Homework 14

Chapter 30

Problem 12

Induction is $L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi (\frac{d}{2})^2}{l}$

constant: μ_0, l

changing: diameter $d' = 2.5 d$

$N' = ?$ in general $l_{wire} = 2\pi r N = \frac{2\pi d}{2} N \Rightarrow N = \frac{l}{\pi d}$

try to get L in terms of d (changing term) and μ_0, l (constants)

now $L_1 = \frac{\mu_0 N_1^2 \pi (\frac{d_1}{2})^2}{l_1}$

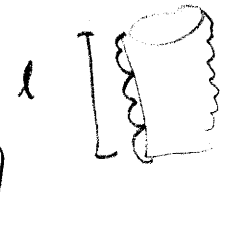
$L_2 = \frac{\mu_0 N_2^2 \pi (\frac{d_2}{2})^2}{l_2}$

$\frac{L_2}{L_1} = \frac{\frac{\mu_0 N_2^2 \pi d_2^2}{4 l_1}}{\frac{\mu_0 N_1^2 \pi d_1^2}{4 l_2}} = \frac{l_2 N_2^2 d_2^2}{l_1 N_1^2 d_1^2} = \frac{l_1}{l_2}\left(\frac{N_2 d_2}{N_1 d_1}\right)^2$

now $l_{wire} = N_1 \pi d_1 = N_2 \pi d_2$

$\frac{N_2}{N_1} = \frac{d_1}{d_2}$

$\frac{L_2}{L_1} = \frac{l_1}{l_2}\left(\frac{d_1}{d_2}\frac{d_2}{d_1}\right)$ what is $l_1 : l_2$?

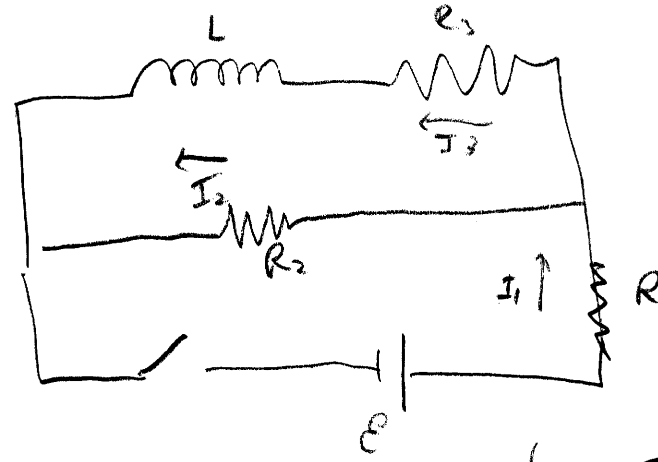


l is diameter of wire × # of turns!

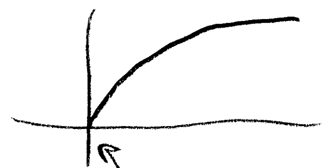
diameter of wire is constant D

$= \frac{D N_1}{D N_2} = \frac{d_2}{d_1} = \boxed{2.5}$

30-26



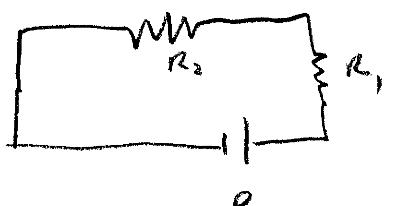
a) Current in RL circuits:



at moment switch is closed, current through L is = 0

$\Rightarrow \boxed{I_3 = 0}$

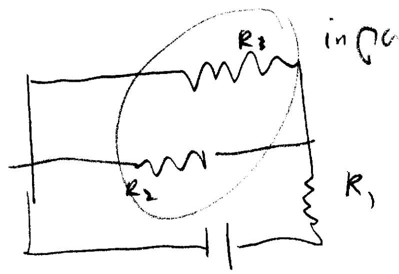
so we essentially have:



