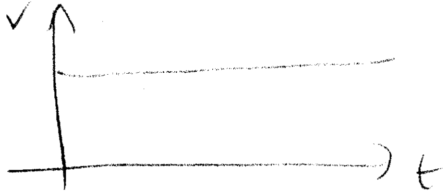
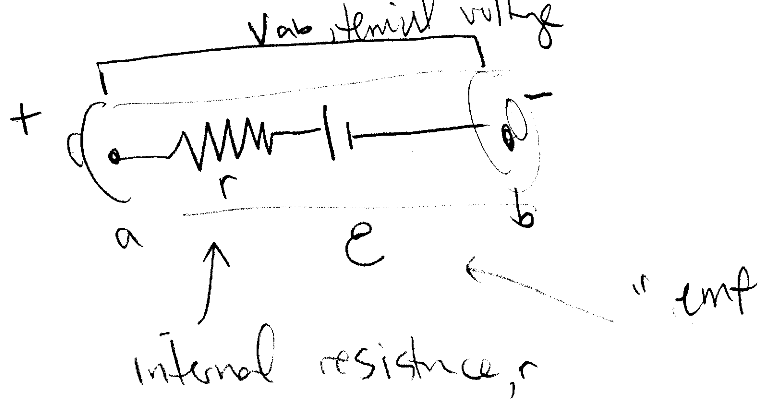
Ch 26 - DC circuits

recall



DC or AC ?

More on batteries: internal resistance:



- batteries maintain a potential difference
- batteries do not nec. put out a fixed current

resistance causes a voltage drop:

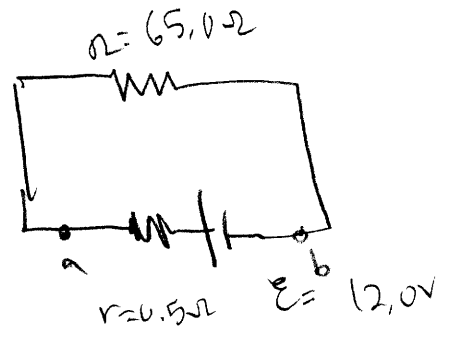
$V = Ir$ (Ohms law)

so $V_{ab} = \mathcal{E} - Ir$

↑ what the battery is rated at

emf: units of volts (it's a pot'l diff)
stands for electromotive force   discuss

Example (26-1) A 65.0 Ω resistor...



find a) current in the circuit
b) terminal voltage
c) power dissipated in R and r

$V = IR$

$I = \frac{V}{R}$ $\quad V = V_{ab} = \varepsilon - Ir$

$I = \frac{\varepsilon - Ir}{R}$

$IR = \varepsilon - Ir$

$I(R+r) = \varepsilon$

$I = \frac{\varepsilon}{R+r} = 0.183\text{ A}$

($IR = \varepsilon - Ir$; $I(R+r) = \varepsilon$; $I = \frac{\varepsilon}{R+r}$)

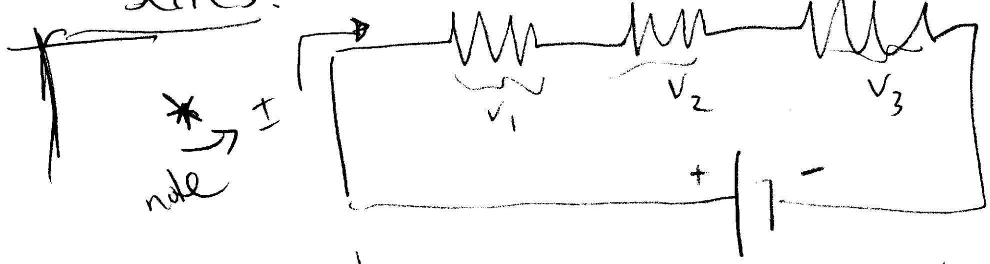
b) $V_{ab} = \varepsilon - Ir = 11.9\text{ V}$

c) $P_R = I^2 R = 2.18\text{ W}$

$P_r = I^2 r = 0.02\text{ W}$

Resistors in series + parallel:

Series:



resistance causes a drop in potential $V = IR$

$V = V_1 + V_2 + V_3$

$= IR_1 + IR_2 + IR_3$

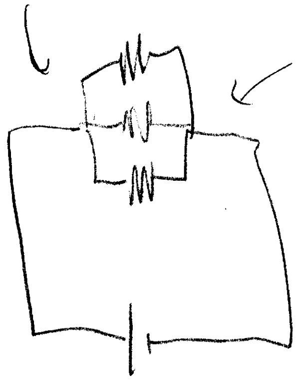
$= I(R_1 + R_2 + R_3)$ ← Req

$= I R_{eq}$

$R_{eq} = R_1 + R_2 + R_3$

→ * I the same - otherwise current would "pile up"
think water pipe

in Parallel:



because charge is conserved, $I = I_1 + I_2 + I_3$ but $I = \frac{V}{R}$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$ parallel resistors

one final tool needed to analyze circuits:

