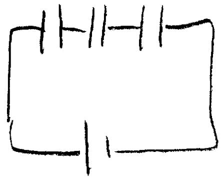
HW 5 - Capacitors

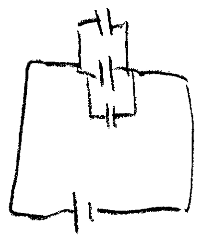
Q6
24-12, 30
23-19

Chapter 24

Q6



or



?

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

I want to use one of these, because I can find C_{equ} + sub it in.

when connected to a battery, V is constant, so I use this

$$U = \frac{1}{2}CV^2$$

$\Rightarrow U \propto C$

for series $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

for $\parallel$ $C_{eq} = C_1 + C_2 + C_3$ ← larger

$\therefore$ more energy if connected in ~~ser~~ parallel.

Problem 23-12

$$U = \frac{1}{2}CV^2 = \frac{1}{2}kC_0(Ed)^2 = \frac{1}{2}kC_0\left(\frac{E_0}{k}d\right)^2$$

$$= \frac{1}{2}\frac{C_0(E_0 d)^2}{k}$$

$$= \frac{1}{k}\left(\frac{1}{2}C_0V_0^2\right) = \frac{1}{k}U_0$$

can't do this because V is kept constant by the battery

$$U = \frac{1}{2}\frac{Q^2}{C} = k\frac{1}{2}\frac{Q^2}{C_0}$$

wrong question

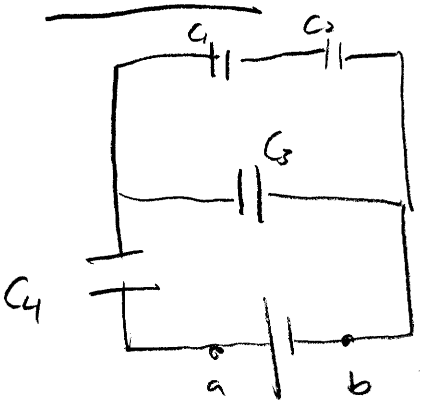
Problem 23-12

The capacitnce of a coaxial caple was found in class:

$$C = \frac{2\pi\epsilon_0 l}{\ln(R_a/R_b)}$$

Cap/length $= \frac{C}{l} = \frac{2\pi\epsilon_0}{\ln(R_a/R_b)}$ $R_a = 5.0$ mm, $R_b = 1.0$ mm plug n chug.

Problem 23-30



$C_1 = C_2 = C_3 = 16\,\mu F$

$C_4 = 28.5\,\mu F$

$Q_2 = 12.4\,\mu C$

$C = \frac{Q}{V}$

find Q_1, Q_3, Q_4, V_1, V_2, V_3, V_4, and V_{ab}

Since C_1 and C_2 are in series, then charge is equal $\boxed{Q_1 = Q_2 = 12.4\,\mu C}$

$V = \frac{Q}{C}$, so $V_1 = \frac{Q_1}{C_1}$ $V_2 = \frac{Q_2}{C_2}$ so $\boxed{V_1 = V_2 = 0.775\,V}$

$V_1 = \frac{12.4\,\mu C}{16\,\mu F} = V_2$ $(Q_1 = Q_2, C_1 = C_2)$

$V_1 + V_2 = \boxed{V_3 = 1.55\,V}$

$Q = CV \Rightarrow Q_3 = C_3 V_3 = (16\,\mu F)(1.55\,V) = \boxed{24.8\,\mu C} = Q_3$

The top combination (C_1, C_2, C_3) is in series with C_4, so the charge on C_4 is equal to the charge on the combination: $Q_4 = Q_3 + Q_1$ (or $Q_4 = Q_3 + Q_2$)

$Q_4 = 24.8\,\mu C + 12.4\,\mu C = \boxed{37.2\,\mu C = Q_4}$

$V_4 = \frac{Q_4}{C_4} = \frac{37.2\,\mu C}{28.5\,\mu F} = \boxed{1.31\,V = V_4}$

$V_{ab} = V_4 + V_3 = (V_4 + V_1 + V_2) = 1.31\,V + 1.55\,V = \boxed{2.86\,V = V_{ab}}$

Problem 23-19

To find the potential, I need to first find the E-field in both regions:

<u>outside ($r>r_0$)</u>

$$EA = \frac{Q}{\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}}$$

inside ($r<r_0$)

$$E\,4\pi r^2 = \frac{\int dq}{\epsilon_0} = \frac{1}{\epsilon_0}\int \rho\, dV = \frac{1}{\epsilon_0}\int_0^r \frac{Q}{\frac{4}{3}\pi r_0^3}4\pi r^2 dr$$

$$= \frac{1}{\epsilon_0} Q \frac{r^3}{r_0^3}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Qr}{r_0^3}\hat{r}}$$

a) $$-V_{r>r_0} = \cancel{V_\infty}^{\,0} - V_r = -\int_r^\infty \frac{Q}{4\pi\epsilon_0 r^2}dr$$

$$V_r = \frac{Q}{4\pi\epsilon_0}\int_r^\infty \frac{1}{r^2}dr$$

$$= \frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_r^\infty$$

$$\boxed{V_{r>r_0} = \frac{Q}{4\pi\epsilon_0 r}}$$

b) $$-V_{r<r_0} = \cancel{V_\infty}^{\,0} - V_{r<r_0} = -\int_r^\infty \vec{E}\cdot d\vec{r} = -\int_r^{r_0}\frac{1}{4\pi\epsilon_0}\frac{Qr}{r_0^3}dr - \int_{r_0}^\infty \frac{Q}{4\pi\epsilon_0 r^2}dr$$

$$V_{r<r_0} = \frac{Q}{4\pi\epsilon_0 r_0^3}\int_r^{r_0} r\,dr + \frac{Q}{4\pi\epsilon_0 r_0}$$

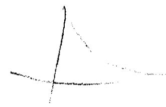
$$= \frac{Q}{4\pi\epsilon_0 r_0^3}\left(\frac{r_0^2}{2} - \frac{r^2}{2}\right) + \frac{Q}{4\pi\epsilon_0 r_0}$$

$$= \frac{Q}{4\pi\epsilon_0 r_0}\left[\frac{1}{2} - \frac{r^2}{2r_0^2} + 1\right]$$

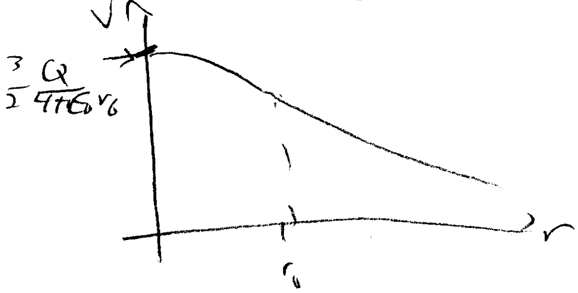
$$\boxed{V = \frac{Q}{4\pi\epsilon_0 r_0}\left[\frac{3}{2} - \frac{r^2}{2r_0^2}\right]}$$

c) To plot V

$r \geq r_0$: $V = \frac{Q}{4\pi\epsilon_0 r} \propto \frac{1}{r}$



$r < r_0$ $V = \frac{Q}{4\pi\epsilon_0 r_0}\left(\frac{3}{2} - \frac{r^2}{2r_0^2}\right) \propto \text{Const} - r^2$



to plot E:

$r \geq r_0$: $E \propto \frac{1}{r^2}$

$r < r_0$ $E \propto r$

$E = 0$ at $r = 0$