DC or AC?

More on batteries: internal resistance:

\[ \varepsilon - IR \] (Ohm's law)

so \( V_{ab} = \varepsilon - IR \)

\( V_{ab} \): voltage difference between points a and b

emf: units of volts (it is a potential difference)

Example (26-1): A 65.0 \( \Omega \) resistor.

\( E = 12.0 \text{V} \)

- a) Current in the circuit
- b) Terminal voltage
- c) Power dissipated in \( R \) and \( R \)
\[ V = IR \]
\[ I = \frac{V}{R} \]
\[ V = V_a = E - IR \]
\[ I = \frac{E - IR}{R} \]
\[ IR = E - IR \]
\[ I(R + r) = E \]
\[ I = \frac{E}{R + r} = 0.183 \, \text{A} \]

b) \[ V_a = E - IR = 11.9 \, \text{V} \]

c) \[ P_R = I^2 R = 0.18 \, \text{W} \]
\[ P_r = I^2 r = 0.02 \, \text{W} \]

Resistors in series + parallel:

Series:

\[ V = V_1 + V_2 + V_3 = I R_1 + I R_2 + I R_3 = I (R_1 + R_2 + R_3) \]

\[ R_{eq} = R_1 + R_2 + R_3 \]

\[ I \] the same - otherwise current would "pile up"
In Parallel:

Because charge is conserved, \( I = I_1 + I_2 + I_3 \) but \( I < \frac{V}{R} \)

\[
\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}
\]

\[
\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
\]

One final tool needed to analyze circuits:
- Kirchhoff's Rules

\( \sum \) application of charge & energy conservation

Junction rule: at any junction, the sum of all currents entering a junction must equal the sum of all currents leaving it.

\( I_{in} = I_{out} \)

Loop rule: the sum of changes in potential energy around any closed loop must be zero.

"What goes up must come down"

Practice:
discuss:

which is brightest? 2nd brightest?

what if I disconnect S?

given

\[ R_3 = 4\, \text{k}\Omega \quad R_1 = 5\, \text{k}\Omega \quad R_2 = 7\, \text{k}\Omega \]

\[ V = 12.0\, \text{V} \]

\[ I = \frac{V}{R_3 + R_4 + R_5} = \frac{1}{R_3 + R_4 + R_5} \]

\[ I = \frac{1}{V} \frac{R_3 + R_4 + R_5}{R_3 + R_4 + R_5} = \frac{1}{R_3 + R_4 + R_5} 

\[ V_3 = \frac{I}{R_3 + R_4 + R_5} R_3 = 7.0\, \text{V} \]

\[ V_1 = 12.0\, \text{V} - 7.0\, \text{V} = 5.0\, \text{V} \]

\[ I = \frac{V}{R_1} = \frac{5.0\, \text{V}}{500\, \text{k}\Omega} = 1.0 \times 10^{-2}\, \text{A} = 10\, \text{mA} \]
Strategy for complex circuits:

1. Label the currents
2. Identify unknowns (4 more # of (eqns))
3. Apply junction rule
4. Apply loop rule
5. Do algebra

Practice:

```
8V  2A
\downarrow  \uparrow
120Ω  25Ω
\downarrow  \uparrow
1V  110Ω
```

Find the current through the middle resistor:

```
I_1 = \frac{8V}{120\Omega} = 0.0667 A
I_2 = \frac{2A}{25\Omega} = 0.08 A
I_3 = \frac{1V}{110\Omega} = 0.0091 A
```

Junction rule:

```
I_2 = I_1 - I_3
```

Loops:

1. Left loop:
   \[ 58V - I_1 R_1 - I_2 R_2 - I_3 R_3 = 0 \]
2. Right loop:
   \[ 3V - I_2 R_4 + I_3 R_2 - I_2 R_5 = 0 \]

From the loops:

\[ I_2 = \frac{58V - I_3 (R_1 + R_2 + R_3)}{(R_1 + R_2)} \]
\[ I_2 = \frac{3V + I_3 R_2}{R_4 + R_5} \]

\[ \frac{56V - I_3 (R_1 + R_2 + R_3)}{(R_1 + R_2)} = \frac{3V + I_2 R_3}{(R_4 + R_5)} \]

\[ (56V - I_3 (R_1 + R_2 + R_3))(R_4 + R_5) = (3V + I_2 R_3) (R_1 + R_2) \]

\[ I_3 = \ldots \]

Q: How does a battery charger work? (discuss)

\[ \text{RC Circuits} \]

Resistor, capacitor

Voltage + charge on cap. change w/ time

Consider a RC circuit:

\[ \begin{align*}
\text{Voltage} & = V \\
\text{Charge} & = Q \\
\text{Current} & = I \\
\text{Resistance} & = R \\
\text{Capacitance} & = C
\end{align*} \]

\[ \begin{align*}
V(t) & = IR(t) + \frac{Q(t)}{C} \\
\text{Initial condition} & = V(0) = V_0 \\
\text{Boundary condition} & = Q(0) = 0
\end{align*} \]

\[ \frac{d}{dt} \left[ \frac{Q}{C} \right] = \frac{1}{R} \left[ \frac{V(t)}{C} \right] \]

\[ \frac{dQ}{dt} = \frac{V(t)}{RC} \]

\[ Q(t) = \frac{V(t)}{RC} t + Q_0 \]

\[ V(t) = V_0 + \frac{Q_0}{C} \]

\[ \text{t = c time} \]

\[ \text{c = c constant} \]

\[ \text{Q = Q changing} \]

\[ \frac{dQ}{dt} = \frac{V}{RC} \]

\[ \text{rearrange} \]

\[ \text{used in:} \]

- Wires/links
- Traffic lights
- Camera flash
- Pacemakers
- etc.
\[ EC = \frac{dQ}{dt} \]

\[ EC - Q = \frac{dQ}{dt} \]

\[ \frac{dQ}{RC} = \frac{dQ}{EC - Q} \]

Integrating:

\[ \frac{1}{RC} \int_0^t dt = \int_0^Q \frac{dQ}{EC - Q} \]

\[ \frac{t}{RC} = \ln \left( \frac{EC - Q}{EC} \right) \]

\[ = -\ln \left( \frac{EC - Q}{EC} \right) - \left[ -\ln EC \right] \]

\[ -\frac{t}{RC} = \ln \left( 1 - \frac{EC}{Q} \right) \]

Raise both sides:

\[ e^{-\frac{t}{RC}} = 1 - \frac{Q}{EC} \]

\[ Q = CE \left( 1 - e^{-\frac{t}{RC}} \right) \]

Capacity charging up

\[ V_C = \frac{Q}{C} = E \left( 1 - e^{-\frac{t}{RC}} \right) \]

Define \( T = RC \) time to reach \( 1 - e^{-1} = 0.63 \) or 63% of its full charge and voltage

Units: \( RC \) = \( \text{SE} = \left( \frac{V}{A} \right) \left( \frac{Q}{V} \right) = \frac{C}{\text{s}} = s \)

Now consider discharge:
no bulky

rate is negative
because

$\frac{dQ}{dt} \cdot R = \frac{Q}{C}$

$\int \frac{dQ}{Q} = - \int \frac{dt}{RC}$

$\ln \left( \frac{Q}{Q_0} \right) = - \frac{t}{RC}$

$\frac{Q}{Q_0} = e^{-t/RC}$

$Q = Q_0 \cdot e^{-t/RC}$

$V = V_0 \cdot e^{-t/RC}$

$\int = - \frac{dQ}{dt} = \frac{Q_0}{RC} \cdot e^{-t/RC} = I_0 \cdot e^{-t/RC}$