Kirchhoff's Rules

↳ application of charge + energy conservation

Junction rule: at any pt, the sum of all currents entering a junction must equal the sum of all currents leaving

"$I_{in} = I_{out}$"

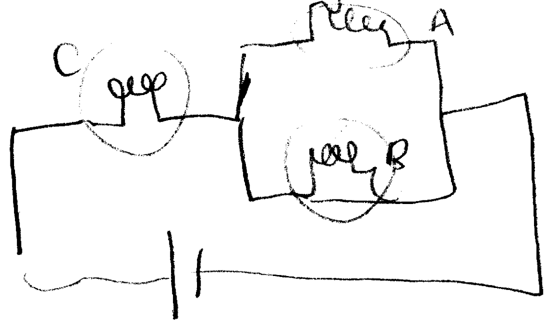
loop rule: the sum of changes in potential energy around any closed loop must be zero

"what goes up must come down"

practice:

→

discuss:



which is brightest? 2nd brightest?

what if I disconnect S?

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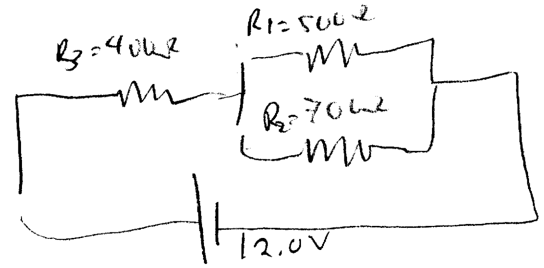
given



find current through the 500 Ω resistor

* give them time to solve!

solution:

$\overset{?}{V_1} = I_1 R_1$

$V_1 = 12.0\text{V} - \overset{?}{V_3} \quad (-V_{ab} + V_3 - V_1 = 0)$

$V_3 = I_3 R_3 \quad I = ?$

$V = I R_{eq} \quad R_{eq} = R_3 + R_{12} \quad \frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$

$= R_3 + \frac{R_1 R_2}{R_1 + R_2}$

$I = \frac{R_{eq}}{V} = \frac{1}{12.0\text{V}}\left(R_3 + \frac{R_1 R_2}{R_1 + R_2}\right)$

$V_3 = \frac{1}{12.0\text{V}}\left(R_3 + \frac{R_1 R_2}{R_1 + R_2}\right) R_3 = 7.0\text{V}$

$V_1 = 12.0\text{V} - 7.0\text{V} = 5.0\text{V}$

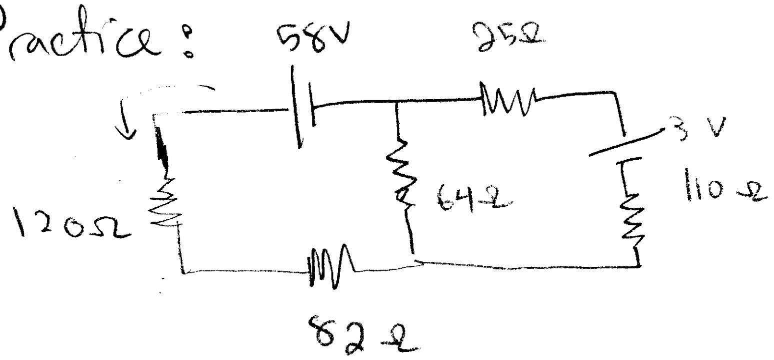
$I_1 = \frac{V_1}{R_1} = \frac{5.0\text{V}}{500\,\Omega} = 1.0 \times 10^{-2}\text{A} = 10\text{mA}$

Strategy for complex circuits:

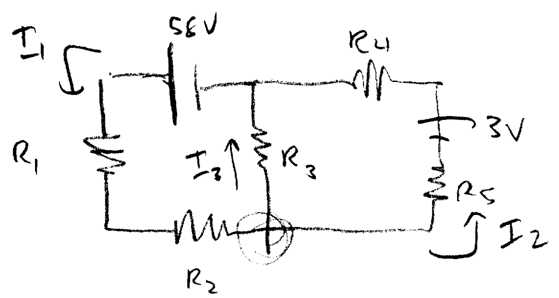
1. label the currents
2. Identify unknowns (+ then # of eqns)
3. Apply junctn rule
4. Apply loop rule
5 Do algebra

} see ex 26-9

Practice:



find the current through the middle resistor



find I_1, I_2, I_3

(unknowns)

junctn rule: ~~$I_2 = I_1 - I_3$~~

$I_1 = I_2 + I_3$

} 3 eqn, 3 unkns

① loop rule: left loop: $58V - I_1R_1 - I_1R_2 - I_3R_3 = 0$

② right loop: $3V - I_2R_4 + I_3R_3 - I_2R_5 = 0$

① $58V - (I_2 + I_3)(R_1 + R_2) - I_3R_3 = 0 \rightarrow 58V - I_2(R_1 + R_2) - I_3(R_1 + R_2 + R_3) = 0$

