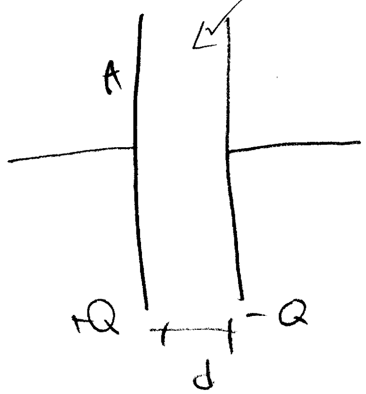
Capacitors

How can we store energy?

capacitors store electric charge

typical capacitor:



charge can't get across easily, so potential diffrnce builds up as excess q grows

So $Q \propto V$

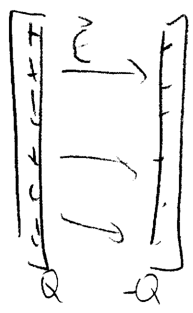
box after — $\boxed{Q = CV}$

C is capacitance

unit? $C = \frac{Q}{V}$ $\frac{\text{Coulomb}}{\text{Volt}}$ = farad

in general, C depends on the system: size, position, materials

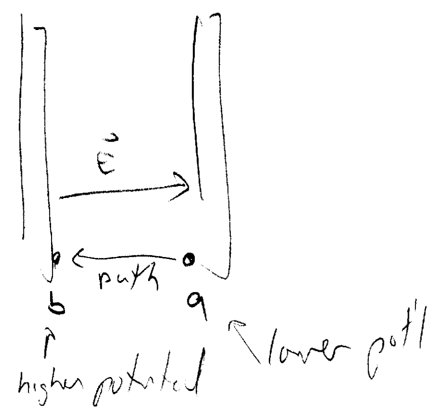
we can use what we know to find C for different geometries:



$V = V_b - V_a = -\int \vec{E} \cdot d\vec{l}$

$= -Ed\cos 180^\circ$

$V = Ed$



what is E? recall

$E = \frac{\sigma}{\epsilon_0}$ $\quad \sigma = \frac{\text{charge}}{\text{area}} = \frac{Q}{A}$

$E = \frac{Q}{A\epsilon_0}$

$V = \frac{Q}{A\epsilon_0} d$

$Q = CV,$

so $C = \frac{Q}{V}$

$= \frac{\cancel{Q}}{\left(\frac{\cancel{Q}d}{A\epsilon_0}\right)} = \frac{A\epsilon_0}{d}$ for // plate cap.

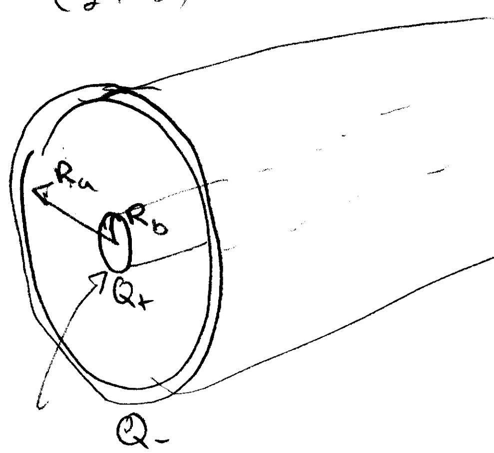
have class solve

Example 1: You want to design a cap. that can store $1 \frac{V}{C}$
You have two metal plates of $A = 1\ cm^2$. How far apart should they be?

$C = \frac{A\epsilon_0}{d} \Rightarrow d = \frac{A\epsilon_0}{C} = \frac{(0.01\ m)^2\ \epsilon_0}{1\ V/C} =$

Example 2 Cylindrical Cap. (24-2)

find C



$V = V_b - V_a$

(typically V_b is chosen to be ⊕ve, so that +ve value results)

$V = -\int_a^b \vec{E} \cdot d\vec{l}$ $\quad E = \frac{1}{2\pi\epsilon_0} \frac{Q}{l R}$ (from before, using Gauss' law)

$V = -\int_a^b \frac{1}{2\pi\epsilon_0} \frac{Q}{l R}\, dR$

$= -\frac{Q}{2\pi\epsilon_0 l} \int_a^b \frac{dR}{R}$

$= \frac{-Q}{2\pi\epsilon_0 l} \ln \frac{R_b}{R_a} = \frac{Q}{2\pi\epsilon_0 l} \ln \frac{R_a}{R_b}$

