Capacitors

How can we store energy?

Capacitors store electric charge.

Charge can't get across easily, so potential difference builds up as excess of q's.

Typical capacitor:

\[ Q = CV \]

So, \( \boxed{Q = CV} \)

Unit? \( C = \frac{Q}{V} \) Coulomb/ Volt = Farad

\( C \) is capacitance.

In general, \( C \) depends on the system: size, position, materials.

We can use what we know to find \( C \) for different geometries.

\[ V = \int (V_b - V_a) = - \int E \cdot dA \]

\[ = -E \cdot d \cos 180^\circ \]

\[ V = Ed \]

What is \( E \)? recall

\[ E = \frac{\varepsilon}{\varepsilon_0} \]

\[ \varepsilon = \frac{\text{charge}}{\text{area}} = \frac{q}{A} \]

\[ E = \frac{q}{A \varepsilon_0} \]

\[ V = \frac{q \cdot d}{A \varepsilon_0} \]
Q = CV, \\
so \( C = \frac{Q}{V} \)

\[ C = \frac{A_0}{(\frac{A_{d1}}{A_{d0}})} = \frac{A_0}{d} \quad \text{for II plate cap.} \]

**Example 1:** You want to design a cap that can store \( \frac{V}{C} \). You have two metal plates of \( A_0 = 1 \text{ cm}^2 \). How far apart should they be?

\[ C = \frac{A_0}{d} \implies d = \frac{A_0}{C} = \frac{(101\text{ cm})^2}{E_0} = \frac{1}{V/C} \]

**Example 2:** Cylindrical Cap.

Find \( C \)

\[ V = V_b - V_a \]

(typically \( \phi \) in chosen to be \( \frac{1}{2} \pi \), so that time value results)

\[ V = -\int_{a}^{b} E \cdot dx \quad E = \frac{1}{2\pi\epsilon_0 LR} \frac{Q}{LR} \]

\[ V = -\int_{a}^{b} \frac{1}{2\pi\epsilon_0 LR} \frac{Q}{LR} dR \]

\[ = -\frac{Q}{2\pi\epsilon_0 LR} \left[ \frac{R_{b}^{1}}{R_{b}} - \ln \frac{R_{b}}{R_{a}} \right] \]

\[ = -\frac{Q}{2\pi\epsilon_0 L} \ln \frac{R_{b}}{R_{a}} = \frac{Q}{2\pi\epsilon_0 L} \ln \frac{R_{a}}{R_{b}} \]
\[ C = \frac{Q}{V}, \text{ so } C = \frac{Q}{\frac{Q}{2\pi \varepsilon_0 R_b} \ln \left( \frac{R_a}{R_b} \right)} \]

\[ C = \frac{2\pi \varepsilon_0 R_b}{\ln \left( \frac{R_a}{R_b} \right)} \]

See ex 24-3, 24-4 for other geometries.

**Capacitors in circuits**

- **Useful**
- **Symbol:** \( \begin{array}{c} \hline \hline \end{array} \) or \( \begin{array}{c} \hline \hline \end{array} \) (butting \( \begin{array}{c} \hline \hline \end{array} \) )

**Series vs Parallel:**

**Series:** in a row

could replace all 3 w/a single cap so that

\[ Q = C_{eq} V \]

\[ V = V_1 + V_2 + V_3 \]

\[ Q = C_1 V_1 \quad Q = C_2 V_2 \quad Q = C_3 V_3 \quad (Q_{eq} \text{ equal}) \]

so \( V_1 = \frac{Q}{C_1} \quad V_2 = \frac{Q}{C_2} \quad ... \)

\[ V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \quad \text{and} \quad V = \frac{Q}{C_{eq}} \]
\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \]

Parallel:

V is the same for all

total charge from battery is

distributed among caps:

\[ Q = Q_1 + Q_2 + Q_3 \]

\[ C_{eq}V = Q_1V + Q_2V + Q_3V \]

\[ C_{eq} = C_1 + C_2 + C_3 \]

Example: Find \( C_{eq} \):

\( C_2, C_3 \text{ in } \parallel \):

\[ C_{eq} = C_2 + C_3 \]

\( C_{23}, C_1 \text{ in } \bigcirc \):

\[ \frac{1}{C_{eq}} = \frac{1}{C_{23}} + \frac{1}{C_1} \]

\[ C_{eq} = \frac{(C_{23}C_1)}{C_3 + C_1} \]

\[ \frac{(C_2 + C_3)C_1}{C_2 + C_3 + C_1} = \frac{2C^2}{3C} = \frac{2}{3}C \]
Capacitors Store Electric Energy

\[ W = \int_Q V \, dq \]

\[ W = \frac{1}{2} CV^2 \]

Example: (24-9) Capacitor (11 plate) is charged with \( Q \) and then disconnected from a battery. The plates are pulled apart so that \( \text{separates} \) goes from \( d \) to \( 2d \). How does the energy change? \( C = \frac{\varepsilon_0 A}{d} \Rightarrow C' = \frac{EA}{2d} = \frac{C}{2} \quad \text{Q constant} \)

\[ U' = \frac{1}{2} \frac{Q^2}{C'} = \frac{1}{2} \frac{Q^2}{(\frac{1}{2}C)} = 2U \]
Dielectrics

- Insulating material used in capacitors instead of air
- Why?
  - Better insulates than air
  - Can make narrower space than plates

Experimentally: \[ C = kC_0 \]
- Capacitance if vacuum fills space
- Capacitance with dielectric "dielectric constant"

Example: \( \parallel \) plate

\[
C_0 = \frac{\varepsilon_0 A}{\delta} \\
C = k\varepsilon_0 A \\
\frac{d}{d}
\]

Define \( k\varepsilon_0 = \varepsilon \)
- \( \varepsilon \) permittivity of material
- \( \varepsilon_0 \) permittivity of free space

How is the electric field affected?

\[
E_0 = \frac{V_0}{d} \quad \text{if the cap is charged and then isolated (} Q \text{ constant, } V \text{ changing)}
\]

\[
V_0 = \frac{Q}{C_0} \quad \rightarrow \quad V = \frac{Q}{C} = \frac{Q}{kC_0} = \frac{V_0}{k}
\]

\[
E = \frac{V}{d} = \frac{V_0}{d} = \frac{V_0}{k} = \left(\frac{E_0}{k}\right)
\]

\( \Rightarrow \) Electric field reduced when dielectric is inserted
molecular description of dielectric:

now put in dielectric:

\[ V = Ed \text{ for } n \text{ plates} \]

\[ C = \frac{Q}{V} = \frac{Q}{Ed} \text{ if } E \downarrow, \text{ } V \uparrow \]

What if dielectric only partly fills?

Can treat like two sep. cap.

\[ V \text{ is the same, so they are in parallel} \]
One more note on energy storage:

\[ U = \frac{1}{2} CV^2 \]

for a flat capacitor, \( C = \frac{\varepsilon_0 A}{d} \), \( V = E \cdot d \)

\[ U = \frac{1}{2} \left( \frac{\varepsilon_0 A}{d} \right) (E_d)^2 \]

\[ = \frac{1}{2} \varepsilon_0 E^2 Ad \]

\( Ad \) is the volume

\[ \frac{U}{\text{volume}} = \frac{1}{2} \varepsilon_0 E^2 \quad \text{"energy density"} \]

With a dielectric,

\[ U = \frac{1}{2} \varepsilon E^2 = \frac{1}{2} \varepsilon E^2 \]