② $3V - I_2(R_4 + R_5) + I_3R_3 = 0$

from ① $I_2 = \dfrac{58V - I_3(R_1 + R_2 + R_3)}{(R_1 + R_2)}$

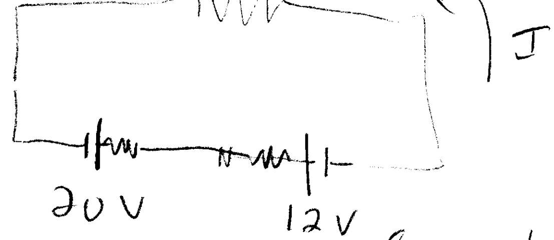
for ② $I_2 = \frac{3V + I_3 R_3}{R_4 + R_5}$

$$\frac{58V - I_3(R_1 + R_2 + R_3)}{(R_1 + R_2)} = \frac{3V + I_3 R_3}{(R_4 + R_5)}$$

$$[58V - I_3(R_1 + R_2 + R_3)](R_4 + R_5) = (3V + I_3 R_3)(R_1 + R_2)$$

$I_3 = \ldots$

Q How does a battery charger work? (discuss)



← charged ✓

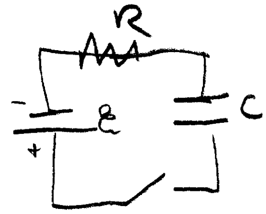
RC Circuits

↑ resistor ↑ capacitor

used in - windshield wipers
- traffic lights
camera flashes
pacemakers
etc

Voltage + charge on cap. change w/ time

Consider a RC circuit:



close switch at $t = 0$

Loop rule $\mathcal{E} - IR - \frac{Q}{C} = 0$

$\mathcal{E} = IR + \frac{Q}{C}$ R, C constants; I, Q changing

$\mathcal{E} = \frac{dQ}{dt} R + \frac{Q}{C}$

rearrange

$$\mathcal{E} C = \frac{dQ}{dt} RC + Q$$

$$\mathcal{E} C - Q = \frac{dQ}{dt} RC$$

$$\frac{dt}{RC} = \frac{dQ}{\mathcal{E} C - Q}$$

integrate

$$\frac{1}{RC}\int_0^t dt = \int_0^Q \frac{dQ}{\mathcal{E}C - Q}$$

$$\frac{t}{RC} = \ln(\mathcal{E}C - Q)(-1)\Big|_0^Q$$

$$= -\ln(\mathcal{E}C - Q) - (-\ln \mathcal{E}C)$$

$$-\frac{t}{RC} = \ln\left(\frac{\mathcal{E}C - Q}{\mathcal{E}C}\right)$$

$$-\frac{t}{RC} = \ln\left(1 - \frac{Q}{C\mathcal{E}}\right)$$

raise both sides

$$e^{-t/RC} = 1 - \frac{Q}{C\mathcal{E}}$$

$$Q = C\mathcal{E}(1 - e^{-t/RC})$$

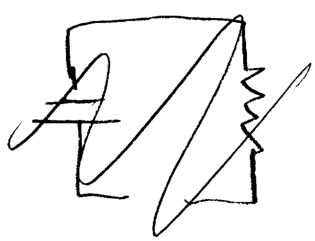
$$V_C = \frac{Q}{C} = \mathcal{E}(1 - e^{-t/RC})$$

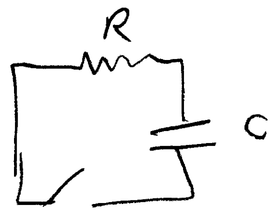
capacitor charging up

define $\tau = RC$ time to reach $1 - e^{-1} = 0.63$ or 63% of its full charge and voltage

units: RC: $\Omega \cdot F = \left(\frac{V}{A}\right)\left(\frac{Q}{V}\right) = \frac{C}{C/s} = s$

now consider discharge:

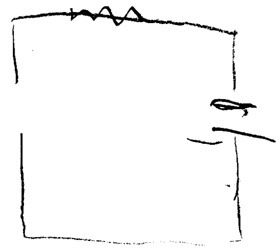




no battery

$$Q/C - IR = 0$$

$$IR - \frac{Q}{C} = 0$$



$$-IR = Q/C$$

rate is negative because Q → $$-\frac{dQ}{dt} R = Q/C$$

★ loss of charge in cap = neg of current through resistor

$$I = -\frac{dQ}{dt}$$

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_0^t \frac{dt}{RC}$$

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$$

$$\frac{Q}{Q_0} = e^{-t/RC}$$

$$Q = Q_0 e^{-t/RC}$$

$$V_C = V_0 e^{-t/RC}$$

$$I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$