$C = \frac{Q}{V}$, so $C = \dfrac{\cancel{Q}}{\frac{\cancel{Q}}{2\pi\epsilon_0 l} \ln \frac{R_a}{R_b}}$

$$\boxed{C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{R_a}{R_b}\right)}}$$

see ex 24-3, 24-4 for other geometries

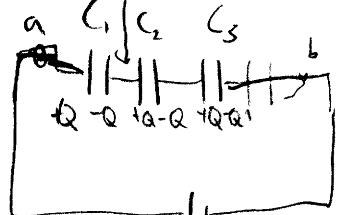
Capacitors in circuits

- Useful
- Symbol: ⊣⊢ or ⊣(

(battery ⊣|⊢)
       + −

Series vs parallel:

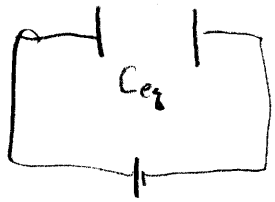
neutral regions between, so charge moves + Q is same for each



Series: in a row

could replace all 3 w/ a single cap so that

$Q = C_{eq} V$



$V = V_1 + V_2 + V_3$

$Q = C_1 V_1 \quad Q = C_2 V_2 \quad Q = C_3 V_3$ (Qs equal)

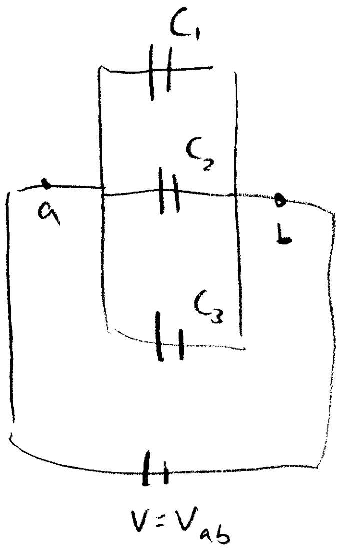
so $V_1 = \frac{Q}{C_1} \quad V_2 = \frac{Q}{C_2} \ldots$

$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$ and $V = \frac{Q}{C_{eq}}$

so $\frac{\cancel{Q}}{C_{eq}} = \cancel{Q}\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}$$

Parallel:



V is the same for all

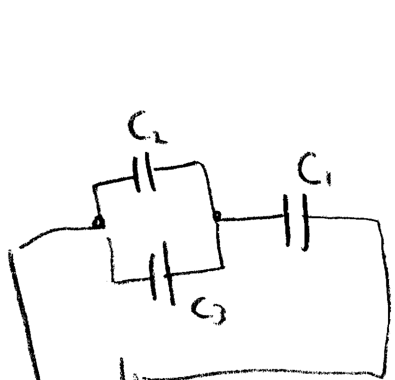
total charge 'from' battery is distributed among cap.:

$Q = Q_1 + Q_2 + Q_3$

$C_{eq}\cancel{V} = C_1\cancel{V} + C_2\cancel{V} + C_3\cancel{V}$

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

example find C_{eq}:



C_2, C_3 in $\parallel$:

$C_{23} = C_2 + C_3$

C_{23}, C_1 in series:

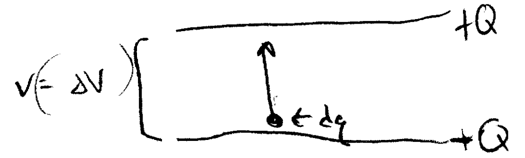
$\frac{1}{C_{eq}} = \frac{1}{C_{23}} + \frac{1}{C_1}$

let $C = C_1 = C_2 = C_3$

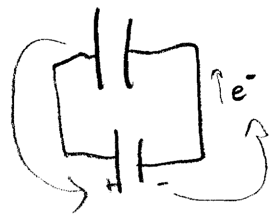
$C_{eq} = \frac{(C_{23})(C_1)}{C_{23}+C_1} = \frac{(C_2+C_3)C_1}{C_2+C_3+C_1} = \frac{2C^2}{3C} = \frac{2}{3}C$

Capacitors Store Electric Energy

Work done to charge = energy stored



Charging cap = moving charge from one plate to the other



$dW = V\,dq$

$W = \int_0^Q V\,dq$ but $V = \frac{q}{C}$

$= \int_0^Q \frac{q}{C}\,dq$

$= \frac{1}{C}\int_0^Q q\,dq$

$\boxed{W = \frac{1}{C}\frac{1}{2}Q^2} = U$

Can rewrite using $Q = CV$

$\boxed{U = \frac{1}{2}\frac{Q^2}{C}} = \frac{1}{2}\frac{(CV)^2}{C} = \boxed{\frac{1}{2}CV^2}$

$= \frac{1}{2}\frac{Q^2}{Q/V} = \boxed{\frac{1}{2}QV}$

3 ways of getting the same thing

Example: (24-9) Capacitor (∥ plate) is charged with Q and then disconnected from a battery. The plates are pulled apart so that sep. goes from d to 2d. How does the energy stored change?

