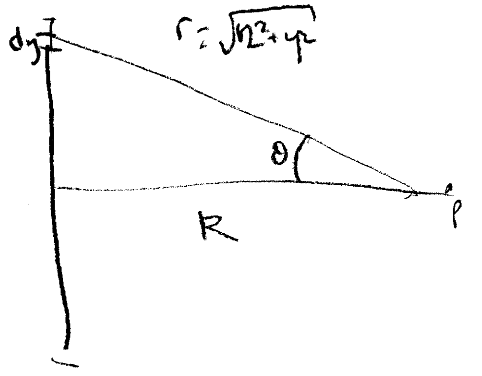
$$= \frac{\mu_0}{4R^2}(I_1 - I_2)$$

$$= \frac{\mu_0}{4R}(0.65I - 0.35I)$$

$$\boxed{\vec{B} = \frac{0.3\,\mu_0 I}{4R} \text{ into the page}}$$

28-40

a)



note:
not infinite

Similar to class example, which we solved two ways

$$B = \frac{\mu_0 I}{4\pi}\int \frac{dy(1)\sin\theta}{r^2} \qquad \sin\theta = \frac{R}{r} = \frac{R}{\sqrt{(R^2+y^2)}}$$

$$= \frac{\mu_0 I}{4\pi}\int_{-d/2}^{d/2} \frac{dy}{(R^2+y^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi R}\left[\frac{y}{(R^2+y^2)^{1/2}}\right]_{-d/2}^{d/2}$$

$$= \frac{\mu_0 I}{4\pi R}\left[\frac{d/2}{(R^2+(d/2)^2)^{1/2}} + \frac{d/2}{(R^2+(d/2)^2)^{1/2}}\right]$$

$$= \frac{\mu_0 I d}{4\pi R(R^2+(d/2)^2)^{1/2}}$$

b)

$$B = \frac{\mu_0 I}{2\pi R} d\,(4R^2+d^2)^{-1/2}$$

$$= \frac{\mu_0 I}{2\pi R}\frac{d}{d}\left(\frac{4R^2}{d^2}+1\right)^{-1/2}$$ do binomial expansion

$\frac{R}{d}$ small as $d \to \infty$

$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \ldots$

$$= \frac{\mu_0 I}{2\pi R}\left(1 + \left(-\frac{1}{2}\right)\frac{4R^2}{d^2} + \ldots\right)$$

to first order:

$$B = \frac{\mu_0 I}{2\pi R}$$ ✓ same as infinite wire