← $R_{eq} = R_1 + R_2$

$$I_1 = I_2 = \frac{V}{R_{eq}} = \frac{\mathcal{E}}{R_1 + R_2}$$

b) the voltage drop across the inductor is $-L\frac{dI}{dt}$
After a long time this rate slows to zero, so the effect of L on the voltage is that it is, for all purposes, not even there.



in parallel!

junction: $I_1 = I_2 + I_3$ ①

loops: $\mathcal{E} - I_1R_1 - I_3R_3 = 0$ ②

$I_2R_2 - I_3R_3 = 0$ ③

3 eq, 3 unknowns

③ $I_2R_2 = I_3R_3$

$I_2 = I_3\frac{R_3}{R_2}$ → in ① $I_1 = I_3\left(\frac{R_3}{R_2} + 1\right) = I_3\left(\frac{R_3 + R_2}{R_2}\right)$

set equal

② : $I_1 = \frac{\mathcal{E} - I_3R_3}{R_1} =$

$$\frac{\mathcal{E} - I_3R_3}{R_1} = I_3\left(\frac{R_3 + R_2}{R_2}\right)$$

$$\frac{\mathcal{E}}{R_1} = I_3\left(\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1\right)$$

$$I_3 = \frac{\mathcal{E}}{R_1}\left(\frac{1}{\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1}\right)$$

$$I_1 = I_3\left(\frac{R_3 + R_2}{R_2}\right)$$

$$I_2 = I_1 - I_3$$

30-31 $C = 1350 \times 10^{-12}\,F$

$f = 550 \times 10^3\,Hz$

a) if $C = 1600 \times 10^3\,Hz$, $C = ?$

$$w = \sqrt{\frac{1}{LC}} = 2\pi f$$

assume L the same

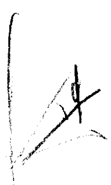
$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$\frac{f_1}{f_2} = \frac{\sqrt{\frac{1}{LC_1}}}{\sqrt{\frac{1}{LC_2}}} = \sqrt{\frac{C_2}{C_1}}$$

$$\frac{f_1^2}{f_2^2} = \frac{C_2}{C_1} \Rightarrow C_2 = C_1\frac{f_1^2}{f_2^2} = (1350 \times 10^{-12}F)\left(\frac{550}{1600}\right) = 0.16\,nF$$

30-57 $L = 25\,mH$ $R = 0.8\,\Omega$ $C = ?$ $f = 360\,Hz$

$V = 45V$



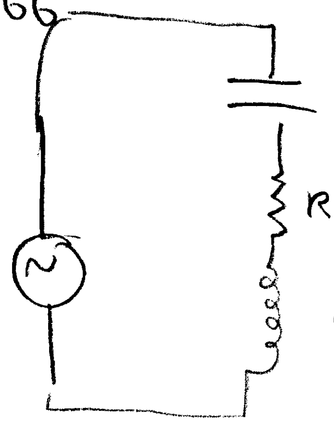
phase is ϕ and $\tan\phi = \frac{X_L - X_C}{R}$ for $\phi = 0$, $X_L = X_C$

$$X_L = \omega L = 2\pi f L \qquad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$C = \frac{1}{4\pi^2 f^2 L} = 7.8\,\mu F$$

30-66



$V = V_0 \sin(2\pi f_0 t + \phi)$

a) resonant frequency $\omega_0 = \sqrt{\frac{1}{LC}}$ occurs when avg current is a max (see 30-9)

$\omega = 2\pi f_0$

$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{(55mH)(1nF)}} = 21\ kHz$

b) $V_0 = 2V$ @ $f = f_0$ $\quad V_{C0} = ?$

$V_{C_0} = I_0 X_c \qquad I_0 = \frac{V_0}{R} \qquad X_c = \frac{1}{\omega C} = \frac{1}{2\pi f_0 C}$

$V_{C0} = \frac{V_0}{R}\frac{1}{2\pi f_0 C} = \frac{(2V)}{(35\Omega)2\pi(21kHz)(1nF)} = 420V$

c) $\frac{V_{C0}}{V_0} = \frac{420V}{2.0V} = 210$