$C = \frac{\epsilon_0 A}{d} \Rightarrow C' = \frac{\epsilon A}{2d} = \frac{C}{2}$ Q constant

use $U' = \frac{1}{2}\frac{Q^2}{C'} = \frac{1}{2}\frac{Q^2}{(C/2)} = 2U$

Dielectrics

↳ insulating material used in capacitors instead of air

<u>why?</u> - better insulator than air

- can make narrower space btwn plates

experimentally: $C = K C_0$

capacitance with dielectric

"dielectric constant"

capacitance if vaccum fills space

example: // plate

$$C_0 = \frac{\epsilon_0 A}{d}$$

$$C = \frac{k \epsilon_0 A}{d}$$

define $k\epsilon_0 = \epsilon$ permitivity of material

permitivity of free space

How is the <u>electric field</u> affected?

$$E_0 = \frac{V_0}{d}$$

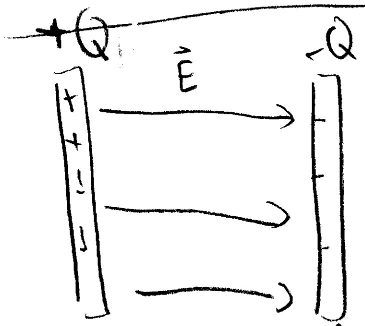
if the cap. is charged and then isolated (Q constant, V changing)

$$V_0 = \frac{Q}{C_0} \longrightarrow V' = \frac{Q}{C} = \frac{Q}{kC_0} = \frac{V_0}{k}$$

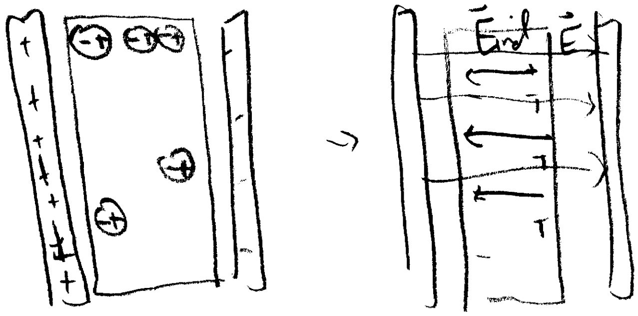
$$E = \frac{V}{d} = \frac{1}{d}\frac{V_0}{k} = \frac{V_0}{d}\frac{1}{k} = \frac{E_0}{k}$$

⇒ Electric field reduced when dielectric is inserted

molecular description of dielectrics:



now put in dielectric:

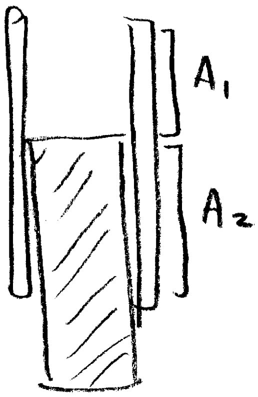


so E is reduced

$V = Ed$ for $\parallel$ plates

$C = \frac{Q}{V} = \frac{Q}{Ed}$ if $E \downarrow$, $V \uparrow$

What if dielectric only partly fills?



Can treat like two sep. cap.

V is the same, so they are in parallel

One more note on energy storage:

$U = \frac{1}{2} C V^2$

for a || plate cap, $C = \frac{\epsilon_0 A}{d}$ $V = E d$

$U = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(E d)^2$

$= \frac{1}{2} \epsilon_0 E^2 \underbrace{Ad}$

Ad is the volume

$u = \frac{U}{\text{volume}} = \frac{1}{2} \epsilon_0 E^2$ "energy density"

With a dielectric,

$u = \frac{1}{2} K \epsilon_0 E^2 = \frac{1}{2} \epsilon E^2$